EECS 120 Signals & Systems
Gastpar

University of California, Berkeley: Fall 2005
October 3, 2005

Practice Midterm 1

Problem 1 *(Short questions.)*

Each of the following is either true or false. If you believe it is true, give a brief argument. If you believe it is false, you can give a brief argument or a counterexample.

(i) The signal $s(t) = \sin(t/1000)$ is a power signal.

(ii) If the system H_1 is linear and the system H_2 is also linear, then the system H defined as $y(t) = H\{x(t)\} = H_2\{H_1\{x(t)\}\}$ is also linear.

(iii) All continuous-time signals $s(t)$ can be expressed as

$$s(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(j\omega)e^{j\omega t}d\omega, \quad \text{where} \quad S(j\omega) = \int_{-\infty}^{\infty} s(t)e^{-j\omega t}dt. \tag{1}$$

(iv) A time-invariant memoryless causal discrete-time system is always linear.

(v) The following is a Fourier transform pair:

$$x(t) = \left|\frac{\sin(t)}{\sqrt{|t|^3}}\right| \quad \stackrel{FT}{\longleftrightarrow} \quad X(j\omega) = \begin{cases} \sqrt{|\omega|^3}, & |\omega| \leq 1 \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

Hint: Do not use too many equations.

Problem 2 *(Convolution and Fourier representations.)* *(20 Points)*

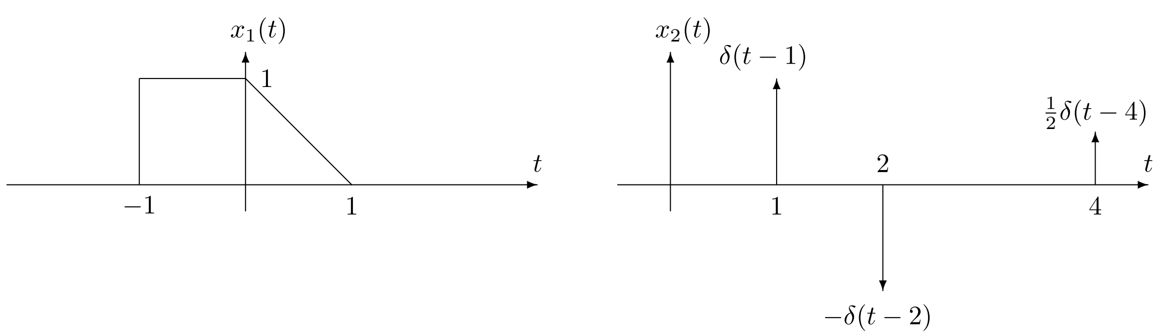
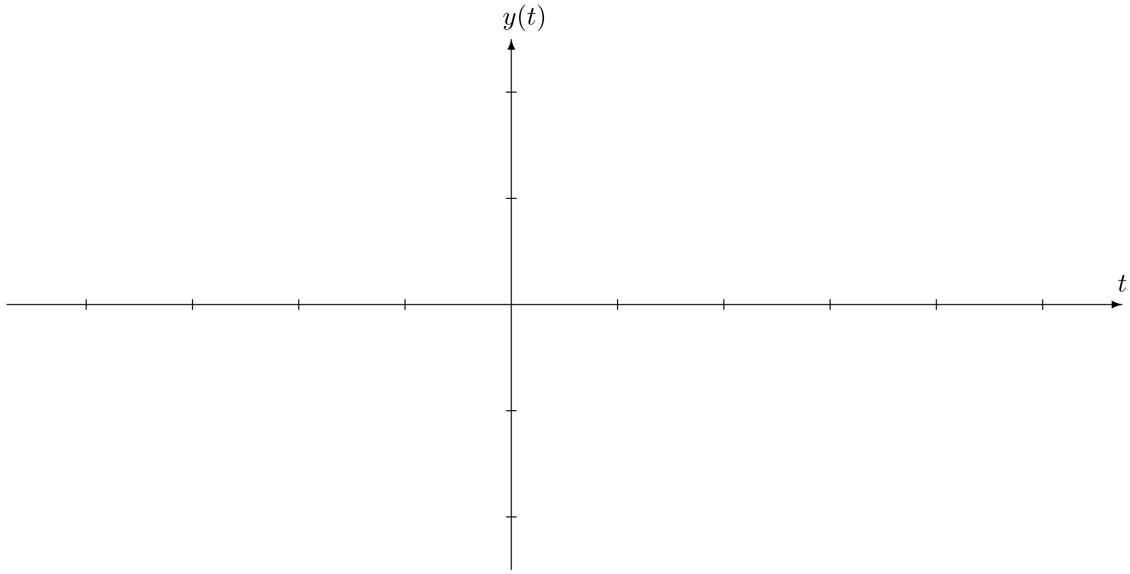


Figure 1: The signals for Problem 2, Part *(i)*.

(i) Consider the two signals shown in Figure 1. Note that the signal $x_2(t)$ consists of three impulse functions,

$$x_2(t) \quad = \quad \delta(t-1) - \delta(t-2) + \frac{1}{2}\delta(t-4). \tag{3}$$

Sketch the convolution of the two signals, that is, sketch the signal $y(t) = (x_1 * x_2)(t)$ in the figure below. Label the axes carefully.



