EECS 120 **Fall 2004**

Homework 2 : Due by 8 p.m. on Monday September 20

1. Parts (b), (d), and (e) of Problem 2.22 on pp. 141 -142 of OWN.
2. (a) Parts (e) and (f) of Problem 2.28 on pg. 144 of OWN.
   (b) Parts (e) and (f) of Problem 2.29 on pp. 144 -145 of OWN.
3. Problem 2.48 on pg. 153 of OWN.
4. Problem 2.53 on pp. 155 -156 of OWN. For part (c) of this problem, you need only do subparts (i) and (v).
5. (a) Consider the constant-coefficient difference equation

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k] \ , \tag{1}$$

where we assume that $a_0 \neq 0$. See equation (2.113) on page 121 of OWN. This equation does not completely specify the output $y[n]$ in terms of the input $x[n]$. We focus on the condition of initial rest at time n_0, i.e. we assume that we consider only inputs such that $x[n] = 0$ for $n < n_0$, and that for such inputs we have $y[n] = 0$ for $n < n_0$. Since we can rewrite the above equation as

$$y[n] = \frac{1}{a_0}\left[\sum_{k=0}^{M} b_k x[n-k] - \sum_{k=1}^{N} a_k y[n-k]\right] \ ,$$

see equation (2.115) on pg. 122 of OWN, we now have, for each input $x[n]$ satisfying $x[n] = 0$ for $n < n_0$, a uniquely defined output $y[n]$, which satisfies $y[n] = 0$ for $n < n_0$. This system is causal, linear, and time invariant (check this for yourself !)

Let a be a $1 \times (N+1)$ array in MATLAB whose i-th entry is the coefficient a_{i-1}, $1 \leq i \leq N+1$. Let b be a $1 \times (M+1)$ array in MATLAB whose i-th entry is the coefficient b_{i-1}, $1 \leq i \leq M+1$. Let x be a $1 \times T$ array in MATLAB whose i-th entry is $x[n_0 + i - 1]$, $1 \leq i \leq T$. The MATLAB function **filter** computes the solution to the constant coefficient difference equation (1) in the condition of initial rest at n_0 for any given input, over the interval of time that the input is specified. Namely, $y = filter(b, a, x)$ returns a $1 \times T$ MATLAB array y whose i-th entry is $y[n_0 + i - 1]$, where $y[n]$ is the solution to (1) with input $x[n]$ with the condition of initial rest at n_0.

Note that even if $x[n] = 0$ for $n \geq n_0 + T$, we would in general have $y[n] \neq 0$ for some values $n \geq n_0 + T$ (and possibly even for all such n). Convince yourself of this ! In particular, read Sec 2.4.2 of OWN and understand the difference between finite impulse response (FIR) systems and infinite impulse response (IIR) system. Thus, the MATLAB function **filter** does not "completely" give the output. It only gives the output over the interval of time on which the input is specified.

Use this MATLAB function to evaluate the output $y[n]$ corresponding to the input $x[n]$ for the following examples. In each case, provide a **stem** plot of $y[n]$ versus n *with the time axis properly labelled.*

i. Consider the constant coefficient difference equation

$$y[n] = \frac{1}{3}x[n] + x[n-1] - \frac{1}{2}x[n-2] \ . \tag{2}$$

in the condition of initial rest at time 0. Suppose the input to this system is $u[n]$. Find the output of the system over the duration of time $0 \leq n \leq 100$, using the MATLAB function **filter**. Draw a **stem** plot of $y[n]$ over $0 \leq n \leq 100$ *with the time axis properly labelled.*

ii. For the difference equation in (2) with the condition of initial rest at time 0 find the output over the interval $0 \leq n \leq 100$ when the input is

$$x[n] = \left[\sin(\frac{\pi n}{10}) + (-1)^n\right] u[n] \ .$$

Draw a **stem** plot of $y[n]$ over $0 \leq n \leq 100$ *with the time axis properly labelled.*

Do NOT submit any M-files. Simply submit a plot for $y[n]$ in each of the two cases.

(b) Consider the linear constant-coefficient differential equation

$$\sum_{k=0}^{N} a_k \frac{d^k}{dt^k} y(t) = \sum_{k=0}^{M} b_k \frac{d^k}{dt^k} x(t) \ , \tag{3}$$

where we assume that $a_N \neq 0$. This equation does not completely specify the output $y(t)$ in terms of the input $x(t)$. See equation (2.109) on pg. 120 of OWN. We focus on the condition of initial rest at time t_0, i.e. we assume that we consider only inputs such that $x(t) = 0$ for $t < t_0$, and that for such inputs we have $y(t) = 0$ for $t < t_0$. The output $y(t)$ solving the differential equation (3) for any such input $x(t)$ can be uniquely determined by solving the equation with the initial conditions

$$y(t_0) = \frac{d}{dt} y(t_0) = \ldots = \frac{d^{N-1}}{dt^{N-1}} y(t_0) = 0 \ .$$

This then defines a causal linear time invariant system. Read section 2.4.1 of the text and convince yourself of this !

Let a be a $1 \times (N+1)$ array in MATLAB whose i-th entry is the coefficient a_{N-i+1}, $1 \leq i \leq N+1$. *Notice the difference from the discrete time case !* Let b be a $1 \times (M+1)$ array in MATLAB whose i-th entry is the coefficient b_{M-i+1}, $1 \leq i \leq M+1$. Let t be a $1 \times T$ MATLAB array with evenly spaced nonnegative entries. The spacing in t can be arbitrary, for instance, the MATLAB command

$$t = [2 : 0.01 : 10];$$

will define a 1×801 array of evenly spaced nonnegative entries with spacing 0.01.

The MATLAB function **impulse** plots the impulse response of the causal LTI system defined by the differential equation (3) with the condition of initial rest at time 0. Namely, the command $impulse(b, a, t)$ returns a plot of the impulse response over the duration of time specified by t.

The MATLAB function **step** plots the unit step response of the causal LTI system defined by the differential equation (3) with the condition of initial rest at time 0. Namely, the command $step(b, a, t)$ returns a plot of the unit step response over the duration of time specified by t.

i. Plot the impulse response of the causal LTI system defined by the linear constant-coefficient differential equation

$$\frac{d^2}{dt^2}y(t) + 2\frac{d}{dt}y(t) + \frac{1}{2}y(t) = x(t)$$

with the condition of initial rest at time 0, over the duration of time $[0, 10]$ with an even spacing of 0.01.

ii. Plot the unit step response of the same causal LTI system system over the same duration of time, with the same spacing.

Do NOT submit any M-files. Simply submit a plot in each of the two cases.