EECS 120 **Fall 2004**

Homework 9 Solutions

(1) Problem 9.35 of OWN:

(a) Let the signal at the bottom node of the block diagram be denoted by $e(t)$. Then we have the following relations

$$\begin{aligned} x(t) - 2\frac{de(t)}{dt} - e(t) &= \frac{d^2e(t)}{dt^2} \\ \frac{d^2e(t)}{dt^2} - \frac{de(t)}{dt} - 6e(t) &= y(t) \end{aligned}$$

Writing the above equations in Laplace domain we have

$$\begin{aligned} X(s) - 2sE(s) - E(s) &= s^2E(s) \\ s^2E(s) - sE(s) - 6E(s) &= Y(s) \end{aligned}$$

Eliminating the auxiliary signal $e(t)$, we get

$$\begin{aligned} H(s) = \frac{Y(s)}{X(s)} &= \frac{s^2 - s - 6}{s^2 + 2s + 1} \\ \Rightarrow s^2Y(s) + 2sY(s) + Y(s) &= s^2X(s) - sX(s) - 6X(s) \end{aligned}$$

Hence we get the following differential equation,

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) = \frac{d^2x(t)}{dt^2} - \frac{dx(t)}{dt} - 6x(t)$$

(b) From the system function $H(s)$ found in the previous part we see that the poles of the system are at $s = -1$. Since the system is given to be causal, and the right most pole of the system is left of the imaginary axis, the system is stable.

(2) Problem 6.29 of OWN:

(i) Fig. 1 sketches the bode plot of the given system function. From the magnitude plot it is clear that high frequency terms are attenuated and hence it is a lowpass filter. Also, from the phase plot we can see that this system has a phase lead for $1/10 < \omega < 10$ and a phase lag elsewhere.

(iii) Fig. 2 sketches the bode plots for this system. Here, as seen from the magnitude plot, the high frequency terms are amplified and hence it is a

