EECS 120 Signals & Systems University of California, Berkeley: Fall 2006
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# Homework 5 Solutions

**Problem 1** *(CTFT.)*

*(a)* To find $A(j\omega)$, we use the multiplication property. Since $a(t) = x(t) \times p(t)$, $A(j\omega) = \frac{1}{2\pi}[X(j\omega) * P(j\omega)]$. We need to find $X(j\omega)$ and $P(j\omega)$. To find $X(j\omega)$ from $x(t)$, we recognize $x(t)$ as being in OWN's Table 4.2 Basic Fourier Transform Pairs. It is a sinc function with $W = 4\pi$. Therefore,

$$X(j\omega) = \begin{cases} 1 & \text{for} |\omega| < 4\pi \\ 0 & \text{for} |\omega| > 4\pi \end{cases} \tag{1}$$

as shown in Figure 1.

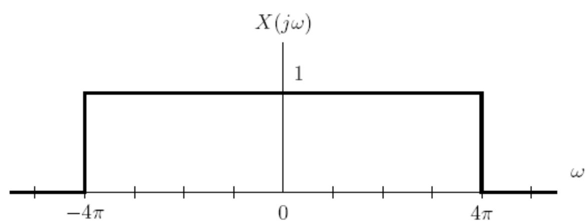



Figure 1: $X(j\omega)$

Because $p(t) = \cos 2\pi t$, $P(j\omega) = \pi[\delta(\omega - 2\pi) + \delta(\omega + 2\pi)]$. Since $P(j\omega)$ is two impulse functions, the convolution of $X(j\omega)$ with $P(j\omega)$ results in the superposition of two copies of $X(j\omega)$, one centered at $\omega = 2\pi$ and the other centered at $\omega = -2\pi$. The resulting $A(j\omega)$ is shown in Figure 2.

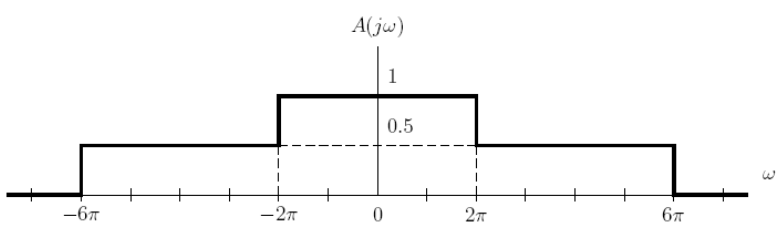



Figure 2: $A(j\omega)$

*(b)* To find $B(j\omega)$, we use the convolution property. Thus, $B(j\omega) = A(j\omega)H(j\omega)$. $A(j\omega)$ is a low pass filter and $H(j\omega)$ is a high pass filter. Multiplying the two together creates a bandpass filter. $A(j\omega)$ cuts off all frequencies for $|\omega| > 6\pi$. $H(j\omega)$ cuts off all frequencies for $|\omega| < 2\pi$. The resulting signal, $B(j\omega)$ is shown in Figure 3.

*(c)* To find $C(j\omega)$, we need to convolve $B(j\omega)$ with $Q(j\omega)$.

$$Q(j\omega) = \begin{cases} 1 & \text{for} |\omega| < 2\pi \\ 0 & \text{for} |\omega| > 2\pi \end{cases} \tag{2}$$

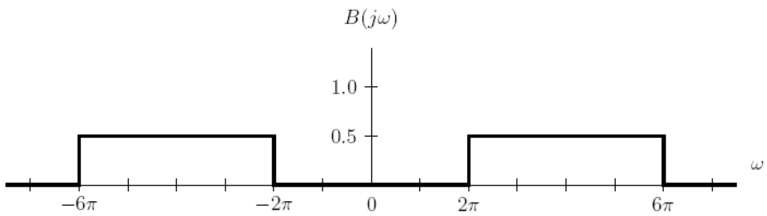



Figure 3: $B(j\omega)$

$Q(j\omega)$ is shown in Figure 4.

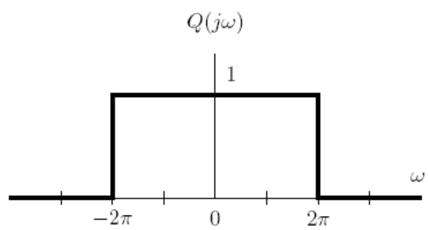



Figure 4: $Q(j\omega)$

Thus, $C(j\omega)$ can be drawn as in Fiture 5.

