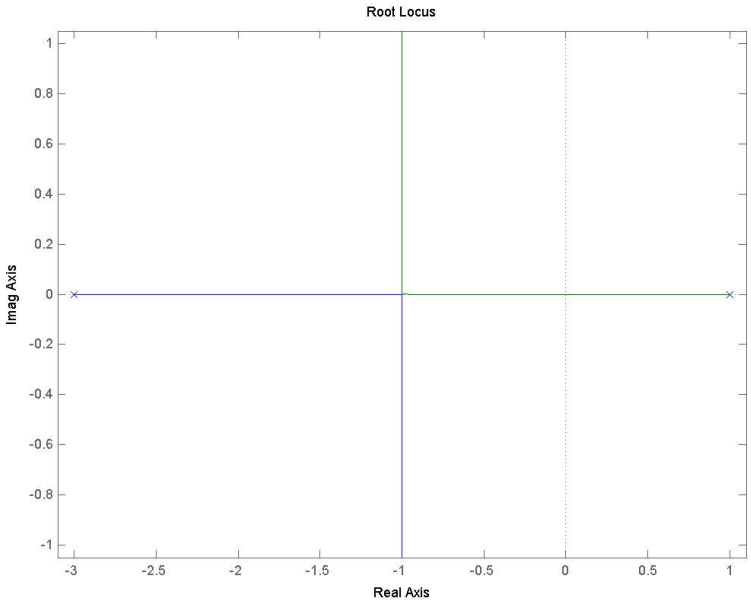
EECS 120 **Fall 2004**

Homework 10 Solutions

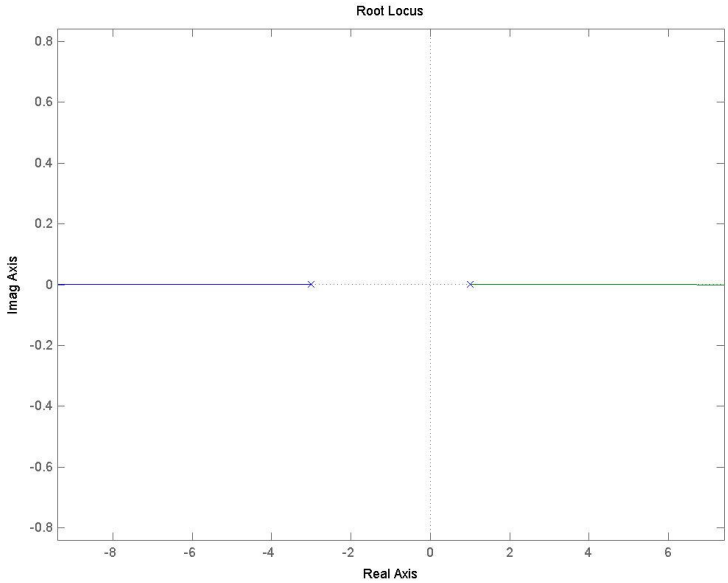
Carry Over Problem from HW9.

