EECS 120 **Fall 2004**

Homework 12 : Due by 8 p.m. on Monday December 13

1. Parts (g) and (h) of Problem 11.30 on pg. 874 of OWN.

2. Problem 11.38 on pp. 882 -883 of OWN.

3. Parts (a), (b), (c) and (d) of Problem 11.59 on pp. 906 -907 of OWN.

4. This notation was introduced in problem 4 of Demo 14, but is being repeated here for convenience :

   Let $x[n]$ be a discrete time signal with z-transform $X(z)$. Given an integer $M \geq 1$, let $(\downarrow M)$ denote the operation of decimation or downsampling by a factor of M, i.e. we write $y[n] = (\downarrow M)x[n]$ if
   $$y[n] = x[nM] \quad \text{for all } n \ .$$
   Let $(\uparrow M)$ denote the operation of expansion by M (sometimes this is also called upsampling although in this course, following the textbook, we have reserved the word "upsampling" for expansion followed by low pass filtering). Thus, we write $z[n] = (\uparrow M)x[n]$ if
   $$z[n] = \begin{cases} x[n/M] & \text{if } n \text{ is an integer multiple of } M \\ 0 & \text{otherwise} \end{cases} \ .$$

   (a) Verify that if $M > 1$, the operation of decimation or downsampling by M, i.e. $(\downarrow M)$ is a linear operator but is not time invariant.

   (b) Verify that if $M > 1$, the operation of expansion by M, i.e. $(\uparrow M)$ is a linear operator but is not time invariant.

   (c) Given a discrete time signal $x[n]$, let $x_{even}[n]$ and $x_{odd}[n]$ denote respectively the signals
   $$x_{even}[n] = \begin{cases} 0 & \text{if } n \text{ is odd} \\ x[n] & \text{if } n \text{ is even} \end{cases} \ , \quad \text{and } x_{odd}[n] = \begin{cases} 0 & \text{if } n \text{ is even} \\ x[n] & \text{if } n \text{ is odd} \end{cases} \ .$$
   You are allowed to add and/or subtract signals at will. You are allowed to use boxes that can perform $(\uparrow 2)$ and boxes that can perform $(\downarrow 2)$. You can use as many such boxes as you like. Show how to create a single-input single-output system that produces the signal with z-transform $X_{odd}(z^2)$ when its input has z-transform $X(z)$.

5. Use MATLAB to plot the Nyquist diagram for the following rational z-transforms :

   (a)
   $$R(z) = \frac{z^2 + 2z + 2}{z^2 - 4}$$

   (b)
   $$R(z) = \frac{z^2 + 2z + 2}{(z^2 - 2z + 2)(z - 2)}$$

This is just a matter of drawing a 2-d plot of the real and imaginary parts of the complex function $R(z)$ as z ranges over the circle of radius 1 in the complex plane.

In each case, explain in detail, in words, why the plot looks like it does, i.e. describe in detail the relation between the shape of the plot and the pole-zero structure of the rational z-transform $R(z)$. You will likely have to do some direct numerical calculation of $R(z)$ for specific values of z in order to justify why each plot looks the way it does.

For this problem you should submit the two Nyquist plots and the detailed description associated to each plot.