EECS 120 Signals & Systems — University of California, Berkeley: Fall 2005
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Handout 1: System Properties

1 Memoryless

Remark: Memory is an important system property. Suppose you have to implement a certain system function on a chip. The amount of memory needed for this is crucial in the design of your chip. *Memoryless* is the extreme case where you need no memory at all to implement your function.

Definition (discrete-time): A system is *memoryless* if the current output value $y[n]$ can be determined from knowing *only* the value of the current input $x[n]$.

Examples: Memoryless: $y[n] = 2x[n]$, and $y[n] = \cos\left(\sqrt{x[n]}\right)$. Not memoryless: $y[n] = x[n+1] + x[n] + x[n-1]$, and $y[n] = x[n]e^{x[n-1]}$, but also $y[n] = nx[n]$.

Definition (continuous-time): A system is *memoryless* if the current output value $y(t)$ can be determined from knowing *only* the value of the current input $x(t)$.

Examples: Memoryless: $y(t) = 2x(t)$, and $y(t) = \cos\left(\sqrt{x(t)}\right)$. Not memoryless: $y(t) = x(t+\tau) + x(t)$, and $y(t) = x(t)e^{x(t-\tau)}$, but also $y(t) = tx(t)$.

Remark: We use the definition of memoryless as in Lee and Varaiya, p. 64. This is in contrast to the definition of memoryless in Oppenheim and Willsky with Nawab, in particular, the conclusion at the end of the example on top of p. 48.

2 Invertibility

Definition (discrete-time): A system H is *invertible* if there exists a system H^{inv} with the property that $H^{inv}\{H\{x[n]\}\} = x[n]$ for *any* signal $x[n]$.

Examples: Invertible: $y[n] = x[n] + 0.5x[n-1]$. Not invertible: $y[n] = (x[n])^2$.

Definition (continuous-time): A system H is *invertible* if there exists a system H^{inv} with the property that $H^{inv}\{H\{x(t)\}\} = x(t)$ for *any* signal $x(t)$.

Examples: Invertbile: $y(t) = 2^{-x(t)}$. Not invertible: $y(t) = (x(t))^4$.

3 Causality

Definition (discrete-time): A system is *causal* if the current output value $y[n]$ does *not* depend on future inputs $x[n+1], x[n+2], x[n+3], \ldots$.

Examples: Causal: $y[n] = x[n] + 2x[n-1]$, and $y[n] = x[n]e^{x[n-1]}$. Not causal: $y[n] = x[n] + 2x[n+1]$, and $y[n] = x[n]e^{x[n+1]}$.

Definition (continuous-time): A system is *causal* if the current output value $y(t)$ does *not* depend on future inputs $x(t+\tau)$, for all $\tau > 0$.

Examples: Causal: $y(t) = x(t)e^{x(t-\tau)}$. Not causal: $y(t) = x(t)e^{x(t+1)}$.

4 Stability

A discrete-time signal $x[n]$ is said to be bounded if *all* its values are smaller than infinity. More precisely, $x[n]$ is bounded if there exists a constant M_x such that $|x[n]| \leq M_x < \infty$, for all n.

Definition (discrete-time): A system is *stable* if, for *any* bounded input signal $x[n]$, the corresponding output signal $y[n]$ can be guaranteed to be bounded.

Examples: Stable: $y[n] = x[n] - 0.9y[n-1]$. Unstable: $y[n] = x[n] - 1.1y[n-1]$.

A continuous-time signal $x(t)$ is said to be bounded if *all* its values are smaller than infinity. More precisely, $x(t)$ is bounded if there exists a constant M_x such that $|x(t)| \leq M_x < \infty$, for all t.

Definition (continuous-time): A system is *stable* if, for *any* bounded input signal $x(t)$, the corresponding output signal $y(t)$ can be guaranteed to be bounded.

Examples: Stable: $y(t) = x(t) + x(t-\tau)$. Unstable: $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$ and $y(t) = \frac{dx(t)}{dt}$.