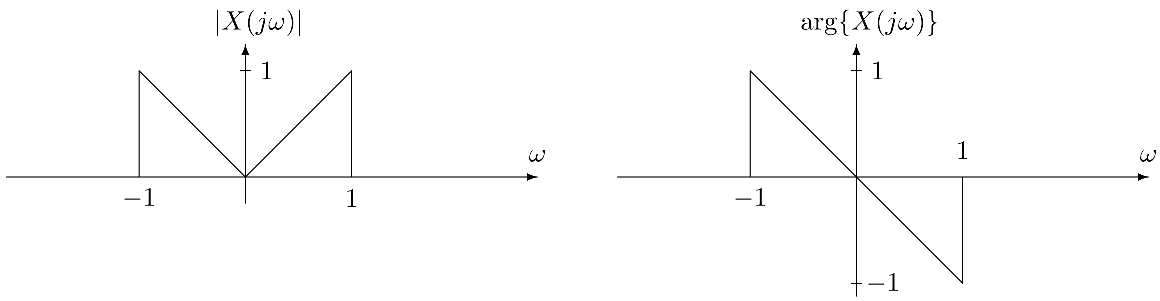


Figure 2: The spectrum for Problem 2, Part *(ii)*.

(ii) The spectrum of the continuous-time signal $x(t)$ is shown in Figure 2. Determine the signal $x(t)$.

Problem 3 *(Linear time-invariant system.)*

A linear time-invariant system with input $x(t)$ and output $y(t)$ satisfies

$$a^2 y(t) + 2a\frac{dy(t)}{dt} + \frac{d^2y(t)}{dt^2} \quad = \quad x(t). \tag{4}$$

(a) Find the frequency response $H(j\omega)$ of the considered system.

(b) For $a = 1/2$, sketch the magnitude of the frequency response $H(j\omega)$. Is the system rather high-pass or rather low-pass? Justify your answer.

(c) For what values of a is the system stable? Justify your answer. *Remark.* If you cannot solve the math, don't worry. Just describe *clearly and concisely* how you would proceed, and you will get partial credit.

Problem 4 *(Filtering.)*

The signal $x(t)$ with spectrum $X(j\omega)$ as shown in Figure 3 is passed through a linear time-invariant

(LTI) system with impulse response

$$h(t) = 2\text{sinc}(2t), \tag{5}$$

where, as defined in class,

$$\text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}. \tag{6}$$

Denote the output of the system by $y(t)$. Calculate the error between $x(t)$ and $y(t)$, given by

$$\int_{-\infty}^{\infty} |x(t) - y(t)|^2 \, dt. \tag{7}$$

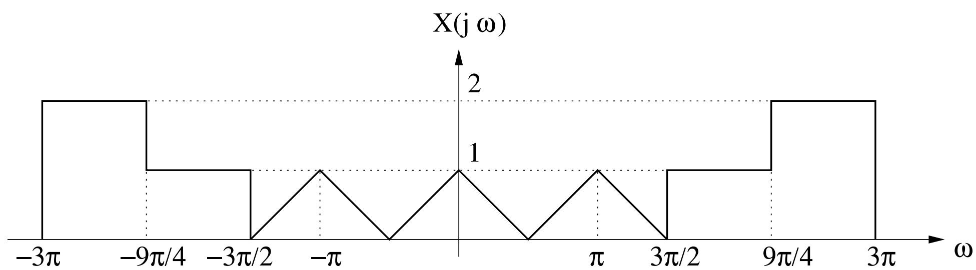


Figure 3: The spectrum of the signal $x(t)$.

Problem 5 *(Multiplication of Polynomials.)*

Multiplying two polynomials is a cumbersome task. For example, if

$$f(x) = 3x^2 + x + 2, \tag{8}$$

and

$$g(x) = x + 4, \tag{9}$$

then we find

$$h(x) = f(x)g(x) = 3x^3 + 13x^2 + 6x + 8. \tag{10}$$

However, we can define the coefficient signals for each polynomial, as follows:

$$f[n] = 3\delta[n-2] + \delta[n-1] + 2\delta[n], \text{ and} \tag{11}$$
$$g[n] = \delta[n-1] + 4\delta[n]. \tag{12}$$

Then, the coefficient signal $h[n]$ of the polynomial $h(x)$ is given by $h[n] = (f * g)[n]$.

(i) Sketch the coefficient signals $f[n]$ and $g[n]$ versus n. Evaluate the convolution and confirm that this indeed gives the coefficient signal $h[n]$ of the polynomial $h(x)$.

(ii) For the polynomials

$$a(x) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n x^n, \text{ and} \tag{13}$$

$$b(x) = \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n x^n, \tag{14}$$

find their product, i.e., find the polynomial $c(x) = a(x)b(x)$.

Problem 6 *(Fourier series.)*

(i) We are given the following information about a signal $x(t)$.

1. $x(t)$ has period 2π.
2. $x(t)$ has a Fourier series expansion with coefficients a_k.
3. $a_k = 0$ if $|k| > 2$.

Write down the Fourier Series expansion of $x(t)$, simplifying as much as possible.

(ii)

We are given more information about $x(t)$.

4. $x(t)$ is real and odd.
5. $x(t-\pi) = -x(t)$

Find a_0 and a_2.

(iii)

We are given another fact about $x(t)$.

6. $\frac{1}{2\pi}\int_0^{2\pi} |x(t)|^2 dt = 2$

Find a_1.

(iv)

Graph $x(t)$ for t in $[0, 2\pi]$. Carefully label the time axis and amplitudes.

Problem 7 *(System properties.)*

Consider the continuous time system whose output $y(t)$ for the input $x(t)$ is given by

$$y(t) = x(t - (\int_t^{t+1} x(u)du)^2).$$

Is the system:

- linear
- causal
- stable

In each case, give a brief argument or counterexample.