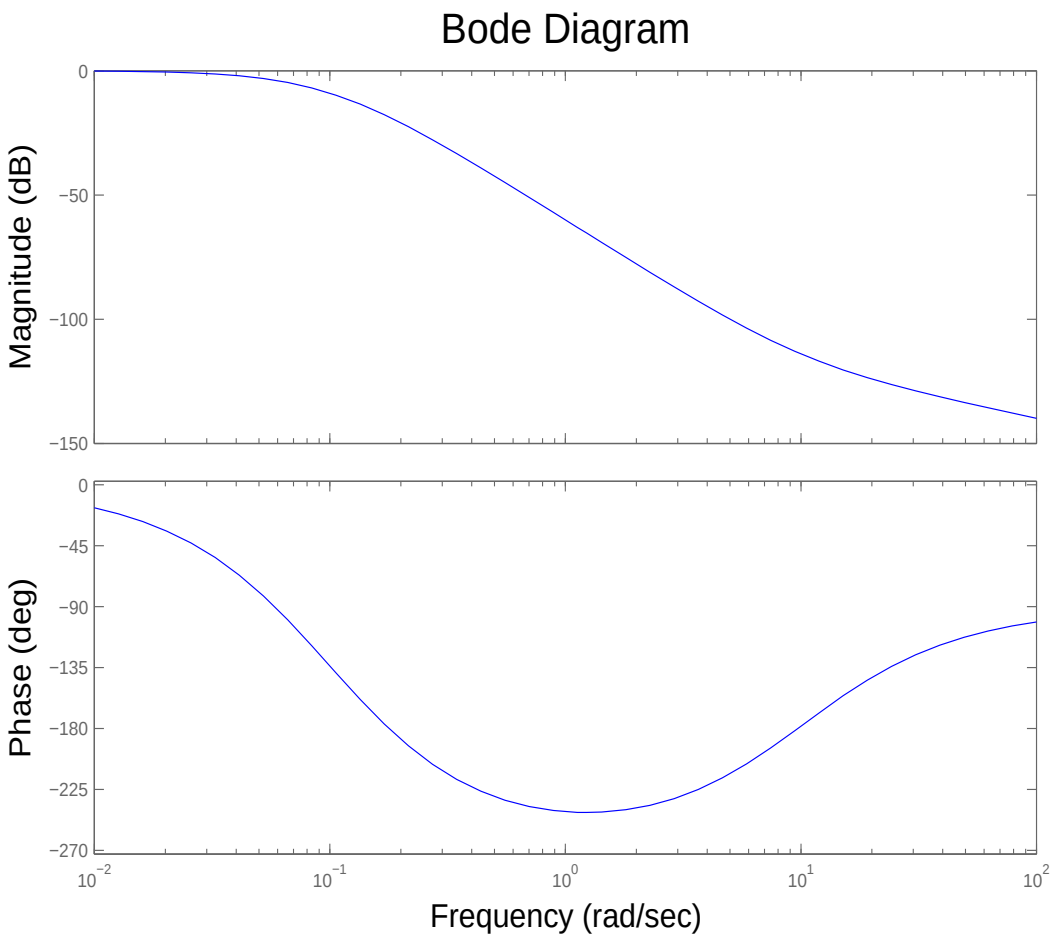

Figure 1: Bode plots for system function in part 2(i).

high pass filter. Also, from the phase plot, we can conclude that the phase of the system leads for $\omega < 10$ and lags for $\omega > 10$.

(3) Problem 9.50 of OWN:

(a) **False**. This statement is true only if in addition we know that the system is causal. For example, consider and system with impulse response, $h(t) = e^t u(-t)$. It is easy to verify that $h(t)$ is absolutely integrable and hence the system is stable. But, the pole of this system is at $s = 1$. Note that this system is an anti-causal system.

(b) **True**. Let $s(t)$ be the step response of the system. Causality of the system implies that $s(t) = 0$ for $t < 0$. Since the Laplace transform of $u(t)$

