

**Homework 10 : Due by 8 p.m. on Monday November 29**

*Carry over problem :*

Please do problem 4 of Homework 9 as part of this problem set.

*Regular problems :*

1. Parts (g) and (j) of Problem 11.28 on pg. 873 of OWN.
2. Part (e) of Problem 11.29 on pg. 873 of OWN.
3. Parts (a), (b), (c), and (d) of Problem 11.32 on pp. 874 -875 of OWN.
4. Problem 11.56 on pp. 903 -904 of OWN.
5. Consider the basic continuous time linear feedback system as in Fig. 11.10 (b) on pg. 835 of OWN, with our standing assumptions that the plant  $H(s)$  is *proper* (i.e. the degree of its numerator polynomial is no bigger than that of its denominator polynomial), the controller  $G(s)$  is proper, and there are no pole-zero cancellations in the product  $G(s)H(s)$ .

Write

$$G(s)H(s) = \frac{b_M s^M + b_{M-1} s^{M-1} + \dots + b_0}{a_N s^N + a_{N-1} s^{N-1} + \dots + a_0} .$$

Under our assumptions we have  $N \geq M$ . Let  $a$  be a  $1 \times (N + 1)$  array in MATLAB whose  $i$ -th entry is the coefficient  $a_{N-i+1}$ ,  $1 \leq i \leq N + 1$ . Let  $b$  be a  $1 \times (M + 1)$  array in MATLAB whose  $i$ -th entry is the coefficient  $b_{M-i+1}$ ,  $1 \leq i \leq M + 1$ .

The MATLAB command **rlocus**( $b, a$ ) returns a plot of the root loci of the equation

$$1 + KG(s)H(s) = 0$$

for  $K \geq 0$ . Thus, under our assumptions, it gives the plot of the root loci for  $K \geq 0$  of the poles of the closed loop system function

$$Q(s) = \frac{H(s)}{1 + KG(s)H(s)} .$$

The command **rlocus**(- $b, a$ ) returns a plot of the root loci of the equation

$$1 + KG(s)H(s) = 0$$

for  $K \leq 0$ . Thus, under our assumptions, it gives the plot of the root loci for  $K \leq 0$  of the poles of the closed loop system function

$$Q(s) = \frac{H(s)}{1 + KG(s)H(s)} .$$

Solve problem 11.37 on pg. 882 of OVN using MATLAB. Thus, for part (a) you should submit two root locus plots (one for  $K > 0$  and one for  $K < 0$ ) and explain how they justify the statement claimed in part (a) of the problem. Similarly, for part (b) you should submit two root locus plots (one for  $K > 0$  and one for  $K < 0$ ) and explain how they justify the statement claimed in part (b). Finally, for part (c) you will have to submit four root locus plots (two for  $K > 0$  and two for  $K < 0$ ) and explain how they justify the statement claimed in part (c) of the problem.

*Note* : Although the problem writes  $G(s) = K$ , you will need to take  $G(s) = 1$  to conform to our notation.

*Important* : Please briefly explain in words the structure of the root loci in each of the plots. This is particularly important for the plots when there are three closed loop poles, because the scale on the real axis in the plots produced by MATLAB is not good enough to make it clear what is going on with the two smaller magnitude poles.