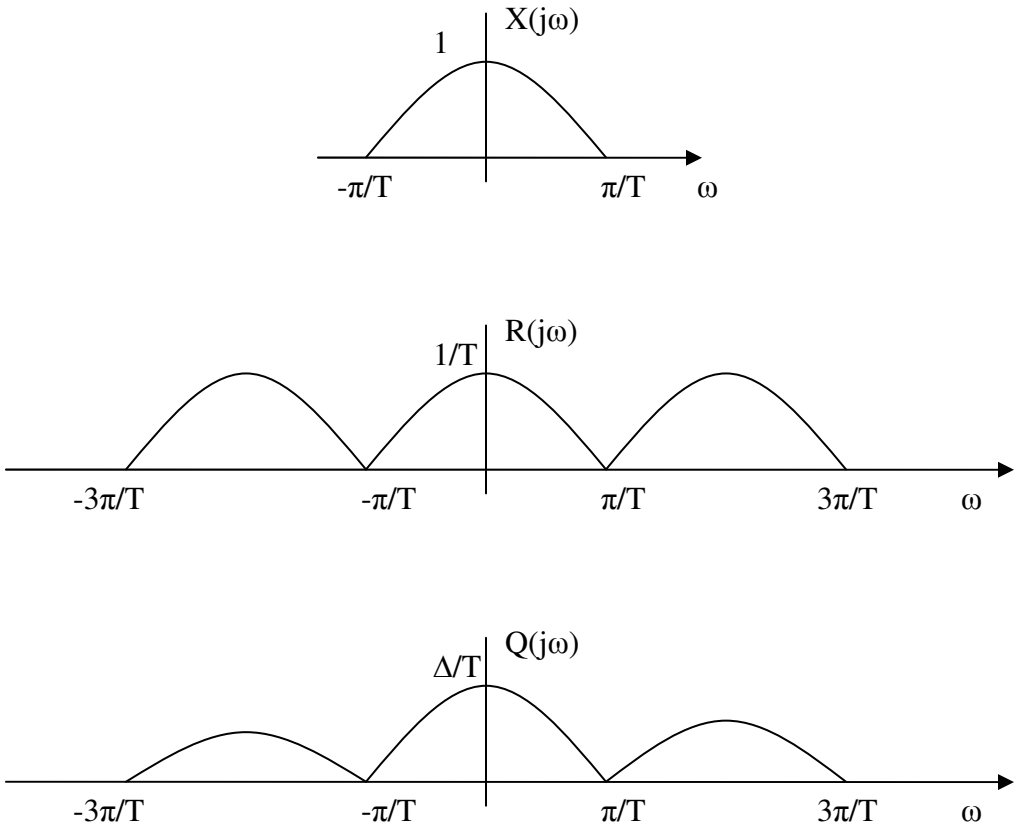
EECS 120 **Fall 2004**

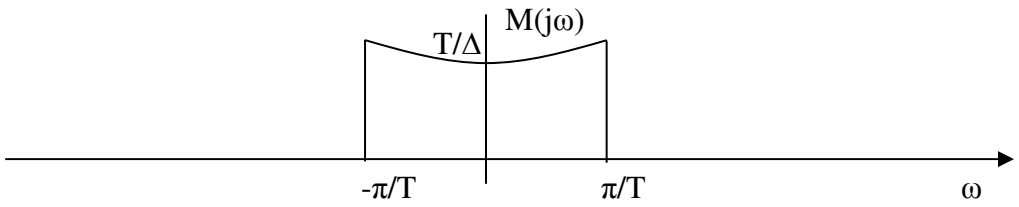
Demo 9 Solutions

1. (a)



(b) Maximum value of Δ : $\frac{2\pi}{\Delta} > \frac{\pi}{T}$, $\Delta < 2T$

(c) M(jω) = $T\frac{\omega}{2\sin(\omega\frac{\Delta}{2})}$, for |ω| < $\frac{\pi}{T}$, 0 else



2. (a) $r(t) = x_1(t)\cos(\omega_c t) + x_2(t)\sin(\omega_c t)$

$y_1(t) = [(x_1(t)\cos(\omega_c t) + x_2(t)\sin(\omega_c t))\cos(\omega_c t)] * h(t)$