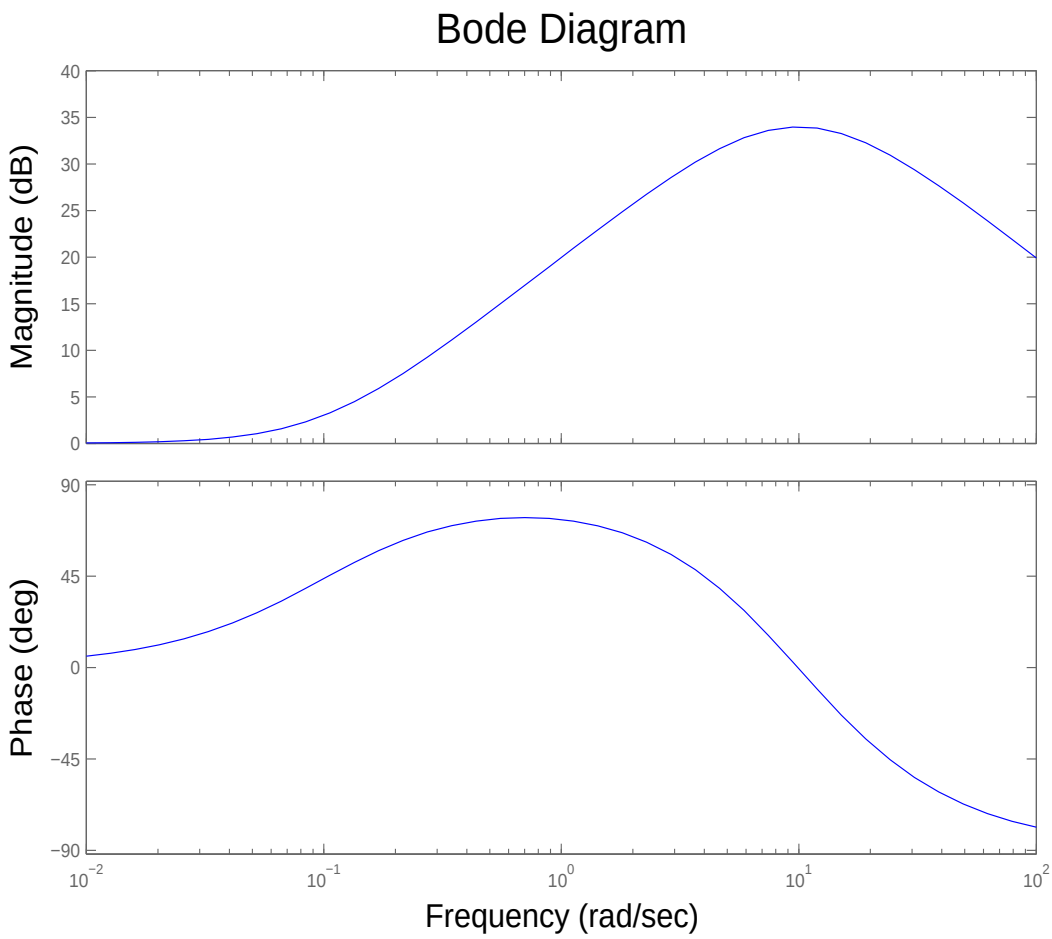

Figure 2: Bode plots for system function in part 2(c).

is $\frac{1}{s}$, we have $S(s) = \frac{H(s)}{s}$. Therefore, by the initial value theorem,

$$\begin{aligned} s(0^+) &= \lim_{s\to\infty} sS(s) \\ &= \lim_{s\to\infty} H(s) \\ &= 0 \end{aligned}$$

where the last equation follows from the fact that $H(s)$ has more poles than zeros. Thus, we have shown that $s(t)$ is continuous at 0.

(c) **False**. The unit step has Laplace transform 1/s with ROC $\Re e\{s\} > 0$. For the step response of the system to make sense we have to assume that the ROC of the system function has a non-empty intersection with $\Re e\{s\} > 0$, so we will assume this. Since the system function has more poles than zeros,

the impulse response is a sum of causal and/or anticausal exponentials. The assumption that the ROC of the system function has a non-empty intersection with $\Re e\{s\} > 0$, is equivalent to the assumption in the time domain that the convolution of the unit step with the impulse response of the system is well defined for all time. Now, looking at the time domain, we see that this convolution must be a continuous function of its argument. Therefore, the step response must be continuous at $t = 0$.

(d) **False**. A stable and causal system must have all its poles in the left half plane. The zeros need not lie in the left half of the s-plane. Consider the following example

$$\begin{aligned} H(s) &= \frac{s-1}{(s+1)(s+2)} \\ &= \frac{3}{s+2} - \frac{2}{s+1} \\ \Rightarrow h(t) &= [3e^{-2t} - 2e^{-t}]u(t) \end{aligned}$$

In this example, the system is stable and causal inspite of having a zero at $s = 1$ (i.e., in the right half place). Hence the given statement is false.

(5) Matlab problem:

(a) Since the poles of this system are in the left half plane (at $s = -1 - \pm\sqrt(0.5)$), the ROC for this causal LTI system will include the imaginary axis and hence the system is stable. Fig. 3 shows the bode plots for this system. Fig. 4 shows the straight line approximation to the magnitude Bode plot and Fig. 5 shows the straight line approximation to the phase Bode plot.

(b) The poles for this system are at $s = \pm 1$. Therefore, the ROC for this causal LTI system will be $\Re e\{s\} > 1$, and hence will not include the imaginary axis. Therefore this system will not be stable. The system function for which this system will be stable is given by

$$\begin{aligned} H(s) &= \frac{1}{s^2-1} \qquad \text{ROC: } -1 < \Re e\{s\} < 1 \\ &= \frac{1}{2}\left[\frac{1}{s-1} - \frac{1}{s+1}\right] \qquad \text{ROC: } -1 < \Re e\{s\} < 1 \end{aligned}$$