11.24

(b) K > 0

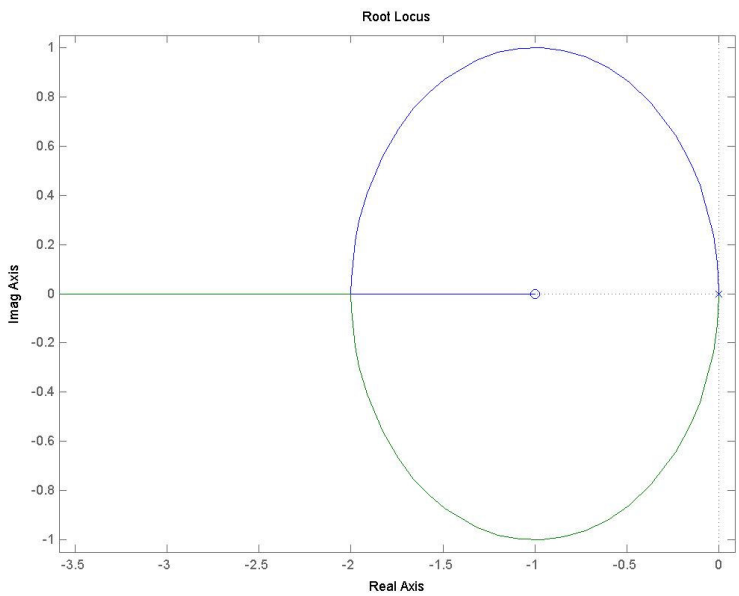


K < 0

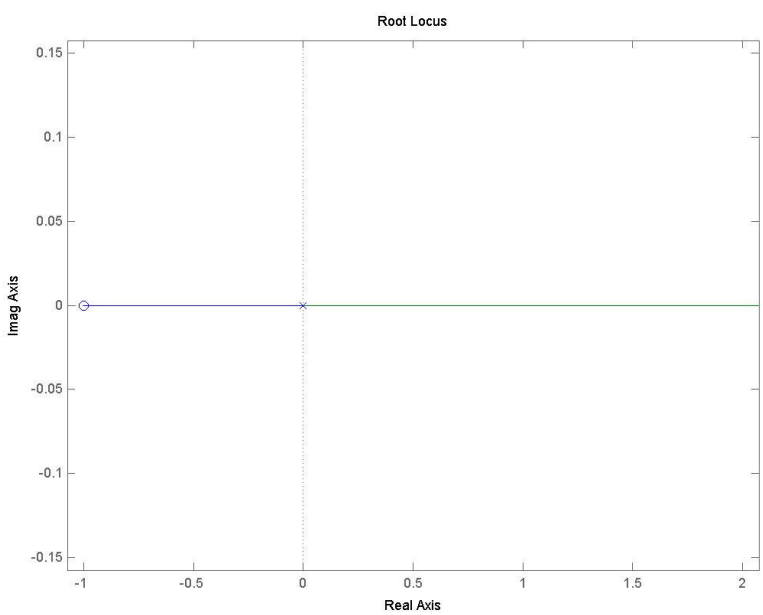


(d)

K > 0


Root Locus
Imag Axis
Real Axis
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-3.5
-3
-2.5
-2
-1.5
-1
-0.5
0

K < 0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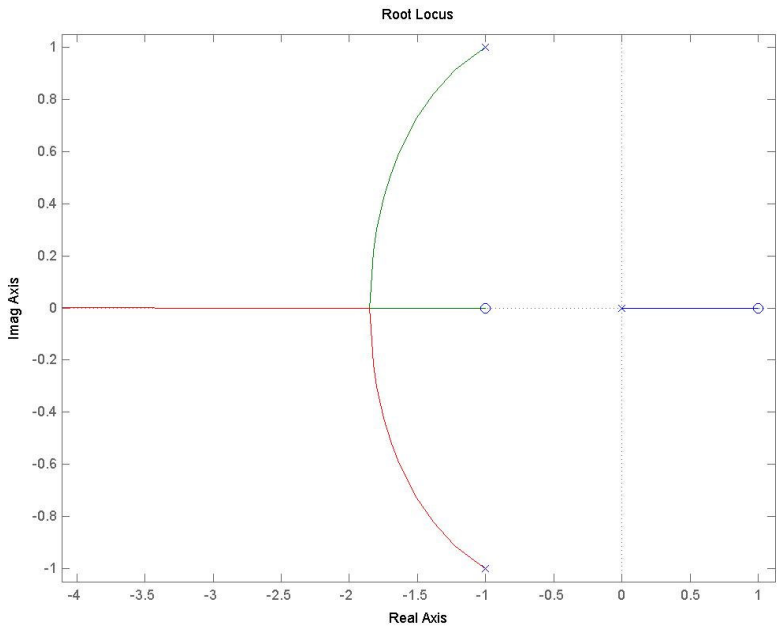
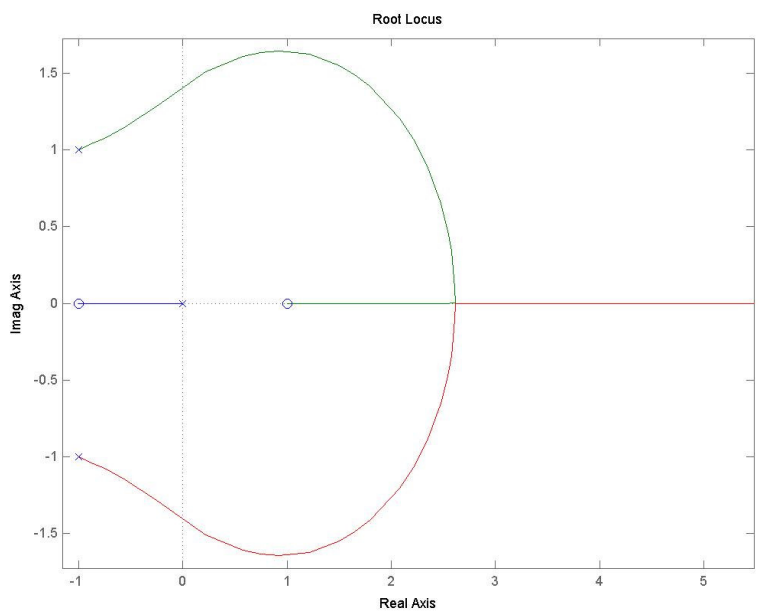
Imag Axis
Real Axis
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-1
-0.5
0
0.5
1
1.5
2

