Demo 14

1. Parts (e) and (i) of Problem 11.30 on pg. 874 of OWN.

2. Problem 11.43 on pg. 885 of OWN.

3. Problem 11.60 on pp. 907 -908 of OWN.

4. Let $x[n]$ be a discrete time signal with z-transform $X(z)$. Given an integer $M \geq 1$, let $(\downarrow M)$ denote the operation of decimation or downsampling by a factor of M, i.e. we write $y[n] = (\downarrow M)x[n]$ if
$$y[n] = x[nM] \quad \text{for all } n \ .$$
Let $(\uparrow M)$ denote the operation of expansion by M (sometimes this is also called upsampling although in this course, following the textbook, we have reserved the word "upsampling" for expansion followed by low pass filtering). Thus, we write $z[n] = (\uparrow M)x[n]$ if
$$z[n] = \begin{cases} x[n/M] & \text{if } n \text{ is an integer multiple of } M \\ 0 & \text{otherwise} \end{cases} \ .$$

   (a) Let $y[n] = (\downarrow M)x[n]$. Verify that
$$Y(z) = \frac{1}{M}\sum_{k=0}^{M-1} X(z^{\frac{1}{M}} e^{j2\pi\frac{k}{M}}) \ .$$

   (b) Let $z[n] = (\uparrow M)x[n]$. Verify that
$$Z(z) = X(z^M) \ .$$

   (c) Let $u[n] = (\uparrow M)(\downarrow M)x[n]$. Verify that
$$U(z) = \frac{1}{M}\sum_{k=0}^{M-1} X(ze^{j2\pi\frac{k}{M}}) \ .$$

   (d) Suppose we first downsample $x[n]$ by a factor of M and then pass the result through a discrete time filter with transfer function $G(z)$. Verify that the output of the filter is the same as the signal we would get if we first pass $x[n]$ through the discrete time filter with transfer function $G(z^M)$ and then downsample the output of this filter by a factor of M. We may write this as
$$G(z)(\downarrow M) = (\downarrow M)G(z^M) \ .$$
   In the study of multirate signal processing, this is called the *first Noble identity.*

(e) Suppose we first pass $x[n]$ through a discrete time filter with transfer function $G(z)$ and then expand the output of the filter by a factor of M. Verify that the signal we get this way is the same as the one we would get if we were to first expand the signal $x[n]$ by a factor of M and then pass this through the discrete time filter with transfer function $G(z^M)$. We may write this as

$$(\uparrow M)G(z) = G(z^M)(\uparrow M) \ .$$

In the study of multirate signal processing, this is called the *second Noble identity*.