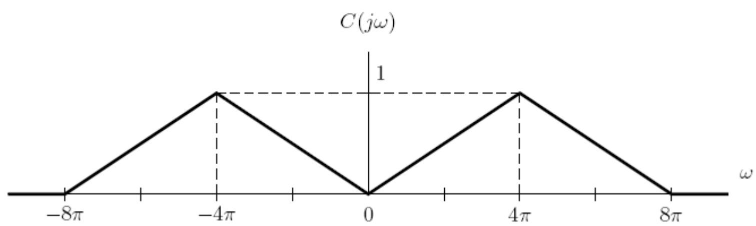



Figure 5: $C(j\omega)$

*(d)* To compute $c(t)$, we multiply $b(t)$ with $q(t)$. $B(j\omega)$ is the sum of two ideal frequency-shifted unity-gain filters. Filters in the frequency domain become sinc functions in the time domain. In addition, a frequency shift of $\omega_0$ corresponds to multiplying by $e^{-j\omega_0 t}$ in the time domain. Hence,

$$b(t) = \frac{1}{2}e^{-j4\pi t}\frac{\sin(2\pi t)}{\pi t} + \frac{1}{2}e^{j4\pi t}\frac{\sin(2\pi t)}{\pi t} = \cos(4\pi t)\frac{\sin(2\pi t)}{\pi t}. \tag{3}$$

Therefore

$$c(t) = b(t)q(t) = \cos(4\pi t)\frac{\sin^2(2\pi t)}{(\pi t)^2}. \tag{4}$$

**Problem 2** *(DTFT.)*

*(a)* OWN 5.21(e)

$x[n] = \left(\frac{1}{2}\right)^{|n|}\cos(\frac{\pi}{8}(n-1))$

$$
\begin{aligned}
X(e^{j\omega}) &= \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \\
&= \sum_{n=-\infty}^{\infty} (0.5)^{|n|}\cos[\pi(n-1)/8]e^{-j\omega n} \\
&= \sum_{n=-\infty}^{-1} (0.5)^{-n}0.5(e^{j\pi(n-1)/8}+e^{-j\pi(n-1)/8})e^{-j\omega n} + \sum_{n=0}^{\infty}(0.5)^{n}0.5(e^{j\pi(n-1)/8}+e^{-j\pi(n-1)/8})e^{-j\omega n} \\
&= 0.5\sum_{n=-\infty}^{-1} e^{-j\pi/8}(0.5e^{-j\pi/8}e^{j\omega})^{-n} + 0.5\sum_{n=-\infty}^{-1} e^{j\pi/8}(0.5e^{j\pi/8}e^{j\omega})^{-n} + 0.5\sum_{n=0}^{\infty} e^{-j\pi/8}(0.5e^{j\pi/8}e^{-j\omega})^{n} + \\
&\quad 0.5\sum_{n=0}^{\infty} e^{j\pi/8}(0.5e^{-j\pi/8}e^{-j\omega})^{n} \\
&= 0.5\sum_{n=1}^{\infty} e^{-j\pi/8}(0.5e^{-j\pi/8}e^{j\omega})^{n} + 0.5\sum_{n=1}^{\infty} e^{j\pi/8}(0.5e^{j\pi/8}e^{j\omega})^{n} + 0.5\sum_{n=0}^{\infty} e^{-j\pi/8}(0.5e^{j\pi/8}e^{-j\omega})^{n} + \\
&\quad 0.5\sum_{n=0}^{\infty} e^{j\pi/8}(0.5e^{-j\pi/8}e^{-j\omega})^{n} \\
&= \frac{0.25e^{-j\pi/4}e^{j\omega}}{1-0.5e^{-j\pi/8}e^{j\omega}} + \frac{0.25e^{j\pi/4}e^{j\omega}}{1-0.5e^{j\pi/8}e^{j\omega}} + \frac{0.5e^{-j\pi/8}}{1-0.5e^{j\pi/8}e^{-j\omega}} + \frac{0.5e^{j\pi/8}}{1-0.5e^{-j\pi/8}e^{-j\omega}}
\end{aligned}
$$

*(b)* OWN 5.22(a)

$$
\begin{aligned}
x[n] &= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega \\
&= \frac{1}{2\pi}\int_{-3\pi/4}^{\pi/4} e^{j\omega n}d\omega + \frac{1}{2\pi}\int_{\pi/4}^{3\pi/4} e^{j\omega n}d\omega \\
&= \frac{1}{2\pi}\left[\frac{1}{jn}e^{j\omega n}\right]_{\omega=-3\pi/4}^{-\pi/4} + \frac{1}{2\pi}\left[\frac{1}{jn}e^{j\omega n}\right]_{\omega=pi/4}^{3\pi/4} \\
&= \frac{1}{j2\pi n}\left(e^{-jn\pi/4} - e^{-jn3\pi/4} + e^{jn3\pi/4} - e^{jn\pi/4}\right) \\
&= \frac{1}{\pi n}\left(\sin(3\pi n/4) - \sin(\pi n/4)\right)
\end{aligned}
$$

**Problem 3** *(DTFT)*

*(a)* OWN 5.26(a)

First, by looking at the plots in Fig. P5.26(a), we see that $X_1(e^{j\omega})$ possesses conjugate symmetry. This implies that $x[n]$ is a real valued signal.