Taking the inverse laplace transform for the ROC as mentioned above, we get

$$h(t) = \frac{1}{2}\left[-e^{t}u(-t) - e^{-t}u(t)\right]$$

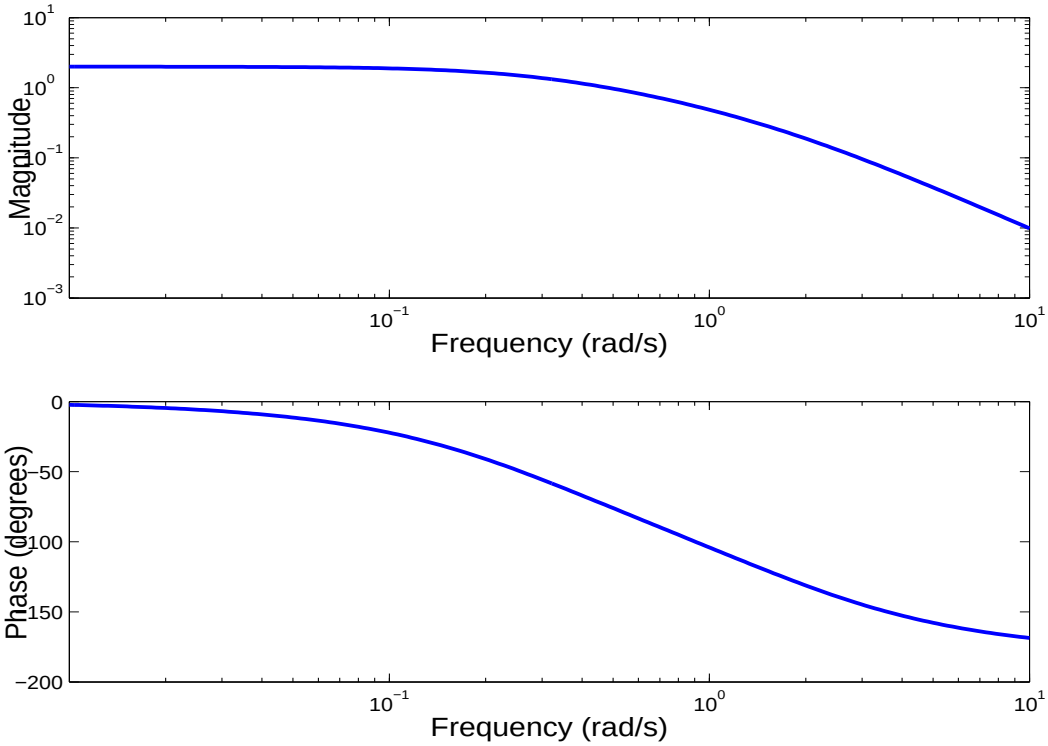

Figure 3: Bode plots for system function in part 5(a).

(c) The poles of this system are on the imaginary axis and hence any ROC cannot contain the imaginary axis. Therefore this system is not stable. For, the same reason there does not exist any LTI system for which it makes sense to call $H(j\omega)$, the frequency response of the system.

(d) Since all the roots of this system are in the left half plane (at $s = -1, -3, -1.5 \pm 2j$), a causal LTI system represented by this differential equation will be stable. Fig. 6 shows the bode plots for this system. Note, that the discontinuity in the phase plot is because of a Matlab effect in which it assumes that the phase $-\pi$ is different from π. Fig. 7 shows the straight line approximation to the magnitude plot and Fig. 8 shows the straight line approximation to the phase plot.