(g)

K > 0

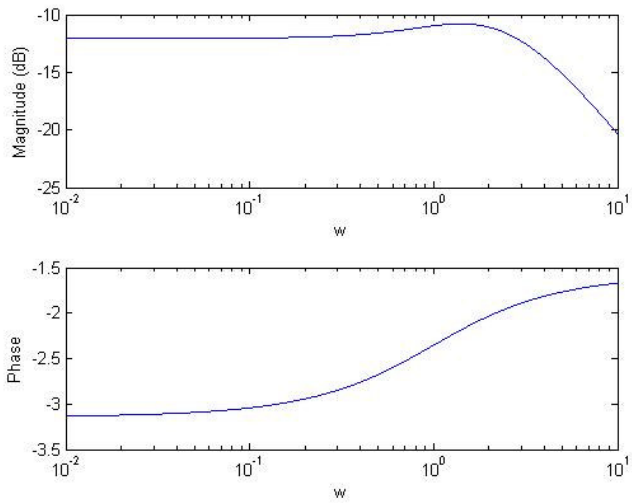

Root Locus
Imag Axis
Real Axis
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-4
-3.5
-3
-2.5
-2
-1.5
-1
-0.5
0
0.5
1

K < 0

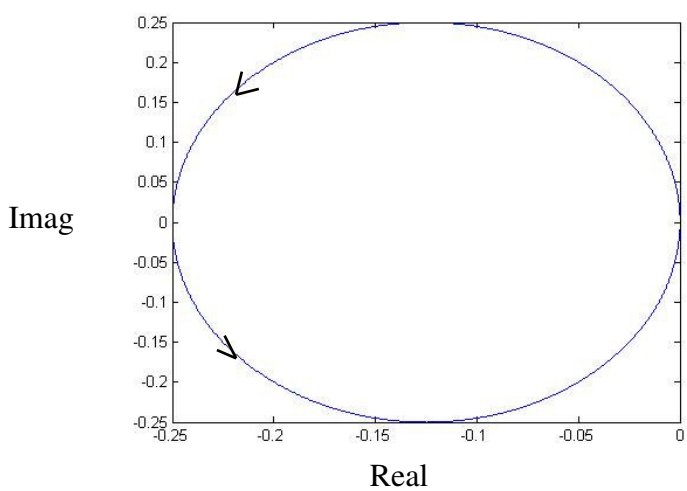

Root Locus
Imag Axis
Real Axis
1.5
1
0.5
0
-0.5
-1
-1.5
-1
0
1
2
3
4
5

1. 11.28 (g) $G(j\omega)H(j\omega) = \dfrac{j\omega + 1}{(j\omega)^2 - 4}$

Bode Plot:



Nyquist Plot:



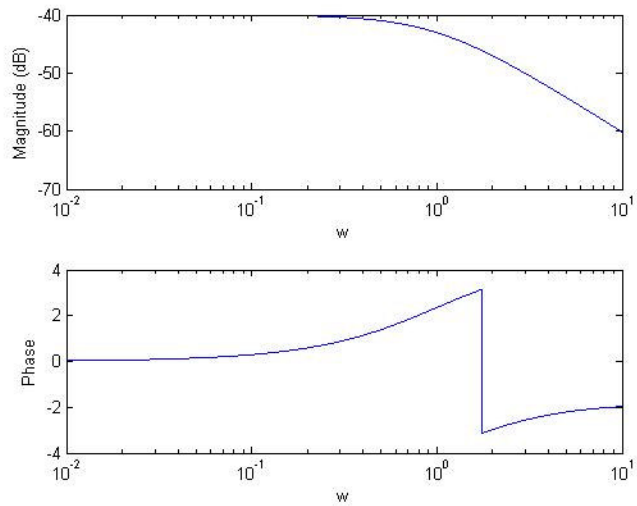
By the Nyquist stability criterion, the number of counter-clockwise encirclements of the point -1/K by the Nyquist plot must equal the number of right-half-plane poles of G(s)H(s). Since G(s)H(s) has 1 right hand pole, the Nyquist plot must encircle the point -1/K once. Thus we get the following condition for the closed loop system to be stable:

$$-\frac{1}{4} < -\frac{1}{K} < 0$$

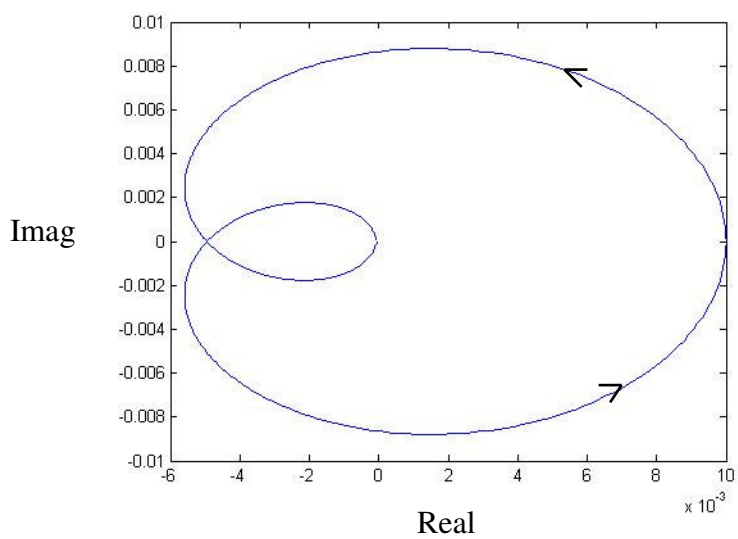
$$K > 4$$

11.28 (j) $G(j\omega)H(j\omega) = \dfrac{j\omega + 1}{(j\omega + 100)(j\omega - 1)^2}$

Bode Plot:



Nyquist Plot:

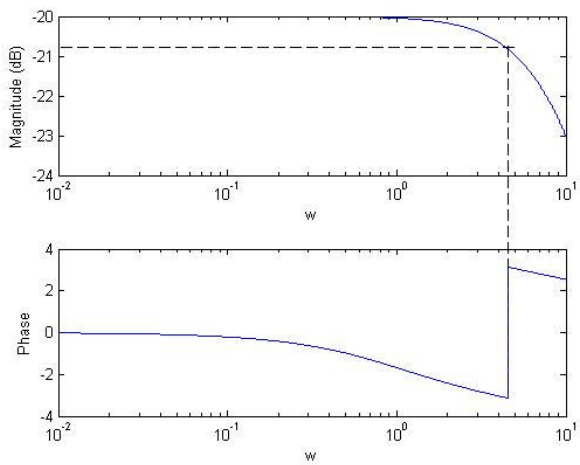


By the Nyquist stability criterion, the number of counter-clockwise encirclements of the point -1/K by the Nyquist plot must equal the number of right-half-plane poles of G(s)H(s). Since G(s)H(s) has 2 right hand pole, the Nyquist plot must encircle the point -1/K twice. This implies:

$$-5x10^{-3} < -\frac{1}{K} < 0, \rightarrow \mathrm{K} > 200$$

2. 11.29 (e)

Bode Plot: |G(jω) H(jω)|



Since the magnitude never reaches 0dB, the phase margin is undefined.

The gain margin is found by looking at the magnitude of G(jω) H(jω) at the frequency where the phase is π radians. In the above case the gain margin is approximately 20dB.

3. 11.32

(a)Closed loop transfer function: $Q(s) = \dfrac{H(s)}{1 + KH(s)G(s)}$

Assume that K ≠ 0, then

$$Q(s) = \frac{N_1(s)D_2(s)}{D_1(s)D_2(s) + KN_1(s)N_2(s)}$$

and there are no cancellations between the numerator and the denominator in this expression. Thus the zeros of Q(s) are the zeros of $N_1(s)$ and of $D_2(s)$, i.e the zeros of H(s) and the poles of G(s).

(b) With K = 0, The closed loop transfer function Q(s) = H(s), thus the poles of Q(s) = the poles of H(s) and the zeros of Q(s) = zeros of H(s).

(c) Show that we can write Q(s) as :

$$\frac{p(s)}{q(s)}\frac{H'(s)}{1+KG'(s)H'(s)}$$

where $H'(s)=\dfrac{\dfrac{N_1(s)}{p(s)}}{\dfrac{D_1(s)}{q(s)}}$ and $G'(s)=\dfrac{\dfrac{N_2(s)}{q(s)}}{\dfrac{D_2(s)}{p(s)}}$

$$\frac{p(s)}{q(s)}\frac{H'(s)}{1+KG'(s)H'(s)}=\frac{p(s)}{q(s)}\frac{\dfrac{N_1(s)}{p(s)}}{\dfrac{D_1(s)}{q(s)}[1+K\dfrac{N_1(s)}{D_1(s)}\dfrac{q(s)}{p(s)}\dfrac{N_2(s)}{D_2(s)}\dfrac{p(s)}{q(s)}]}$$

$$=\frac{\dfrac{N_1(s)}{D_1(s)}}{1+K\dfrac{N_1(s)}{D_1(s)}\dfrac{N_2(s)}{D_2(s)}}=\text{Q(s)}$$

