11.24
(b) $K > 0$

$K < 0$
(d) 

\[ K > 0 \]

\[ K < 0 \]
(g)

\[ K > 0 \]

\[ K < 0 \]
1. 11.28 (g) \( G(j\omega)H(j\omega) = \frac{j\omega + 1}{(j\omega)^2 - 4} \)

Bode Plot:

Nyquist Plot:

By the Nyquist stability criterion, the number of counter-clockwise encirclements of the point \(-1/K\) by the Nyquist plot must equal the number of right-half-plane poles of \(G(s)H(s)\). Since \(G(s)H(s)\) has 1 right hand pole, the Nyquist plot must encircle the point \(-1/K\) once. Thus we get the following condition for the closed loop system to be stable:

\[
-\frac{1}{4} < -\frac{1}{K} < 0
\]

\( K > 4 \)
11.28 (j) \( G(j\omega)H(j\omega) = \frac{j\omega + 1}{(j\omega + 100)(j\omega - 1)^2} \)

By the Nyquist stability criterion, the number of counter-clockwise encirclements of the point -1/K by the Nyquist plot must equal the number of right-half-plane poles of \( G(s)H(s) \). Since \( G(s)H(s) \) has 2 right hand poles, the Nyquist plot must encircle the point -1/K twice. This implies:

\[-5 \times 10^{-3} < -\frac{1}{K} < 0, \quad \Rightarrow K > 200\]
2. 11.29 (e)

Bode Plot: $|G(j\omega)H(j\omega)|$

Since the magnitude never reaches 0dB, the phase margin is undefined.

The gain margin is found by looking at the magnitude of $G(j\omega)H(j\omega)$ at the frequency where the phase is $\pi$ radians. In the above case the gain margin is approximately 20dB.

3. 11.32

(a) Closed loop transfer function: $Q(s) = \frac{H(s)}{1 + KH(s)G(s)}$

Assume that $K \neq 0$, then

$$Q(s) = \frac{N_1(s)D_2(s)}{D_1(s)D_2(s) + KN_1(s)N_2(s)}$$

and there are no cancellations between the numerator and the denominator in this expression. Thus the zeros of $Q(s)$ are the zeros of $N_1(s)$ and of $D_2(s)$, i.e the zeros of $H(s)$ and the poles of $G(s)$.

(b) With $K = 0$, The closed loop transfer function $Q(s) = H(s)$, thus the poles of $Q(s)$ = the poles of $H(s)$ and the zeros of $Q(s)$ = zeros of $H(s)$. 
(c) Show that we can write $Q(s)$ as:

\[
\frac{p(s)}{q(s)} \frac{H'(s)}{1 + KG'(s)H'(s)} = \frac{\frac{N_1(s)}{D_1(s)}}{\frac{q(s)}{p(s)}} \quad \text{and} \quad G'(s) = \frac{\frac{N_2(s)}{D_2(s)}}{\frac{p(s)}{q(s)}}
\]

where $H'(s) = \frac{p(s)}{D_1(s)} \quad \text{and} \quad G'(s) = \frac{q(s)}{p(s)}$

\[
\frac{p(s)}{q(s)} \frac{H'(s)}{1 + KG'(s)H'(s)} = \frac{p(s)}{q(s)} \frac{D_1(s)}{q(s)} [1 + K \frac{\frac{N_1(s)}{D_1(s)} \frac{q(s)}{p(s)} \frac{N_2(s)}{D_2(s)} \frac{p(s)}{q(s)}}]
\]

\[
= \frac{\frac{N_1(s)}{D_1(s)}}{1 + K \frac{\frac{N_1(s)}{D_1(s)} \frac{N_2(s)}{D_2(s)}}} = Q(s)
\]

\[
(d) \quad H(s) = \frac{s+1}{(s+4)(s+2)} \quad G(s) = \frac{s+2}{s+1}
\]

In this case we have $p(s) = s+1, \quad q(s) = s+2$

$H'(s) = \frac{1}{s+4}, \quad G'(s) = 1$

Zeros of $Q(s)$ are the zeros of $p(s)$, the zeros of $H'(s)$, and the poles of $G'(s)$

Zeros of $Q(s)$: $s = -1$

Poles of $Q(s)$ are the zeros of $q(s)$ and the solution of $1 + KG'(s)H'(s) = 0$.

