1. (a) \( y(t) = 2y_0(t) \)

(b) \( y(t) = y_0(t) - y_0(t-2) \)

(c) \( y(t) = y_0(t-1) \)

(d) We have : \( x(t) = x_0(-t) \), \( h(t) = h_0(t) \) so that:

\[
y(t) = \int_{\tau=-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{\tau=-\infty}^{\infty} x_0(-\tau)h_0(t-\tau)d\tau
\]

Not enough information to determine \( y(t) \). Consider two cases for \( x_0(t) \) and \( h_0(t) \) below:

Both of the above cases result in the same \( y_0(t) \) as illustrated in figure P2.47. However, if we let \( x(t) = x_0(-t) \) then the \( y(t) \) will be different for the two cases as illustrated below:
(e) \( x(t) = x_0(-t), h(t) = h_0(-t) \)

\[
y(t) = \int_{\tau = -\infty}^{\infty} x(\tau) h(t - \tau) d\tau = \int_{\tau = -\infty}^{\infty} x_0(-\tau) h_0(\tau - t) d\tau = \int_{\tau = -\infty}^{\infty} x_0(\tau) h_0(-t - \tau) d\tau
\]

\( = y_0(-t) \)

(f) We have: \( x(t) = x(t), h(t) = h(t) \)

\[
x'(t) = \lim_{\Delta \to 0} \frac{x_0(t + \Delta) - x_0(t)}{\Delta}, \quad h'(t) = \lim_{\gamma \to 0} \frac{h_0(t + \gamma) - h_0(t)}{\gamma}
\]

\[
x(t) \ast h(t) = \lim_{\Delta \to 0} \lim_{\gamma \to 0} \frac{y_0(t + \Delta + \gamma) - y_0(t + \gamma) - y_0(t + \Delta) + y_0(t)}{\Delta \gamma}
\]

\( = y'(t) = 0.5[\delta(t) - \delta(t-2)] \)

2. (a)

For System A followed by system B:

\( z[n] = nw[n] = n(1/2)^n u[n] \)

For System B followed by System A:

\( y[n] = z[n] \ast h[n], \quad z[n] = n\delta[n] = 0, \quad y[n] = 0 \)

(b) \( z[n] = w[n] + 2 \)

For System A followed by System B:

\( z[n] = (1/2)^n u[n] + 2 \)

For System B followed by System A:

\( z[n] = \delta[n] + 2 \)

\( y[n] = (1/2)^n u[n] + 2 \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k = (1/2)^n u[n] + 4 \)

3. (a) S1: \( y_1[n] = 0.5x_1[n] - 2.5x_1[n-4] \)

S2: \( y_2[n] = 0.5y_2[n-1] - 0.5y_2[n-3] + x_2[n] \)
\[ y_2[n] = 0.5y_2[n-1] - 0.5y_2[n-3] + 0.5x_1[n] - 2.5x_1[n-4] \]

\[ 2y_2[n] - y_2[n-1] + y_2[n-3] = x_1[n] - 5x_1[n-4] \]

(b) S1:

(c) S2:
4. \( \int_{-\infty}^{\infty} g(\tau)u_1(\tau) d\tau = -g'(0) \)

Define \( g(\tau) = x(t-\tau) \) for some fixed \( t \).

\[ \Rightarrow \int_{-\infty}^{\infty} x(t-\tau)u_1(\tau) d\tau = -x'(t) \]

(b) Show that \( f(t)u_1(t) = f(0)u_1(t) - f'(0)\delta(t) \)

For any \( g(t) \) that is continously differentiable at \( t=0 \):

\[ \int_{-\infty}^{\infty} g(\tau)f(\tau)u_1(\tau) d\tau = -\frac{d}{dt}[g(t)f(t)]|_{t=0} = -g'(0)f(0) - g(0)f'(0) \]

\[ \int_{-\infty}^{\infty} g(\tau)f(0)u_1(\tau) d\tau - \int_{-\infty}^{\infty} g(\tau)f'(0)\delta(\tau) d\tau = \]

\[ -f(0)g'(0) - g(0)f'(0) \]
(c) \( \int_{-\infty}^{\infty} x(\tau) u_2(\tau) d\tau = x^*(0) \)

Let \( g(t) \) be an arbitrary function that is twice differentiable at 0.

\[
\int_{-\infty}^{\infty} g(\tau) f(\tau) u_2(\tau) d\tau = \frac{d^2}{dt^2} [g(t) f(t)] \bigg|_{t=0} = \frac{d}{dt} [g'(t) f(t) + g(t) f'(t)] \bigg|_{t=0} \\
= g''(0) f(0) + g'(0) f'(0) + g'(0) f'(0) + g(0) f''(0) \\
= g''(0) f(0) + 2 g'(0) f'(0) + g(0) f''(0)
\]

This would also result from

\[
f(0) u_2(t) - 2 f'(0) u_1(t) + f''(0) \delta(t)
\]