EECS 120 **Fall 2004**

Sample Final

• The exam is for three hours.

• The maximum score is 100 points. The maximum score for each part of each problem is indicated.

• The exam is closed-book and closed-notes. Calculators, computing, and communication devices are NOT permitted.

• Six double-sided sheets of notes are allowed. These should be legible to normal eyesight, i.e. the lettering should not be excessively small.

• No form of collaboration between students is allowed.

1. *(4 points)* State whether the following is true or false. Give a brief explanation. A correct answer with a correct explanation gets 4 points. A correct answer without a correct explanation gets only 1 point.

   • A causal linear time invariant continuous time system with a rational Laplace transform which has more zeros than poles in the right half plane is stable.

2. *(5 points)*

   Let $x(t) \geq 0$ and $y(t) \geq 0$ be nonnegative continuous time signals, with respective Fourier transforms $X(j\omega)$ and $Y(j\omega)$. A Stanford student claims that

$$\int_{-\infty}^{\infty} \mid X(j\omega) + Y(j\omega) \mid^2 d\omega \geq \int_{-\infty}^{\infty} \mid X(j\omega) \mid^2 d\omega \ .$$

   Is this true or false ? Either explain clearly why this is true or give a counterexample.

3. *(5 + 10 points)* A real valued continuous time signal $x(t)$ is known to be bandlimited : if $X(j\omega)$ denotes the Fourier transform of $x(t)$, then $X(j\omega) = 0$ for $\mid \omega \mid \geq W$.

   The signal is nonuniformly sampled with the pulse train

$$q(t) = \sum_{n=-\infty}^{\infty} \left[\delta(t - nT) + \delta(t - nT - \frac{T}{4}) \right] \ .$$

   To begin with, we *do not* assume that there is any special relation between W and T.

   (a) Let $y(t) = x(t)q(t)$. Determine the Fourier transform of $y(t)$, i.e. $Y(j\omega)$.

   (b) Find the best possible condition you can on the relation between W and T which will ensure that it is possible to perfectly recover $x(t)$ from $y(t)$.

4. *(12 points)*

Consider the linear constant coefficient differential equation

$$\frac{d^2}{dt^2}y(t) - y(t) = \frac{d}{dt}x(t) \ ,$$

thought of as giving the output function $y(t)$ in terms of the input function $x(t)$.

Find a linear time invariant *anticausal* system whose input and output are related by this differential equation.

5. *(10 points)*

A discrete time linear time invariant system is such that the input

$$x[n] = \delta[n] - \delta[n-1] + \frac{1}{4}\delta[n-2] \ ,$$

produces the output

$$y[n] = \frac{1}{2^{n-2}}u[n-2] \ .$$

Is the system stable ? Carefully and clearly explain your answer.

6. *(12 points)*

Let $x[n]$ be a discrete time signal. We first downsample it by a factor of 2, resulting in the signal

$$x_1[n] = x[2n] \ , \quad n \in \mathbf{Z} \ .$$

We then expand the signal $x_1[n]$ by a factor of 3, resulting in the signal

$$x_2[n] = \begin{cases} x_1[n/3] & \text{if } n \text{ is a multiple of 3} \\ 0 & \text{otherwise} \end{cases} \ .$$

Finally, we downsample $x_2[n]$ by a factor of 2, resulting in the signal

$$y[n] = x_2[2n] \ , \quad n \in \mathbf{Z} \ .$$

Find the most general conditions you can on the discrete time Fourier transform of the original signal $x[n]$ under which you are guaranteed that it can be uniquely recovered from the signal $y[n]$.

7. *(3 + 9 points)*

Let $y(t)$ be a continuous time signal which is time limited to $(-3T, 3T)$.

Let

$$x(t) = \sum_{n=-\infty}^{\infty} y(t - nT) \ .$$

(a) Verify that $x(t)$ is well defined and is periodic with period T.

(b) Let a_n denote the Fourier series coefficient of $x(t)$, whose period is taken to be T. Express the a_n in terms of the Fourier series $Y(j\omega)$ of $y(t)$.

8. *(2 + 3 + 3 + 1 + 3 + 3 points)*

Consider the continuous time signal

$$x(t) = u(t+1.5) + u(t+0.5) - u(t-0.5) - u(t-1.5) \ .$$

(a) Sketch $x(t)$.

(b) Determine the Fourier transform of $x(t)$, i.e. $X(j\omega)$.

(c) Let $T = 1/3$. Let

$$y[n] = x(nT) \ , \quad n \in \mathbf{Z} \ .$$

Thus the discrete time signal $y[n]$ is the sequence of samples of $x(t)$ with sampling period $T = \frac{1}{3}$.

Let $Y(z)$ denote the z-transform of $y[n]$. Determine $Y(z)$.

(d) Determine the discrete time Fourier transform of $y[n]$, i.e. $Y(e^{j\omega})$.

(e) Let $h(t)$ denote the signal

$$h(t) = u(t) - u(t - \frac{1}{3}) \ .$$

Let

$$\tilde{x}(t) = \sum_{n=-\infty}^{\infty} y[n]h(t-nT) \ .$$

Thus $\tilde{x}(t)$ can be thought of as the output of a zero order hold with impulse response $h(t)$, whose input is the signal $\sum_{n=-\infty}^{\infty} y[n]\delta(t-nT)$.

Determine the Fourier transform of $\tilde{x}(t)$, i.e. $\tilde{X}(j\omega)$.

(f) Determine $\int_{-\infty}^{\infty} \mid x(t) - \tilde{x}(t) \mid^2 dt$.

9. *(2 + 2 + 5 + 3 + 3 points)*

Consider the rational Laplace transform

$$R(s) = \frac{s-1}{(s^2+2s+2)(s^2+2s+5)} \ .$$

This problem asks some questions about the root locus plot for this rational Laplace transform, i.e. about the locations of the zeros of $1 + KR(s)$ as K ranges over real values. It is *not* necessary for you to determine the entire root locus in order to answer the questions in this problem.

In each part of the problem asking for an explanation, at most 1 point will be given for a correct answer without a correct explanation.

(a) Does the value $s_0 = 0$ lie on the root locus for some value $K > 0$? Explain what criterion you used in arriving at your answer.

(b) Does the value $s_0 = 0$ lie on the root locus for some value $K < 0$? Explain what criterion you used in arriving at your answer.

(c) Clearly state and explain the angle criterion for determining whether a complex number s_0 can be on the root locus plot for $R(s)$.

(d) Does the value $s_0 = 1 + j\frac{3}{2}$ lie on the root locus for some value $K > 0$? Explain how you arrived at your answer.

(e) Does the value $s_0 = j100$ lie on the root locus for some value $K > 0$? Explain how you arrived at your answer.