Poles of $Q(s)$: $s = -2$, and solution to $1 + \frac{K}{s+4} = 0$
Solution to \( 1 + \frac{K}{s+4} = 0 \): \( s + 4 + K = 0 \), \( s = -4 - K \)

4. 11.56

(a) Linearized equation:

\[
L \frac{d^2 \theta(t)}{dt^2} = g \theta(t) - a(t) + Lx(t)
\]

Laplace Transform (for \( a(t) = 0 \))

\[
Ls^2 \Theta(s) = g \Theta(s) + LX(s)
\]

\[
H(s) = \frac{\Theta(s)}{X(s)} = \frac{L}{LS^2 - g} = \frac{1}{s^2 - \frac{g}{L}} = \frac{1}{(s + \sqrt{\frac{g}{L}})(s - \sqrt{\frac{g}{L}})}
\]

So we have a pole in the right half plane at \( s = \sqrt{\frac{g}{L}} \)

(b) For \( a(t) = K \theta(t) \) we have the following block diagram:

Where \( H(s) \) is the transfer function from part (a):

\[
H(s) = \frac{L}{LS^2 - g}
\]
Closed Loop System: \( Q(s) = \frac{H(s)}{1 + KH(s)/L} \)

Poles: \( 1 + KH(s)/L = 0 \),

\[ Ls^2 - g + K = 0 \]

\[ s = \pm \sqrt{\frac{g - K}{L}} \]

For \( K < g \) we always have one pole in the RHP. For \( K \geq g \) we have both poles on the imaginary axis. Thus the system is unstable for all values of \( K \).

If \( x(t) = \delta(t) \), then the output will just be the impulse response of the closed loop system.

Since we know that for \( K \geq g \) we have both poles on the imaginary axis, for these values of \( K \) the pendulum will sway back and forth in an undamped oscillatory fashion.

(c) For \( a(t) = K_1\dot{\theta}(t) + K_2 \frac{d\theta(t)}{dt} \), we have the closed loop transfer function:

\[ Q(s) = \frac{H(s)}{1 + (K_1 + K_2s)H(s)/L} \]

Poles: \( 1 + (K_1 + K_2s)H(s)/L = 0 \)

\[ \frac{Ls^2 - g + K_1 + K_2s}{Ls^2 - g} = 0 \]

\[ s^2 - \frac{g}{L} + \frac{K_1}{L} + \frac{K_2s}{L} = 0 \]

\[ s = \frac{-K_2/L \pm \sqrt{(K_2/L)^2 - 4(K_1 - g)/L}}{2} \]

with \( g = 9.8 \text{m/s}^2 \), and \( L = 0.5 \text{m} \) we set:
\( \zeta = \text{damping ratio} = 1 \)

\( \omega_n = \text{natural frequency} = 3 \text{ rad/s}. \)

From eq. (6.33) we have \( K_2/L = 2\zeta \omega_n = 6, K_2 = 3 \)

and \( (K_1 - g)/L = \omega_n^2 = 9, K_1 = 14.3 \)

For these values of \( K_1 \) and \( K_2 \) the poles are at \( s = -3 \)

5. 11.37

(a)

\( K > 0 \), As can be seen in the root locus, the closed loop pole initially at \( s = 2 \), shifts to the left hand plane as \( K \) gets larger.

K < 0
K > 0, The plot shows that as K gets larger, the closed loop poles shift to the right half plane.

K < 0
(c) For $H(s)$ given by eq. (P11.37-1):

$K > 0$, The closed loop poles stay in the LHP as $K$ gets larger.

$K < 0$
For $H(s)$ given by eq. (P11.37-3):

$K > 0$ Again, we see that the closed loop poles shift to the left hand plane as $K$ gets larger.

$K < 0$