(d) $H(s)=\dfrac{s+1}{(s+4)(s+2)}$, $G(s)=\dfrac{s+2}{s+1}$

In this case we have $p(s)=s+1$, $q(s)=s+2$

$H'(s)=\dfrac{1}{s+4}$, $G'(s)=1$

Zeros of Q(s) are the zeros of p(s), the zeros of H'(s), and the poles of G'(s)

Zeros of Q(s): s = -1

Poles of Q(s) are the zeros of q(s) and the solution of 1+KG'(s)H'(s) = 0.

Poles of Q(s): s = -2, and solution to $1+\dfrac{K}{s+4}=0$

Solution to $1+\frac{K}{s+4}=0$: $s+4+K=0$, s = -4-K

4. 11.56

(a) Linearized equation:

$$L\frac{d^2\theta(t)}{dt^2}=g\theta(t)-a(t)+Lx(t)$$

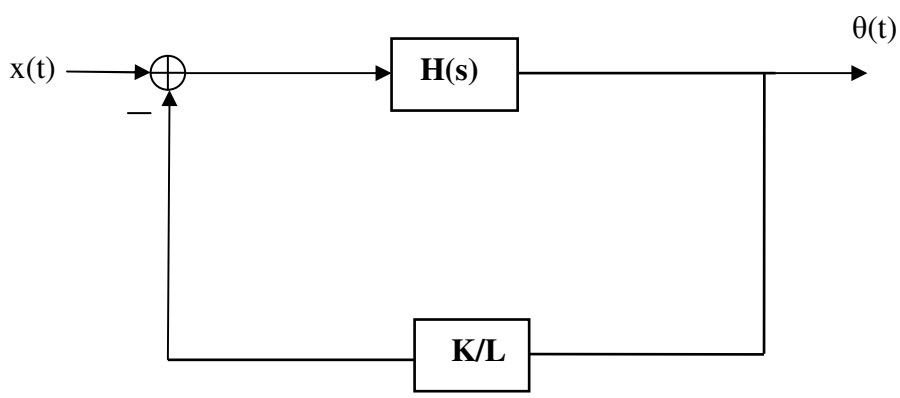
Laplace Transform (for a(t) = 0)

$$Ls^2\theta(s)=g\theta(s)+LX(s)$$

$$H(s)=\frac{\theta(s)}{X(s)}=\frac{L}{LS^2-g}=\frac{1}{s^2-\frac{g}{L}}=\frac{1}{(s+\sqrt{\frac{g}{L}})(s-\sqrt{\frac{g}{L}})}$$

So we have a pole in the right half plane at s = $\sqrt{\frac{g}{L}}$

(b) For $a(t)=K\theta(t)$ we have the following block diagram:



Where H(s) is the transfer function from part (a):

H(s)= $\frac{L}{Ls^2-g}$

Closed Loop System: Q(s) = $\dfrac{H(s)}{1+KH(s)/L}$

Poles: 1+KH(s)/L = 0,

$$Ls^2 - g + K = 0$$

$$s = \pm\sqrt{\frac{g-K}{L}}$$

For K < g we always have one pole in the RHP. For K ≥ g we have both poles on the imaginary axis. Thus the system is unstable for all values of K.

If x(t) = δ(t), then the output will just be the impulse response of the closed loop system.

Since we know that for For K ≥ g we have both poles on the imaginary axis, for these values of K the pendulum will sway back and forth in an undamped oscillatory fashion.

(c) For $a(t) = K_1\theta(t) + K_2\dfrac{d\theta(t)}{dt}$, we have the closed loop transfer function :

Q(s) = $\dfrac{H(s)}{1+(K_1+K_2 s)H(s)/L}$

Poles: $1+(K_1+K_2 s)H(s)/L = 0$

$$\frac{Ls^2 - g + K_1 + K_2 s}{Ls^2 - g} = 0$$

$$s^2 - g/L + K_1/L + K_2 s/L = 0$$

$$s = \frac{-K_2/L \pm \sqrt{(K_2/L)^2 - 4(\frac{K_1 - g}{L})}}{2}$$

with g = 9.8m/s^2, and L = 0.5m we set:

ζ= damping ratio = 1

ω_n = natural frequency = 3 rad/s.

