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$$H(s) = \frac{(s+4)(s^2+5s+6)}{(s+1)(s^2-2s+3)}$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s^3 + 9s^2 + 26s + 24}{s^3 - s^2 + s + 3}$$

$$s^3 Y(s) - s^2 Y(s) + s Y(s) + 3 \cdot Y(s)$$
$$= s^3 X(s) + 9s^2 X(s) + 26s\, X(s) + 24 \cdot X(s)$$

$$\frac{d^3 y(t)}{dt} - \frac{d^2 y(t)}{dt} + \frac{dy(t)}{dt} + 3 \cdot y(t) =$$
$$\frac{d^3 x(t)}{dt} + 9 \cdot \frac{d^2 x(t)}{dt} + 26 \cdot \frac{dx(t)}{dt} + 24 \cdot x(t)$$

$$H(s) = \frac{s^3 - s^2 + s + 3 + 10s^2 + 25s + 21}{s^3 - s^2 + s + 3}$$

$$= 1 + \frac{10s^2 + 25s + 21}{(s+1)(s^2-2s+3)}$$

$$= 1 + \frac{A}{s+1} + \frac{Bs + C}{s^2 - 2s + 3}$$

$$10s^2 + 25s + 21 = A(s^2 - 2s + 3) + (Bs + c)(s+1)$$
$$10s^2 + 25s + 21 = s^2(A + B) + s(-2A + B + C) + (3A + C)$$

$$\begin{cases} 10 = A + B \\ 25 = -2A + B + C \\ 21 = 3A + C \end{cases} \Rightarrow \begin{matrix} 10 = A + B \\ 4 = -5A + B \end{matrix} \Rightarrow 6 = 6A$$

$$\Downarrow$$

$$C = 18 \Leftarrow B = 9 \Leftarrow A = 1$$

$$H(s) = 1 + \frac{1}{s+1} + \frac{9(s+2)}{s^2 - 2s + 3}$$

$$= 1 + \frac{1}{s+1} + \frac{9(s+2)}{(s-1)^2 + (\sqrt{2})^2}$$

$$= 1 + \frac{1}{s+1} + 9 \cdot \frac{(s-1)}{(s-1)^2 + (\sqrt{2})^2} + \frac{27}{(s-1)^2 + (\sqrt{2})^2}$$

$$= 1 + \frac{1}{s+1} + 9 \cdot \frac{(s-1)}{(s-1)^2 + (\sqrt{2})^2} + \frac{27}{\sqrt{2}} \cdot \frac{\sqrt{2}}{(s-1)^2 + (\sqrt{2})^2}$$

$$h(t) = \delta(t) + e^{-t} u(t) + 9 \cdot e^{t} \cos(\sqrt{2}\, t)\, u(t) + \frac{27}{\sqrt{2}} e^{t} \cdot \sin(\sqrt{2}\, t) \cdot u(t)$$

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poles at: -1, $\frac{2 \pm \sqrt{4 - 4 \cdot 3 \cdot 1}}{2 \cdot 1} = 1 \pm j\sqrt{2}$

causal: ROC is a right half plane

$\Rightarrow$ not stable because ROC does not contain $j\omega$-axis

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inverse system: $G(s) = \frac{(s+1)(s^2 - 2s + 3)}{(s+4)(s^2 + 5s + 6)} = \frac{(s+1)(s^2 - 2s + 3)}{(s+4)(s+2)(s+3)}$

poles at $-4, -2, -3$

$\Rightarrow$ stable and causal inverse does exist