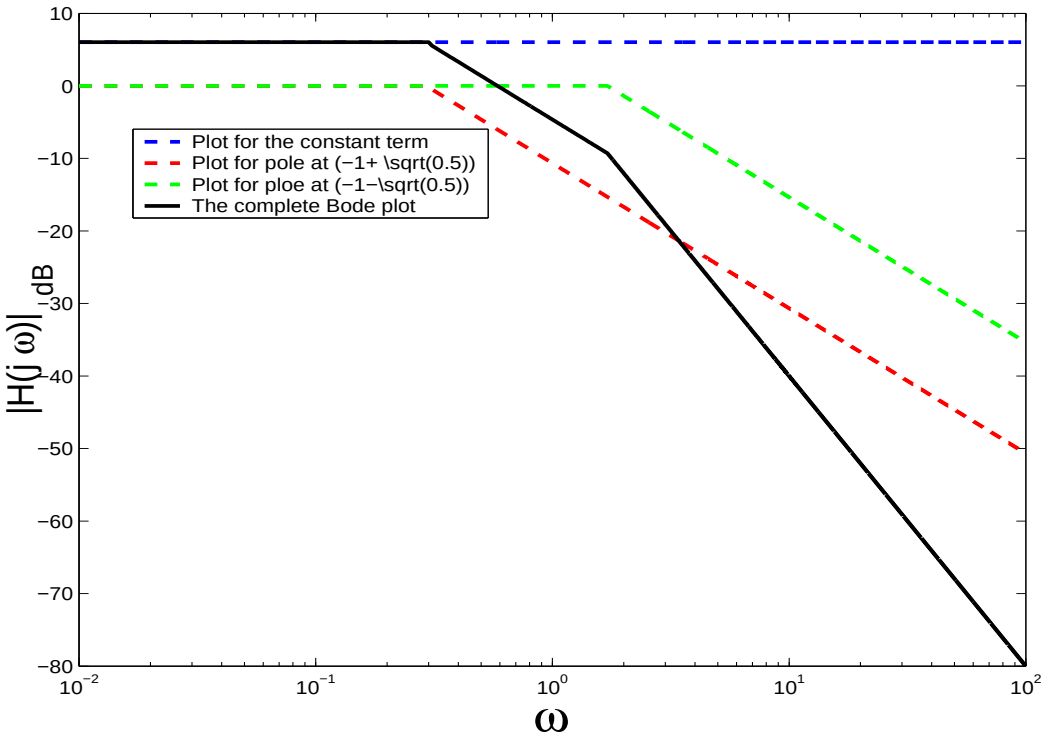

Figure 4: Straight line approximation of the Magnitude plot for the system function in part 5(a). Here the dashed line plots are magnitude plots for individual poles and the solid line plot is the magnitude plot obtained by summing the dashed line plots.

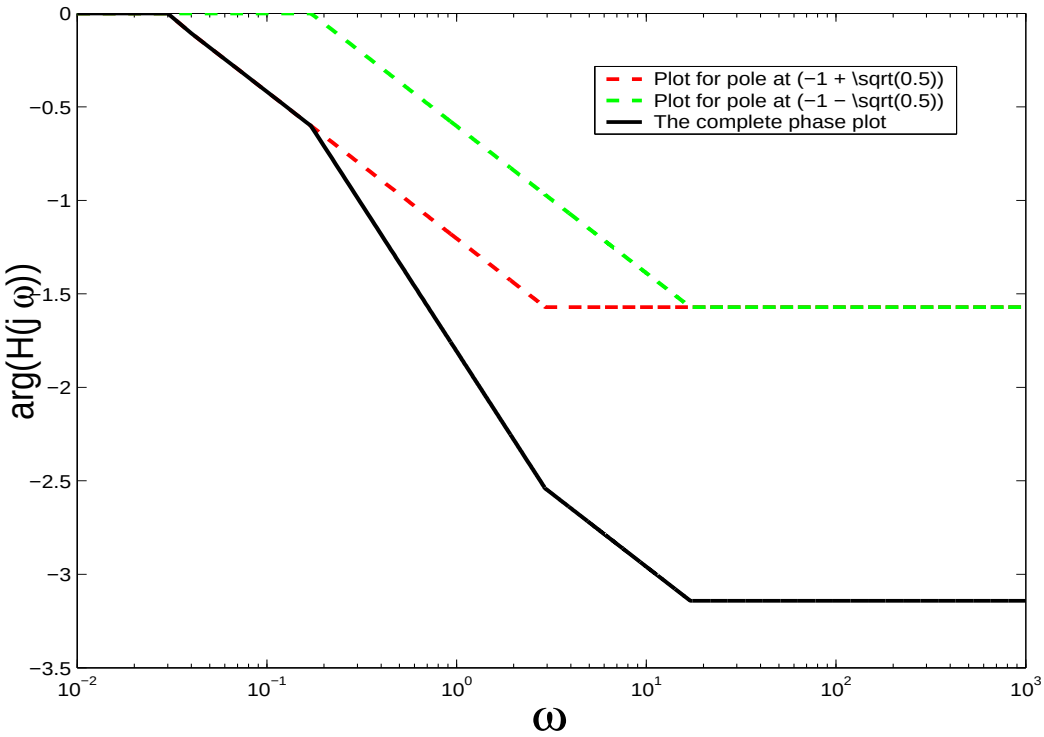

Figure 5: Straight line approximation of the phase plot for the system function in part 5(a). Here the dashed line plots are magnitude plots for individual poles and the solid line plot is the magnitude plot obtained by summing the dashed line plots.

