1. (d) \( H(s) = \frac{s + 2}{s^2 + 7s + 12}, \quad -4 < \text{Re}\{s\} < -3, \) where \( s = \sigma + j\omega \)

Doing partial fraction expansion we get:

\[ H(s) = \frac{A}{s + 4} + \frac{B}{s + 3}, \]

solving for A, B we get: \( A = 2, \quad B = -1 \)

\[ H(s) = \frac{2}{s + 4} + \frac{-1}{s + 3} \]

\[ h(t) = 2e^{-4t}u(t) + e^{-3t}u(-t) \]

(e) \( H(s) = \frac{s + 1}{s^2 + 5s + 6}, \quad -3 < \text{Re}\{s\} < -2, \) where \( s = \sigma + j\omega \)

Doing partial fraction expansion we get:

\[ H(s) = \frac{A}{s + 3} + \frac{B}{s + 2}, \]

solving for A, B we get: \( A = 2, \quad B = -1 \)

\[ H(s) = \frac{2}{s + 3} + \frac{-1}{s + 2} \]

\[ h(t) = 2e^{-3t}u(t) + e^{-2t}u(-t) \]

2.

1. \( x(t)e^{-3t} \) is absolutely integrable.

We know that \( X(s) = \int_{t=-\infty}^{\infty} x(t)e^{-\sigma t}e^{-j\omega t} dt \)

If \( x(t)e^{-3t} \) is absolutely integrable this implies that \( \sigma = 3 \) is in the ROC.

Hence the ROC is:
(TL) \( \text{Re}(s) > 2 \) (TR) \( \text{Re}(s) > -2 \) (BL) \( \text{Re}(s) > 2 \) (BR) \( s = C \), (C is whole complex plane)
2. \( x(t) * (e^{-t}u(t)) \) is absolutely integrable.

Laplace Transform of \( e^{-t}u(t) \) is \( \frac{1}{s+1}, \ -1 < \Re{s} \)

Let the ROC of the convolution above be denoted R. We know that R must contain \( \Re{s} = 0 \) since we are given that the convolution is absolutely integrable. Let the ROC of \( x(t) \) be denoted \( R_x \). So we have that \( \{R_x \cap \{-1 < \Re{s}\}\} \) must contain \( \Re{s} = 0 \)

Then the constraint is that R must contain the line \( \Re{s} = 0 \).

Hence the ROC is:
- (TL) \(-2 > \Re{s} > 2\)
- (TR) \(\Re{s} > -2\)
- (BL) \(\Re{s} < 2\)
- (BR) \(s = C\) (C is whole complex plane)

3. \( x(t) = 0, \ t > 1. \)

\( x(t) \) is left sided. This implies that if \( \Re{s} = \sigma \) is in the ROC, then all values of \( s \) for which \( \Re{s} < \sigma \) will also be in the ROC.

Hence the ROC is:
- (TL) \(\Re{s} < -2\)
- (TR) \(\Re{s} < -2\)
- (BL) \(\Re{s} < 2\)
- (BR) \(s = C\) (C is whole complex plane)

4. \( x(t) = 0, \ t < -1. \)

\( x(t) \) is right sided. This implies that if \( \Re{s} = \sigma \) is in the ROC, then all values of \( s \) for which \( \Re{s} > \sigma \) will also be in the ROC.

Hence the ROC is:
- (TL) \(\Re{s} > 2\)
- (TR) \(\Re{s} > -2\)
- (BL) \(\Re{s} > 2\)
- (BR) \(s = C\) (C is whole complex plane)

3. 

(a)
4.  

(a) \( H_1(s) = \frac{s + 1}{s^2 + 5s + 6} \)

\[
\frac{d^2 y(t)}{dt^2} + 5 \frac{dy(t)}{dt} + 6y(t) = \frac{dx(t)}{dt} + x(t)
\]
(b) \( H_2(s) = \frac{s^2 - 5s + 6}{s^2 + 7s + 10} \)

\[
\frac{d^2 y(t)}{dt^2} + 7 \frac{dy(t)}{dt} + 10y(t) = \frac{d^2 x(t)}{dt^2} - 5 \frac{dx(t)}{dt} + 6x(t)
\]
(e) \( H_3(s) = \frac{s}{(s + 2)^2} \)

\[
\frac{d^2 y(t)}{dt^2} + 4 \frac{dy(t)}{dt} + 4 y(t) = \frac{dx(t)}{dt}
\]

5. (a)

Possible ROCs:
Possible ROCs: