EECS 120 Signals & Systems
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Review Session Problems

Problem 1 *System Properties*

Determine whether the following continuous time system is (i) memoryless, (ii) linear, (iii) time invariant, (iv) causal, (v) stable

$$y(t) = \int_{-\infty}^{2t} x(\tau) d\tau$$

Problem 2 *(Convolution and Fourier representations.)*

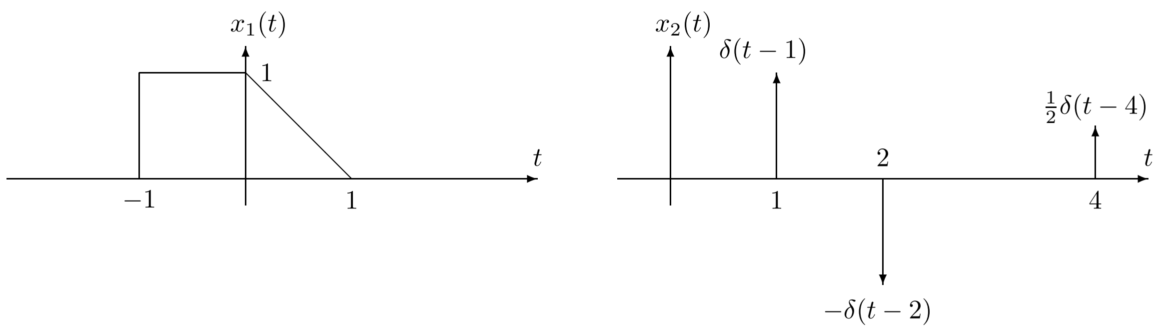


Figure 1: The signals for Problem 2, Part *(a)*.

(a) Consider the two signals shown in Figure 1. Note that the signal $x_2(t)$ consists of three impulse functions,

$$x_2(t) = \delta(t-1) - \delta(t-2) + \frac{1}{2}\delta(t-4) \quad (1)$$

Sketch $x_1(t) * x_2(t)$, the convolution of the two signals. Label the axes carefully.

(b) The spectrum of the continuous-time signal $x(t)$ is shown in Figure 2. Determine the signal $x(t)$.

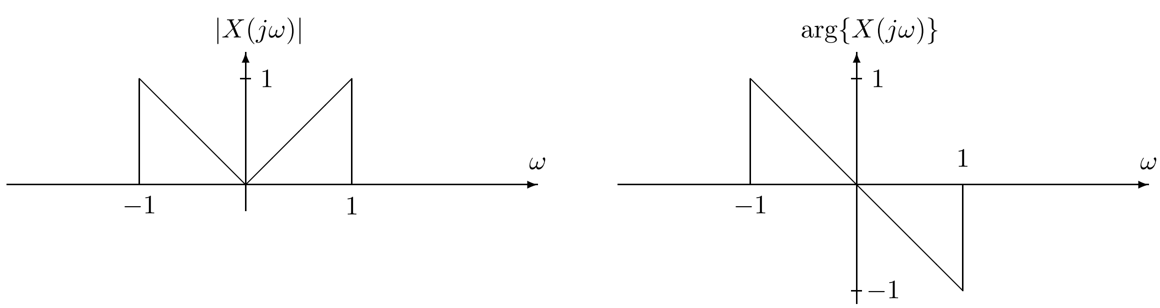


Figure 2: The spectrum for Problem 2, Part *(b)*.

Problem 3 *(Linear time-invariant system.)*

A linear time-invariant system with input $x(t)$ and output $y(t)$ satisfies

$$a^2 y(t) + 2a\frac{dy(t)}{dt} + \frac{d^2y(t)}{dt^2} \quad = \quad x(t). \tag{2}$$

(a) Find the frequency response $H(j\omega)$ of the considered system.

(b) For $a = 1/2$, sketch the magnitude of the frequency response $H(j\omega)$.

(c) Find the output $y(t)$ when the input is $\sin(2t)$.

(d) For what values of a is the system stable? Justify your answer.