$= [(0.5x_1(t) + 0.5x_1(t)\cos(2\omega_c t) + 0.5x_2(t)\cos(2\omega_c t))] * h(t)$

The high frequency components above: $0.5x_1(t)\cos(2\omega_c t)$, $0.5x_2(t)\cos(2\omega_c t)$ get filtered out by the low pass filter h(t). The filter has a gain of 2 to scale back the $0.5x_1(t)$. We are left with:

$y_1(t) = x_1(t)$

Similarly:

$y_2(t) = [(x_1(t)\cos(\omega_c t) + x_2(t)\sin(\omega_c t))\sin(\omega_c t)] * h(t)$

$= [0.5x_1(t)\sin(2\omega_c t) + 0.5x_2(t) - 0.5x_2(t)\cos(2\omega_c t)] * h(t)$

The high frequency components above: $0.5x_1(t)\sin(2\omega_c t)$, $0.5x_2(t)\cos(2\omega_c t)$ get filtered out by the low pass filter h(t). The filter has a gain of 2 to scale back the $0.5x_2(t)$. We are left with:

$y_2(t) = x_2(t)$

3. (a)

Instantaneous frequency =

$$\frac{d}{dt}(\omega_0 t^2 / 2) = \omega_0 t$$

(b)

$$S(j\omega) = \int_{t=-\infty}^{\infty} e^{j\omega_0 t^2/2} e^{-j\omega t} dt = \int_{t=-\infty}^{\infty} e^{j(t\sqrt{\frac{\omega_0}{2}} - \sqrt{\frac{\omega^2}{2\omega_0}})^2} e^{-j\frac{\omega^2}{2\omega_0}} dt$$

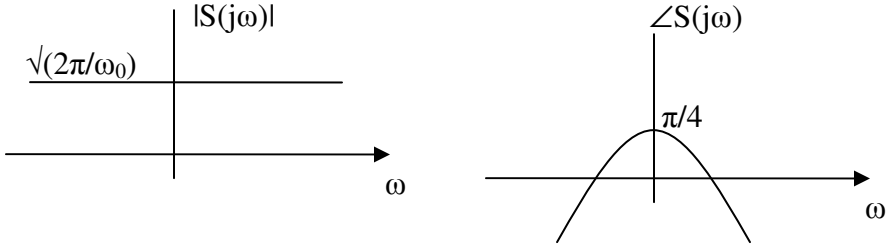
Letting $u = t\sqrt{\frac{\omega_0}{2}} - \sqrt{\frac{\omega^2}{2\omega_0}}$, $du = \sqrt{\frac{\omega_0}{2}} dt$

We have:

$$S(j\omega) = e^{-j\frac{\omega^2}{2\omega_0}}\sqrt{\frac{2}{\omega_0}}\int_{u=-\infty}^{\infty} e^{ju^2}du = e^{-j\frac{\omega^2}{2\omega_0}}\sqrt{\frac{\pi}{\omega_0}}(1+j)$$

$$|S(j\omega)| = \sqrt{\frac{2\pi}{\omega_0}}$$

$$\angle S(j\omega) = \frac{-\omega^2}{2\omega_0} + \frac{\pi}{4}$$



(c) Let r(t) be the signal entering h(t). So we have:

$$r(t) = x(t)e^{-j\frac{\omega_0 t^2}{2}}$$

Let v(t) be the signal exiting h(t). So we have:

$$v(t) = r(t) * h(t) = \int_{\tau=-\infty}^{\infty} r(\tau)h(t-\tau)d\tau = \int_{\tau=-\infty}^{\infty} x(\tau)e^{-j\frac{\omega_0\tau^2}{2}}e^{j\frac{\omega_0(t-\tau)^2}{2}}d\tau$$

Finally, $y(t) = v(t)e^{-j\frac{\omega_0 t^2}{2}} = \int_{\tau=-\infty}^{\infty} x(\tau)e^{-j\frac{\omega_0\tau^2}{2}}e^{j\frac{\omega_0(t-\tau)^2}{2}}e^{-j\frac{\omega_0 t^2}{2}}d\tau$

$$= \int_{\tau=-\infty}^{\infty} x(\tau)e^{-j(\omega_0 t)\tau}d\tau = X(j\omega_0 t)$$