From eq. (6.33) we have $K_2/L = 2\zeta\omega_n = 6$, $K_2 = 3$

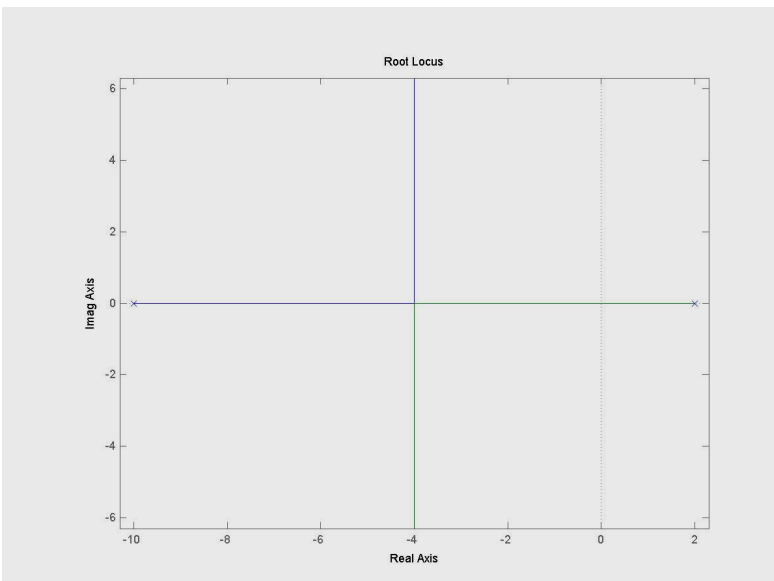
and $(K_1 - g)/L = \omega_n^2 = 9$, $K_1 = 14.3$

For these values of K_1 and K_2 the poles are at $s = -3$

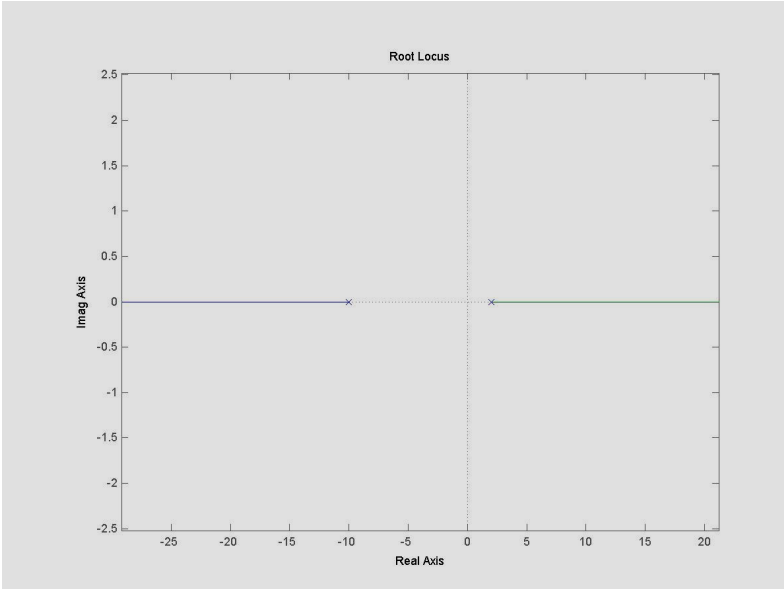
5. 11.37

(a)

K > 0, As can be seen in the root locus, the closed loop pole initially at s = 2, shifts to the left hand plane as K gets larger.

