EECS 120 Signals & Systems — University of California, Berkeley: Fall 2005
Gastpar — August 29, 2005

Homework 1
Due: Thursday, September 8, 2005, at 3pm

Reading OWN Chapter 1 and Chapter 2, Sections 2.1 and 2.2.

Practice Problems *(Suggestions.)* OWN 1.3, 1.11, 1.17, 2.4, 2.11(a)

Problem 1 *(Complex numbers.)*

(a) With two real numbers, a and α, a complex number can be specified in its *polar form* as

$$z = ae^{j\alpha}, \tag{1}$$

where j is the imaginary unit (i.e., the root of -1). In terms of a and α, express the magnitude $|z|$, the phase $\arg(z)$, the real part $Re\{z\}$, the imaginary part $Im\{z\}$, and the complex conjugate z^*.

(b) Express the following complex numbers in Cartesian and polar coordinates (remember to consider the range of α), and draw them in the complex plane:

$$e^{j5\pi}, \frac{1}{3}e^{j\pi/4}, \left(\frac{1}{3}e^{j\pi/4}\right)^*, 2+j, \frac{1+2j}{3+j}, j^{-j}. \tag{2}$$

(c) Express $\frac{a+jb}{c+jd}$ in polar form.

(d) Consider the complex-valued function

$$H(j\omega) = \frac{1}{1-2j\omega}, \tag{3}$$

where ω is a real number. Sketch $|H(j\omega)|$ and $\arg(H(j\omega))$ versus ω for $-5 < \omega < 5$. *Remark:* Writing $H(j\omega)$ is standard engineering practice, even though strictly speaking, it would be more logical to write $H(\omega)$.

Problem 2 *(Elementary functions and their graphs.)*

(a) Consider the complex-valued signal $y(t) = e^{j2\pi t}$.

Express (as functions of t): $Re\{y(t)\}, Im\{y(t)\}, |y(t)|, \arg(y(t))$.

Sketch by hand, in one figure, for $-1 \le t \le 1$, the following functions: $y_1(t) = Re\{y(t)\}, y_2(t) = Im\{y(3t)\}, y_3(t) = Re\{y(2t+0.5)\}, y_4(t) = Re\{y(-2t+0.5)\}$. Clearly label the sketches and their zero-crossings.

(b) Determine the fundamental period of the signal $z(t) = \cos(\frac{\pi}{3}t) + \cos(\frac{2\pi}{5}t)$.

(c) Is $w(t) = \cos(\frac{\pi}{3}t) + \cos(t)$ periodic? If so, determine its fundamental period.

(d) Consider the following function:

$$x(t) = \begin{cases} 1+t, & -1 \le t \le 0, \\ 1-2t, & 0 \le t \le 1/2, \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

Sketch, in one figure, $x(t)$, $x(t/2-3)$, and $x(2-t)$. Carefully label both axes in the plot.

Problem 3 *(Properties of signals.)*

Categorize each of the following signals as an energy signal or a power signal, and find the energy or time-averaged power of the signal: (Please include your derivation of the result.)

(a) The continuous-time signal $x(t)$, defined by

$$x(t) = \begin{cases} 2e^{-3t}, & t \geq 0, \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

(b) The discrete-time signal $y[n]$, defined by

$$y[n] = \begin{cases} 3-n, & 0 \leq n < 3, \\ n-3, & 3 \leq n \leq 6, \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

(c) The continuous-time signal $z(t)$, defined for $-\infty < t < \infty$ by

$$z(t) = \sin(\frac{\pi}{2}t) + 2\cos(5\pi t). \tag{7}$$

Problem 4 *(Periodic discrete-time signals.)*

(a) Let $y_1[n] = \cos(\frac{\pi}{3}n)$ and $y_2[n] = \cos(\frac{3\pi}{5}n)$. What are their fundamental periods? Plot two periods of each signal in Matlab and clearly mark the end of each period.

(b) Determine the fundamental period of the signal $z[n] = y_1[n] + y_2[n]$. Plot two periods of $z[n]$ in Matlab and clearly mark the end of each period.

(c) Is $z[n] = \cos(2n)$ periodic? If so, what is its period?

Problem 5 *(Properties of systems.)*

Are the following systems linear? Are they time-invariant? In each case, give a short justification using the definitions of these properties.

(a) $\mathcal{H}\{x(t)\} = x(at+b)$

(b) $\mathcal{H}\{x(t)\} = x(at^2+b)$

(c) $\mathcal{H}\{x(t)\} = x(at)+b$

(d) $\mathcal{H}\{x(t)\} = \frac{d}{dt}x(t)$

Problem 6 *(Properties of systems.)*

For each of the following systems with input $x(t)$ and output $y(t)$, determine whether the system is (i) memoryless, (ii) stable, (iii) causal, (iv) linear, and (v) time invariant. In each case, give a short justification using the definitions of these properties.

(e) $y(t) = x(t)\frac{e^t+e^{-t}}{2}$

(f) $y(t) = \int_{-\infty}^{t/2} x(3\tau)d\tau$

(g) $y(t) = x(-t/2)$

(h) $y[n] = \sum_{k=-\infty}^{n} x[k+1]$

(i) $y[n] = x[n] \sum_{k=-\infty}^{\infty} \delta[n-3k]$

(j) $y[n] = \begin{cases} x[n], & n \text{ odd}, \\ 0, & \text{otherwise.} \end{cases}$

Problem 7 *(Convolution of step functions.)*

Recall that if an LTI system has impulse response $h(t)$ and input $x(t)$, the output is given by a convolution $y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$. Recall also that the unit step function is defined as

$$u(t) = \begin{cases} 1, & \text{if } t \geq 0, \\ 0, & \text{otherwise.} \end{cases} \tag{8}$$

For the following, find the output given the input $x(t)$ and the system impulse response $h(t)$ specified. Sketch (by hand) the input, the impulse response, and the output.

(a) $h(t) = u(t) - u(t-1)\,,\ x(t) = u(t)$

(b) $h(t) = u(t)\,,\ x(t) = u(t)$

(c) $h(t) = u(t) - u(t-1)\,,\ x(t) = u(t) - u(t-1)$

(d) Explain in words what one should expect when convolving step/pulses with steps/pulses.

Problem 8 *(Convolution via matlab.)*

Many homework sets will have a component in Matlab. For this problem, you will use Matlab to convolve exponentials. You can type 'help conv' in Matlab for help on using the function. For the following, plot $h(t)$ and $x(t)$ in the domain $[-2, 2]$ with .01 between points. Then, plot the resulting convolution. To create the time axis, you can use, for example, 't = [-2:.01:2]'. If the Matlab exercises in this homework require a lot of effort, you should go through a tutorial on Matlab. Please submit your Matlab code along with the plots.

(a) $h(t) = e^{-t}u(t)$ and $x(t) = u(t)$

(b) $h(t) = e^{-t/2}u(t)$ and $x(t) = e^{-t}u(t)$