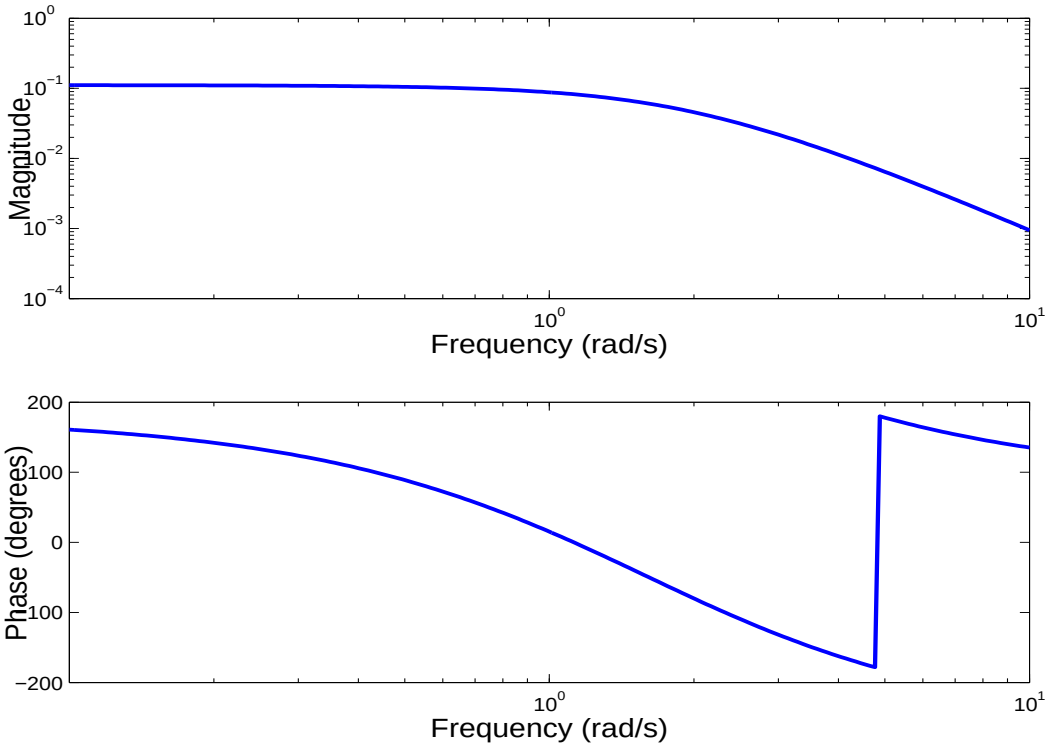

Figure 6: Bode plots for system function in part 5(d).