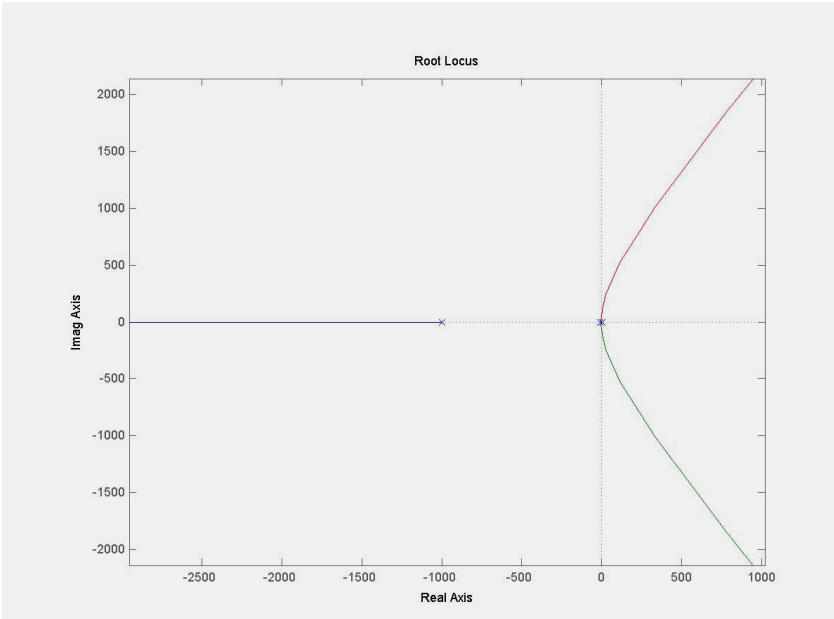


K < 0

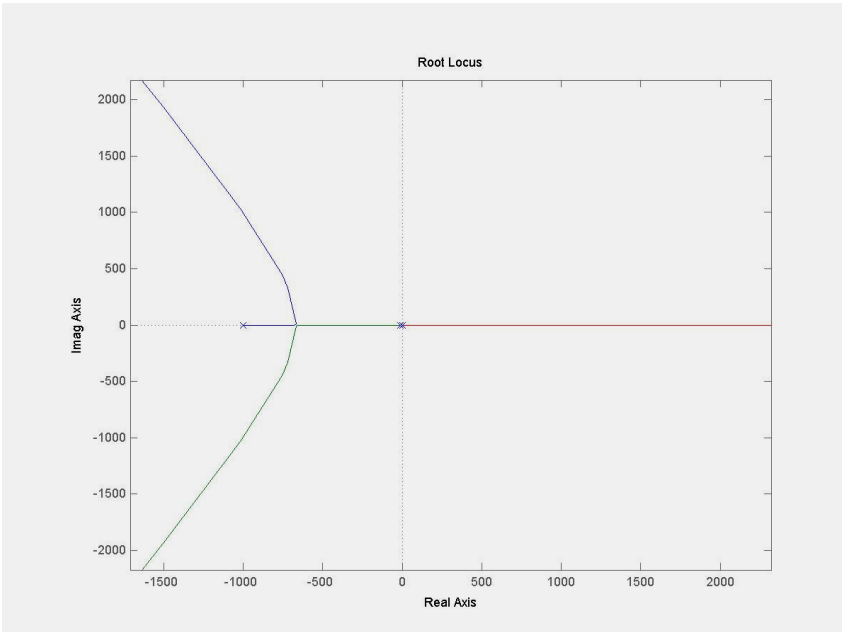


(b)

K > 0, The plot shows that as K gets larger, the closed loop poles shift to the right half plane.

