EECS 120 Signals & Systems — University of California, Berkeley: Fall 2006
Ramchandran — November 16, 2006

Homework 11
Due: Wednesday, November 22, 2006, at 5pm
Homework 11 GSI: Mark Johnson

Remark Due to the Thanksgiving Holiday, this homework set is due on **Wednesday**, and is therefore shorter than usual.

Reading OWN Sections 9.4, 9.8 - 9.9, 11.1 - 11.3

Problem 1 *(Block Diagram Representations.)*

A causal LTI system has the block diagram representation shown in the following figure. Determine a differential equation relating the output signal $y(t)$ and the input signal $x(t)$

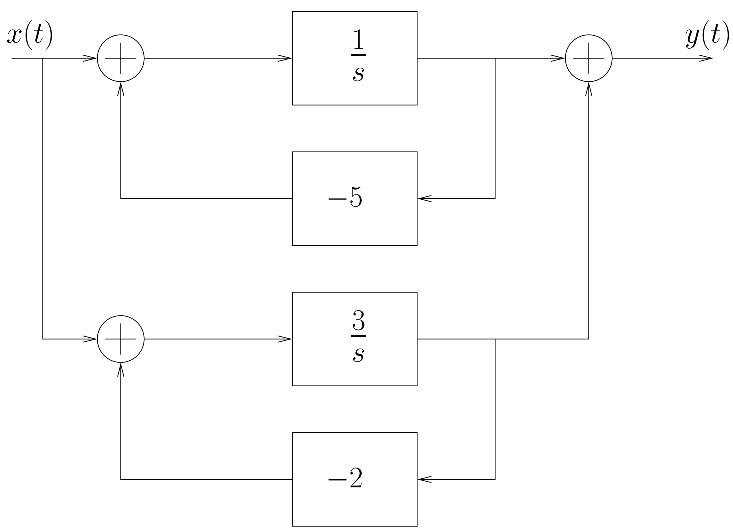

Figure 1: Block diagram for Problem 1

Problem 2 *(Unilateral Laplace Transform.)*

OWN Problem 9.65

Problem 3 *(Pole/Zero Plots)*

Match the pole/zero plots (a)-(e) with the corresponding magnitude responses (1)-(5). In each case, provide a brief justification. (For example: "must have two symmetric peaks, therefore can only be plot (x)".)

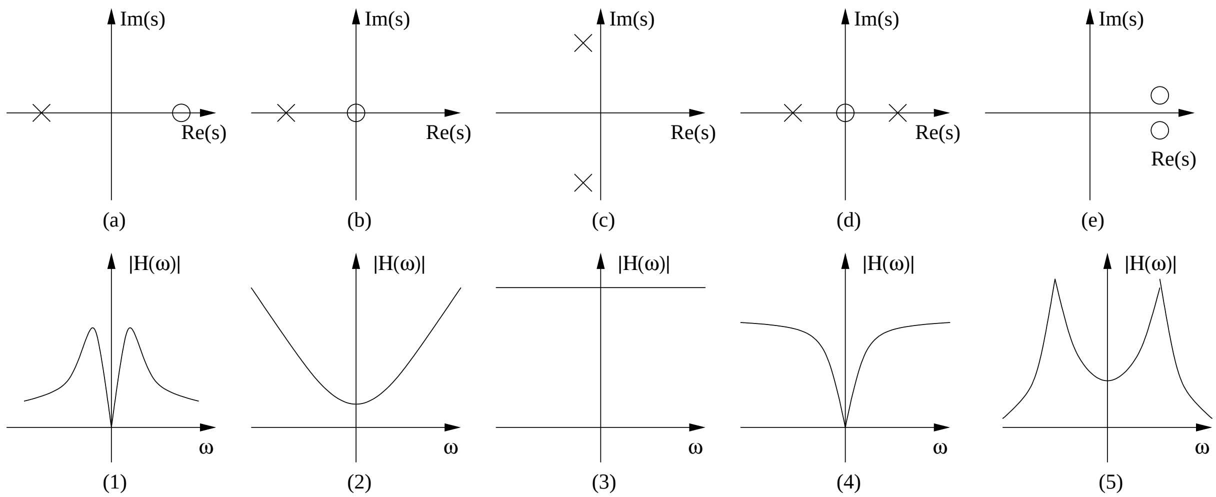

Figure 2: Matching of Pole/Zero Plots and Frequency response.

Problem 4 *(A simple feedback control system)*

One of the key applications of the Laplace transform is the control of feedback systems. Consider the following simple *causal* feedback system.

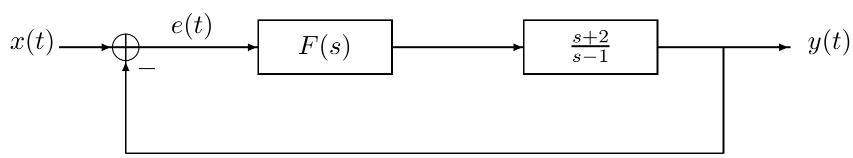

Figure 3: A simple feedback system

The second box, $\frac{s+2}{s-1}$, models an industrial plant. This system is unstable. The task of the engineer is to design the controller $F(s)$ in a clever way. The overall goal of the control is to make the error signal $e(t) = 0$.

(a) Show that the overall transfer function is

$$T(s) = \frac{Y(s)}{X(s)} = \frac{\frac{s+2}{s-1}F(s)}{1 + \frac{s+2}{s-1}F(s)}. \tag{1}$$

and that the Laplace transform of the error signal satisfies

$$E(s) = (1 - T(s))X(s). \tag{2}$$

(b) Suppose that $F(s) = K$, where K is a real number. Sketch the root locus and determine the range of K such that the overall system $T(s)$ is stable. (Recall that we assume the system to be *casual.*)

(c) Assume that K is chosen such that the overall system is stable. For $x(t) = u(t)$, what is the asymptotic value of the error $e(t)$ as $t \to \infty$?

Problem 5 *(Bode Plots)*

As we have seen in class, the Bode plot of a frequency response is simply the plot

$$|H(j\omega)|_{dB} \stackrel{def}{=} 20\log_{10}|H(j\omega)|. \tag{3}$$

(a) In this problem, we use Matlab to confirm Bode's approximation. Consider the system with transfer function

$$H(s) = \frac{1}{1+s/10}. \tag{4}$$

You may use the following Matlab code:

```
w = [ 0.1:0.1:1000 ];
semilogx(w, 20*log10(abs(1./(1 + j*w/10))));
grid;
```

Print out the resulting plot, and add (with a color pen) the approximation that we have seen in class.

Then repeat this exercise for the phase, using the approximation in Handout 3 and the following Matlab code:

```
w = [ 0.1:0.1:1000 ];
semilogx(w, angle(1./(1 + j*w/10)));
grid;
```

(b) Repeat Part *(a)* for the second-order system

$$H(s) = \frac{1}{1+s/20+(s/10)^2}. \tag{5}$$

For the *phase response*, use the approximation for very small ω and for very large ω that we derived in lecture on Thursday.

(c) By hand (without using Matlab), provide the Bode plot of the *magnitude* of the frequency response of the system with transfer function

$$H(s) = \frac{(s+1)(s+1000)}{(s+10)(s+100)}. \tag{6}$$

Describe the system behavior in words.

Hint: Confirm your result using Matlab, but be sure you know how to do it by hand — after all, that's the main point of Bode plots.