4. (a)

From eqn. (5.20) on pg. 369 of OWN:

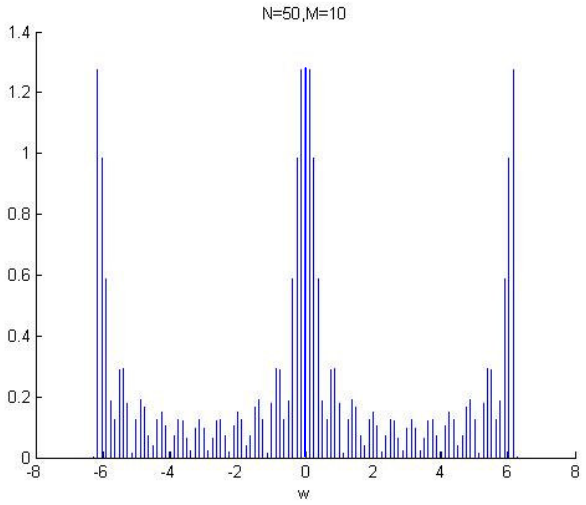
$$P(e^{j\omega}) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - \frac{2\pi k}{N})$$

where $a_k = \frac{1}{N}\sum_{n=0}^{M} e^{-jk(\frac{2\pi}{N})n} = a_k = \frac{1}{N}\frac{1-e^{-jk(\frac{2\pi}{N})(M+1)}}{1-e^{-jk(\frac{2\pi}{N})}}$

$$a_k = \frac{1}{N}\frac{e^{-jk(\frac{\pi}{N})(M+1)}[e^{+jk(\frac{\pi}{N})(M+1)} - e^{-jk(\frac{\pi}{N})(M+1)}]}{e^{-jk(\frac{\pi}{N})}[e^{+jk(\frac{\pi}{N})} - e^{-jk(\frac{\pi}{N})}]}$$

$$a_k = \frac{1}{N}\frac{e^{-jk(\frac{\pi}{N})M}\sin[k(\frac{\pi}{N})(M+1)]}{\sin[k(\frac{\pi}{N})]}$$

plot of $|P(e^{j\omega})|$:

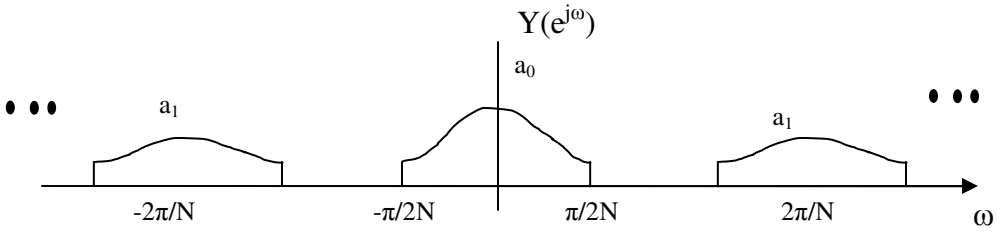


Note that the spectral lines above are spaced by $\frac{2\pi}{N}$

Also note that $P(e^{j\omega})$ is periodic with period 2π

(b)

$$Y(e^{j\omega}) = \sum_{k=0}^{N-1} a_k X(e^{j(\omega - k\frac{2\pi}{N})})$$



(c)

We need : $\frac{2\pi}{N} > 2\omega_M$ to prevent spectral overlap as we slide each copy of the spectrum over.

$\omega_M < \frac{\pi}{N}$, so the two sided bandwidth allowed for x[n] is $\frac{2\pi}{N}$

The solution does not depend on M, because all M does is to change the discrete time Fourier series coefficients of the periodic signal p[n] and the DC Fourier coefficient is never zero.

(d) To recover x[n] bandpass filter y[n] with the following filter:

$$H(e^{j\omega}) = \left\{ \begin{array}{cc} 0, & \omega_M < \omega < 2\pi - \omega_M \\ 1/a_0, & else \end{array} \right\}$$