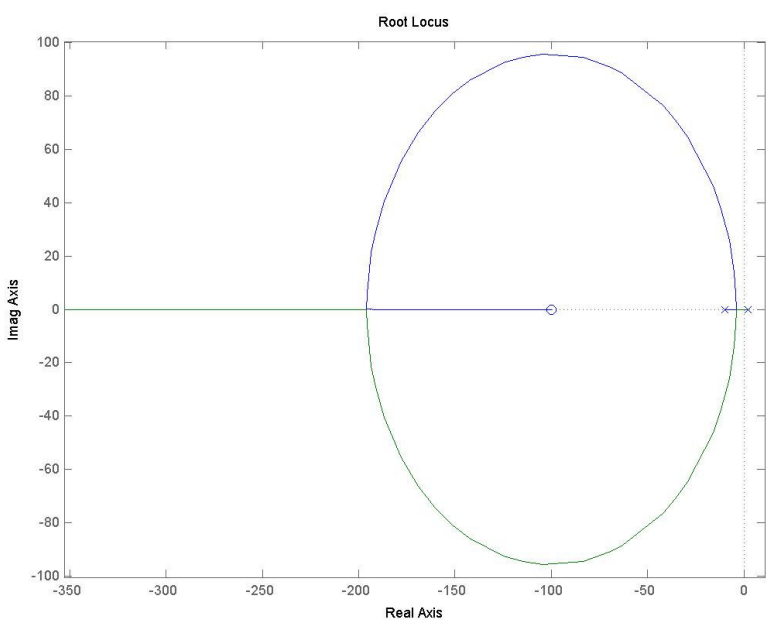


K < 0

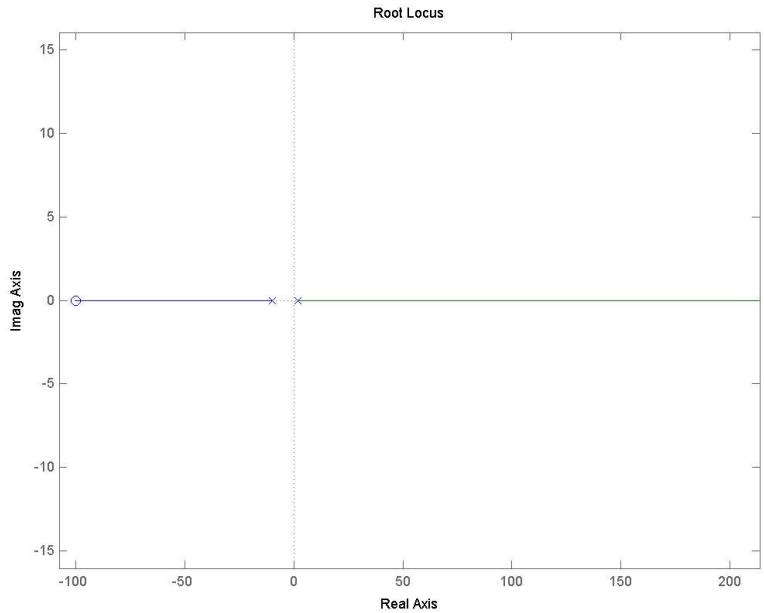


(c) For H(s) given by eq. (P11.37-1):

K > 0, The closed loop poles stay in the LHP as K gets larger.

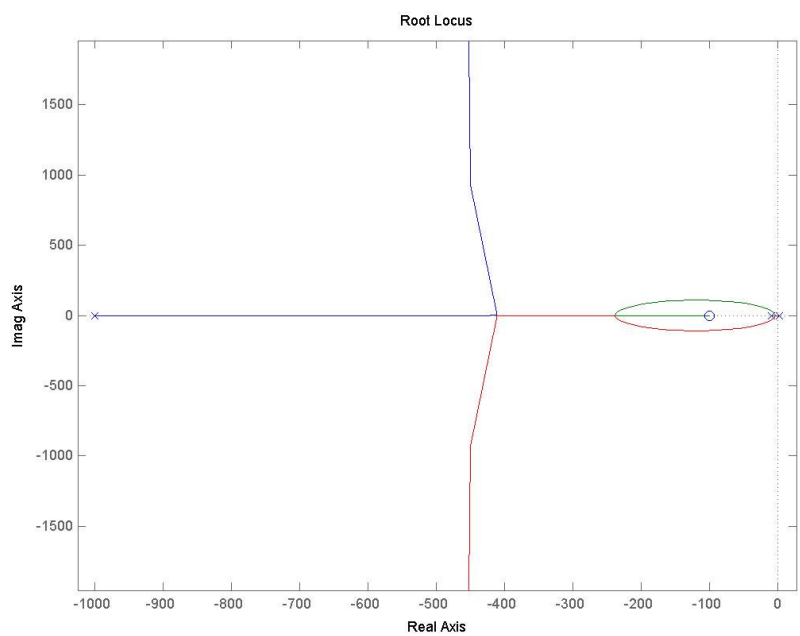


K < 0

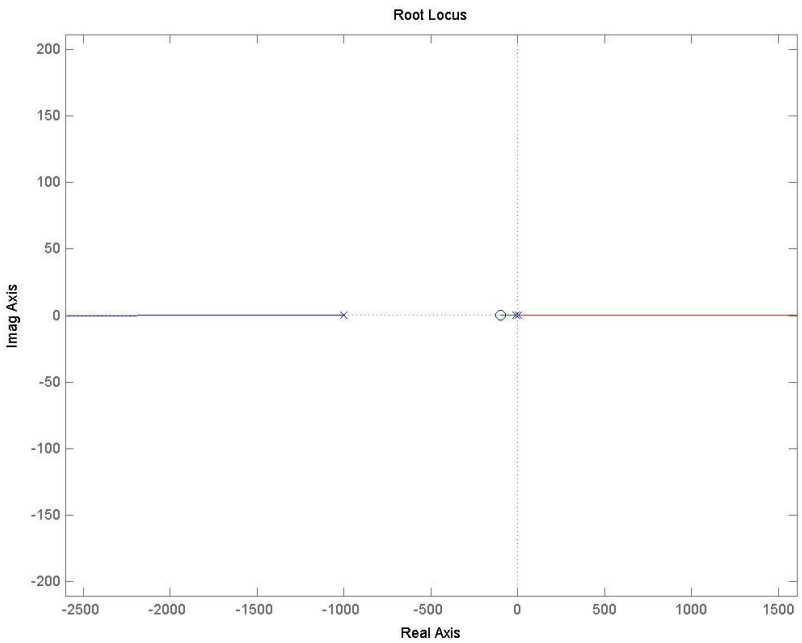


For H(s) given by eq. (P11.37-3):

K > 0 Again, we see that the closed loop poles shift to the left hand plane as K gets larger.



K < 0


Root Locus
Imag Axis
Real Axis
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150
100
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-100
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-200
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0
500
1000
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