Next, we consider a signal $y[n]$ with DTFT $Y(e^{j\omega}) = \Re\{X_1(e^{j\omega})\}$

By the even-odd decomposition property in Table 5.1, we have that $y[n] = \mathcal{E}\{x_1[n]\}$

Then, we observe that we can express $X_2(e^{j\omega})$ as

$$X_2(e^{j\omega}) = Y(e^{j\omega}) + Y(e^{j(\omega-2\pi/3)}) + Y(e^{j(\omega+2\pi/3)})$$

Using the frequency shifting property of the DTFT and the linearity of the DTFT, both in Table 5.1, we have that

$$\begin{aligned} x_2[n] &= \mathcal{E}\{x_1[n]\}\left[1 + e^{j2\pi/3} + e^{-j2\pi/3}\right] \\ &= \mathcal{E}\{x_1[n]\}\left[1 + 2\cos(2\pi/3)\right] \end{aligned}$$

*(b)* OWN 5.50(a)

Using the transform pairs in Table 5.2, and the time-shifting property in Table 5.1, we see that

$$Y(e^{j\omega}) = \frac{1}{1 - \frac{1}{3}e^{-j\omega}}$$

$$X(e^{j\omega}) = \frac{1}{1 - \frac{1}{2}e^{-j\omega}} - \frac{1}{4}\frac{e^{-j\omega}}{1 - \frac{1}{2}e^{-j\omega}} = \frac{1 - \frac{1}{4}e^{-j\omega}}{1 - \frac{1}{2}e^{-j\omega}}$$

We can then compute the frequency response

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1 - \frac{1}{2}e^{-j\omega}}{\left(1 - \frac{1}{3}e^{-j\omega}\right)\left(1 - \frac{1}{4}e^{-j\omega}\right)}$$

To find $h[n]$, we first find the partial fraction expansion of the frequency response

$$H(e^{j\omega}) = \frac{A}{1 - \frac{1}{3}e^{-j\omega}} + \frac{B}{1 - \frac{1}{4}e^{-j\omega}}$$

By setting the two expressions for $H(e^{j\omega})$ equal, cross-multiplying, and equating the constant terms and $e^{-j\omega}$ terms; we can solve for $A = -2$ and $B = 3$.

Using the transform pairs in Table 5.2, we see that

$$h[n] = 3\left(\frac{1}{4}\right)^n u[n] - 2\left(\frac{1}{3}\right)^n u[n]$$

Now, to find the difference equation, we take the equation

$$\frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1 - \frac{1}{2}e^{-j\omega}}{\left(1 - \frac{1}{3}e^{-j\omega}\right)\left(1 - \frac{1}{4}e^{-j\omega}\right)}$$

and cross-multiply to show that

$$Y(e^{j\omega}) - Y(e^{j\omega})\frac{7}{12}e^{-j\omega} + Y(e^{j\omega})\frac{1}{12}e^{-j2\omega} = X(e^{j\omega}) - X(e^{j\omega})\frac{1}{2}e^{-j\omega}$$

Taking the inverse Fourier transform, and using the time shifting property, we obtain

$$y[n] - \frac{7}{12}y[n-1] + \frac{1}{12}y[n-2] = x[n] - \frac{1}{2}x[n-1]$$

**Problem 4** OWN 5.51 (b) *(Systems characterized by linear constant-coecient dierence equations.)*

*(i)* Let us name the intermediate outputs as $p[n]$, $q[n]$ and $w[n]$ as shown in Figure 6.

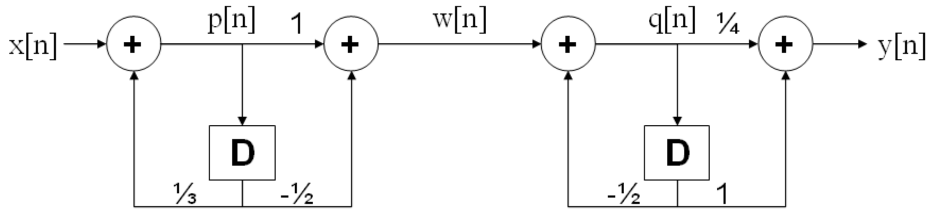



Figure 6: Define intermediate outputs $p[n]$, $q[n]$ and $w[n]$

We first try to derive the relationship between $x[n]$ and $w[n]$.