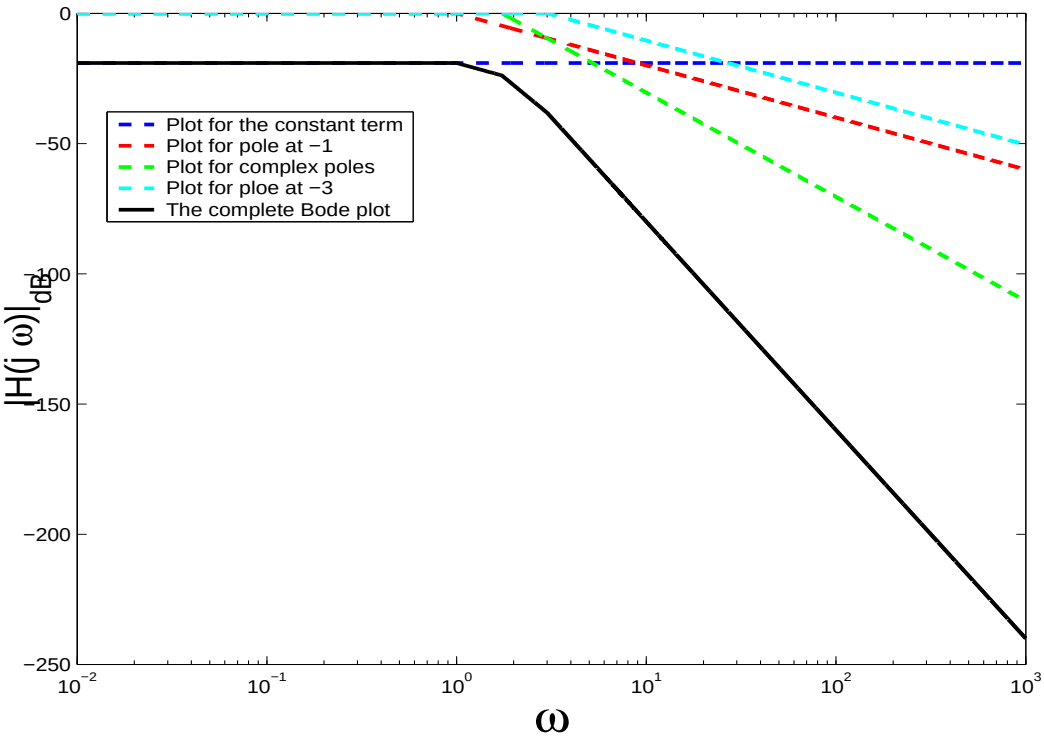

Figure 7: Straight line approximation of the magnitude plot for the system function in part 5(d). Here the dashed line plots are magnitude plots for individual poles and the solid line plot is the magnitude plot obtained by summing the dashed line plots.

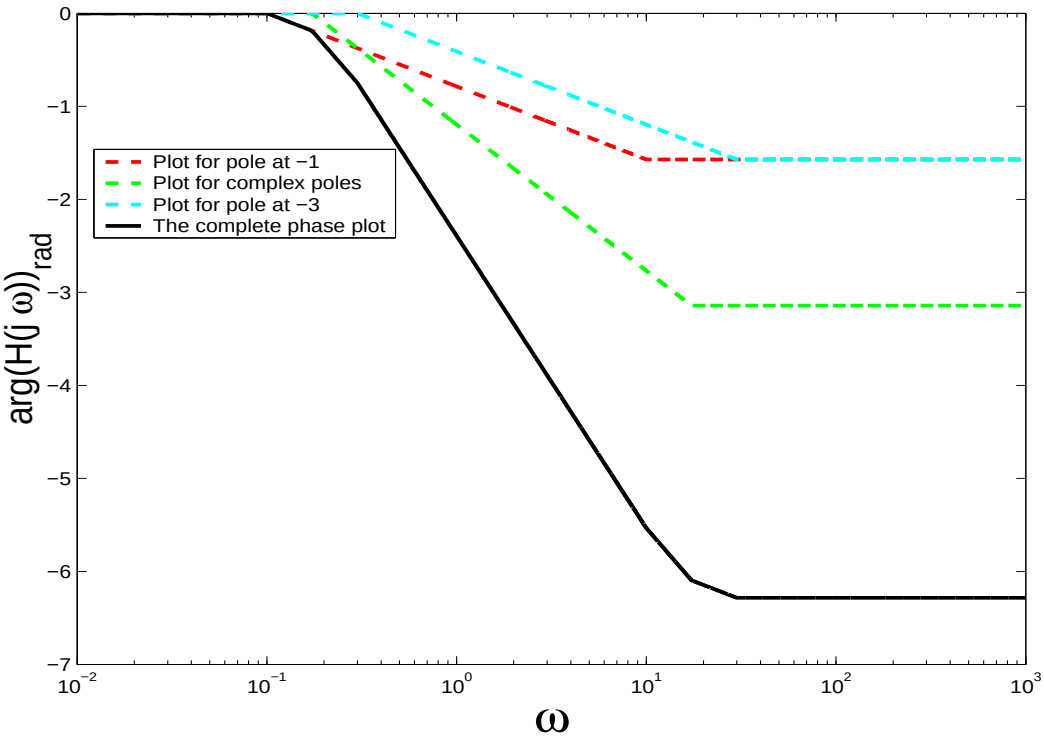

Figure 8: Straight line approximation of the phase plot for the system function in part 5(d). Here the dashed line plots are magnitude plots for individual poles and the solid line plot is the magnitude plot obtained by summing the dashed line plots.