Problem 4 *(Filtering.)*

The signal $x(t)$ with spectrum $X(j\omega)$ as shown in Figure 3 is passed through a linear time-invariant (LTI) system with impulse response

$$h(t) \quad = \quad 2\text{sinc}(2t), \tag{3}$$

where

$$\text{sinc}(t) \quad = \quad \frac{\sin(\pi t)}{\pi t}. \tag{4}$$

Denote the output of the system by $y(t)$. Calculate the error between $x(t)$ and $y(t)$, given by

$$\int_{-\infty}^{\infty} |x(t) - y(t)|^2 \, dt. \tag{5}$$

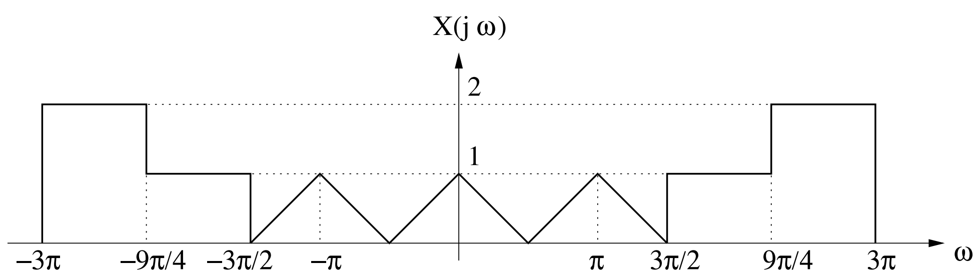


Figure 3: The spectrum of the signal $x(t)$.

Problem 5 *(DTFT.)*

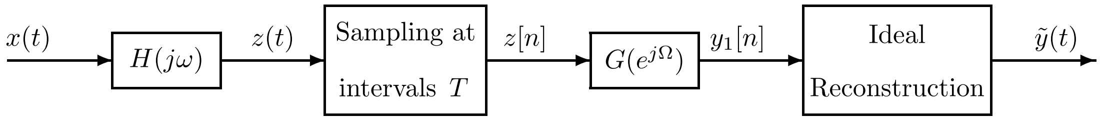
For the discrete-time signal $x[n]$ it is known that $X(e^{j\omega}) = 0$, for $|\omega| > \pi/4$. Determine the range of ω for which the DTFT of $y[n] = \cos(\frac{5\pi}{4}n)x[n]$ must be zero. *Hint:* Select an example spectrum $X(e^{j\omega})$ and sketch the resulting DTFT of $y[n]$.

Problem 6 *(Sampling.)*

We would like to construct a system D that implements a derivative, that is, for an input $x(t)$, the system should give an output $y(t)$ given by

$$y(t) \quad = \quad D\{x(t)\} = \frac{dx(t)}{dt}. \tag{6}$$

It is suggested to use the following system:



where

$$G(e^{j\Omega}) \quad = \quad j\frac{\Omega}{T}, \text{ for } |\Omega| \leq \pi. \tag{7}$$

This system does not exactly implement the desired system D. Instead, it produces an output $\tilde{y}(t)$ which is, in general, not equal to the desired output $y(t)$.

(a) Suppose that

$$H(j\omega) = \begin{cases} 1, & \text{for } |\omega| \leq \pi/T \\ 0 & \text{otherwise,} \end{cases} \tag{8}$$

- Determine and sketch the overall frequency response (magnitude and phase) of the system with input $x(t)$ and output $\tilde{y}(t)$.
- For the test signal $x(t)$ with Fourier transform

  $$X(j\omega) \quad = \quad e^{-|\omega|}, \tag{9}$$

  determine the error between the desired signal, $y(t)$, and the actual system output, $\tilde{y}(t)$, given by

  $$E \quad = \quad \int_{-\infty}^{\infty} |y(t) - \tilde{y}(t)|^2 dt, \tag{10}$$

  as a function of the sampling interval T. What happens as we increase the sampling frequency?

(b) Unfortunately, it is quite difficult to exactly implement ideal frequency filters like $H(j\omega)$ in Part *(a)*. As a simple model of this imperfection, suppose now that

$$H(j\omega) = \begin{cases} 1, & \text{for } |\omega| \leq \pi/T \\ \epsilon & \text{otherwise.} \end{cases} \tag{11}$$

For the same test signal as in Part *(a)*, that is, $X(j\omega) = e^{-|\omega|}$, determine the spectrum $Z_\delta(j\omega)$ of the sampled signal,

$$z_\delta(t) \quad = \quad z(t) \sum_{k=-\infty}^{\infty} \delta(t - kT). \tag{12}$$

- Start with a sketch of $Z_\delta(j\omega)$, carefully labeling the *frequency* axis.
- Which adverse effect corrupts the signal $z_\delta(t)$?
- Then, write out a formula for $Z_\delta(j\omega)$. The simpler your formula, the better.