Homework 4
Due: Thursday, September 28, 2006, at 5pm
Homework 4 GSI: Mark Johnson

(*Submit your grades to ee120staff@gmail.com*)

Reading OWN Chapter 3-4 (Fourier Representations.)

Practice Problems *(Suggestions.)* OWN 4.11, 4.12, 4.19

Problem 1 *(Properties of the CTFS.)*

OWN Problem 3.46, all parts (details below). For Part (a), start out by showing (in 3 lines at most!) that

$$Z[k] = \frac{1}{T}\int_T z(t)e^{-jk\omega_0 t}dt \tag{1}$$

$$= \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} X[n]Y[m]\left(\frac{1}{T}\int_T e^{j(n+m-k)\omega_0 t}dt\right). \tag{2}$$

In this case, do not worry about shifting around infinite sums. Then, evaluate the integral to obtain

$$Z[k] = \sum_{n=-\infty}^{\infty} X[n]Y[k-n]. \tag{3}$$

Then, do Part (b), but **only** for Figure P3.46(a)

Finally, do Part (c).

Problem 2 *(Properties of the DTFS.)*

(a) OWN Problem 3.48, Part (a)

(b) OWN Problem 3.48, Part (e)

Problem 3 *(Properties of the DTFS.)*

(a) OWN Problem 3.48, Part (f)

(b) OWN Problem 3.48, Part (h)

Problem 4 *(FT.)*

(a) OWN Problem 4.22, Part (e)

(b) OWN Problem 4.23, Parts (a) and (b)

Problem 5 *(FT.)*

(a) OWN Problem 4.29, **only** for the signal $x_a(t)$ and $x_c(t)$

(b) OWN Problem 4.41

Problem 6 *(FT.)*

OWN 4.36 (all parts)

Problem 7 *(Frequency response of linear time-invariant systems.)*

Let a system be specified by a differential (or difference) equation. Then, its frequency response can be found easily, as you will establish in this homework problem.

(a) In the continuous-time case, suppose that the LTI system is specified by

$$\sum_{k=0}^{N} a_k \frac{d^k}{dt^k} y(t) \quad = \quad \sum_{m=0}^{M} b_m \frac{d^m}{dt^m} x(t). \tag{4}$$

Solve the differential equation for the input $x(t) = e^{j\omega t}$. As we have seen in class, if the input to a continuous-time LTI system is $x(t) = e^{j\omega t}$, then the output can be expressed as $y(t) = H(j\omega)e^{j\omega t}$. Plug this into the above differential equation to determine $H(j\omega)$. You will obtain an expression for $H(j\omega)$ in terms of the coefficients $a_k, k = 0, \ldots, N$ and $b_m, m = 0, \ldots, M$.

(b) In the discrete-time case, suppose that the LTI system is specified by

$$\sum_{k=0}^{N} a_k y[n-k] \quad = \quad \sum_{m=0}^{M} b_m x[n-m]. \tag{5}$$

Solve the differential equation for the input $x[n] = e^{j\omega n}$. As we have seen in class, if the input to a discrete-time LTI system is $x[n] = e^{j\omega n}$, then the output can be expressed as $y[n] = H(e^{j\omega})e^{j\omega n}$. Plug this into the above differential equation to determine $H(e^{j\omega})$. You will obtain an expression for $H(e^{j\omega})$ in terms of the coefficients $a_k, k = 0, \ldots, N$ and $b_m, m = 0, \ldots, M$.

Problem 8 *(Frequency response of linear time-invariant systems (continued).)*

(a) For the system specified by the differential equation, i.e.,

$$2\frac{dy(t)}{dt} + 6y(t) \quad = \quad x(t), \tag{6}$$

determine the frequency response $H(j\omega)$, and the corresponding impulse response $h(t)$. Then, find the output when the input is $x(t) = \sin(t/4)$. *Hint:* Write $\sin(t/4)$ in terms of functions of the form $e^{j\omega_0 t}$, and recall from class that for such inputs, the output is imply given by $y(t) = H(j\omega_0)x(t)$.

(b) Draw a block diagram (involving only unit time delays, additions, and multiplications by constant factors) of the discrete-time LTI system characterized by the following frequency response:

$$H(e^{j\omega}) \quad = \quad \frac{1 + 2e^{-j\omega} - 3e^{-2j\omega} + e^{-3j\omega}}{2 + 3e^{-j\omega} - 5e^{-2j\omega} + 7e^{-3j\omega} - 9e^{-4j\omega}}. \tag{7}$$

Try using as few delay elements as possible. (Recall that delay elements are memory, and hence somewhat expensive.) *Hint:* Determine the difference equation of the system.