We know that

$$w[n] = p[n] - \frac{1}{2}p[n-1] \tag{5}$$

$$p[n] = x[n] + \frac{1}{3}p[n-1] \tag{6}$$

By taking the Fourier transform, we obtain

$$W(e^{j\omega}) = (1 - \frac{1}{2}e^{-j\omega})P(e^{j\omega}) \tag{7}$$

$$(1 - \frac{1}{3}e^{-j\omega})P(e^{j\omega}) = X(e^{j\omega}) \tag{8}$$

Therefore

$$\frac{W(e^{j\omega})}{X(e^{j\omega})} = \frac{1 - \frac{1}{2}e^{-j\omega}}{1 - \frac{1}{3}e^{-j\omega}} \tag{9}$$

Similarly, we can derive the relationship between $w[n]$ and $y[n]$

$$y[n] = \frac{1}{4}q[n] + q[n-1] \tag{10}$$

$$q[n] = w[n] - \frac{1}{2}q[n-1] \tag{11}$$

By taking the Fourier transform, we obtain

$$Y(e^{j\omega}) = (\frac{1}{4} + e^{-j\omega})Q(e^{j\omega}) \tag{12}$$

$$(1 + \frac{1}{2}e^{-j\omega})Q(e^{j\omega}) = W(e^{j\omega}) \tag{13}$$

Therefore

$$\frac{Y(e^{j\omega})}{W(e^{j\omega})} = \frac{\frac{1}{4} + e^{-j\omega}}{1 + \frac{1}{2}e^{-j\omega}} \tag{14}$$

From Equation (9) and (14)

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{Y(e^{j\omega})}{W(e^{j\omega})} \cdot \frac{W(e^{j\omega})}{X(e^{j\omega})} = \frac{\frac{1}{4} + e^{-j\omega}}{1 + \frac{1}{2}e^{-j\omega}} \cdot \frac{1 - \frac{1}{2}e^{-j\omega}}{1 - \frac{1}{3}e^{-j\omega}} = \frac{\frac{1}{4} + \frac{7}{8}e^{-j\omega} - \frac{1}{2}e^{-2j\omega}}{1 + \frac{1}{6}e^{-j\omega} - \frac{1}{6}e^{-2j\omega}}. \tag{15}$$

Cross-multiplying and taking the inverse Fourier transform we obtain

$$y[n] + \frac{1}{6}y[n-1] - \frac{1}{6}y[n-2] = \frac{1}{4}x[n] + \frac{7}{8}x[n-1] - \frac{1}{2}x[n-2]. \tag{16}$$

*(ii)* From *(i)*

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{\frac{1}{4} + \frac{7}{8}e^{-j\omega} - \frac{1}{2}e^{-2j\omega}}{1 + \frac{1}{6}e^{-j\omega} - \frac{1}{6}e^{-2j\omega}}. \tag{17}$$

*(iii)* Through partial fraction expansion of $H(e^{j\omega})$, we get

$$H(e^{j\omega}) = \frac{\frac{1}{4} + \frac{7}{8}e^{-j\omega} - \frac{1}{2}e^{-2j\omega}}{(1 - \frac{1}{3}e^{-j\omega})(1 + \frac{1}{2}e^{-j\omega})} = 3 - \frac{\frac{13}{20}}{1 - \frac{1}{3}e^{-j\omega}} - \frac{\frac{21}{10}}{1 + \frac{1}{2}e^{-j\omega}} \tag{18}$$

Since the system is causal ( $h[n] = 0$ for $n < 0$ ), taking inverse Fourier transform, we get

$$h[n] = 3\delta[n] - \frac{13}{20}(\frac{1}{3})^n u[n] - \frac{21}{10}(-\frac{1}{2})^n u[n]. \tag{19}$$

**Problem 5** *(Fourier Representations and their interconnections.)*

*(a)* We know from Table 4.2 that the inverse Fourier transform of a rectangular pulse is a sinc.

$$x(t) = \frac{\sin(Wt)}{\pi t}$$

The plot is shown at the end of this problem.

*(b)* $Z(j\omega)$ is equal to $X(j\omega)$ convolved with a train of pulses.

$$Z(\omega) = X(j\omega) \star \sum_{k=-\infty}^{\infty} \delta(\omega - 3Wk)$$

In the time domain, letting $T = \frac{2\pi}{3W}$ and using the transform pair in Table 4.2, we have

$$\begin{aligned} z(t) &= (2\pi)\,(x(t))\left(\frac{T}{2\pi}\sum_{n=-\infty}^{\infty}\delta(t-nT)\right) \\ &= T\sum_{n=-\infty}^{\infty} x(nT)\delta(t-nT) \\ &= T\sum_{n=-\infty}^{\infty} \frac{\sin(n\cdot 2\pi/3)}{\pi nT}\delta(t-nT) \\ &= \sum_{n=-\infty}^{\infty} \frac{\sin(n\cdot 2\pi/3)}{\pi n}\delta(t-nT) \end{aligned}$$

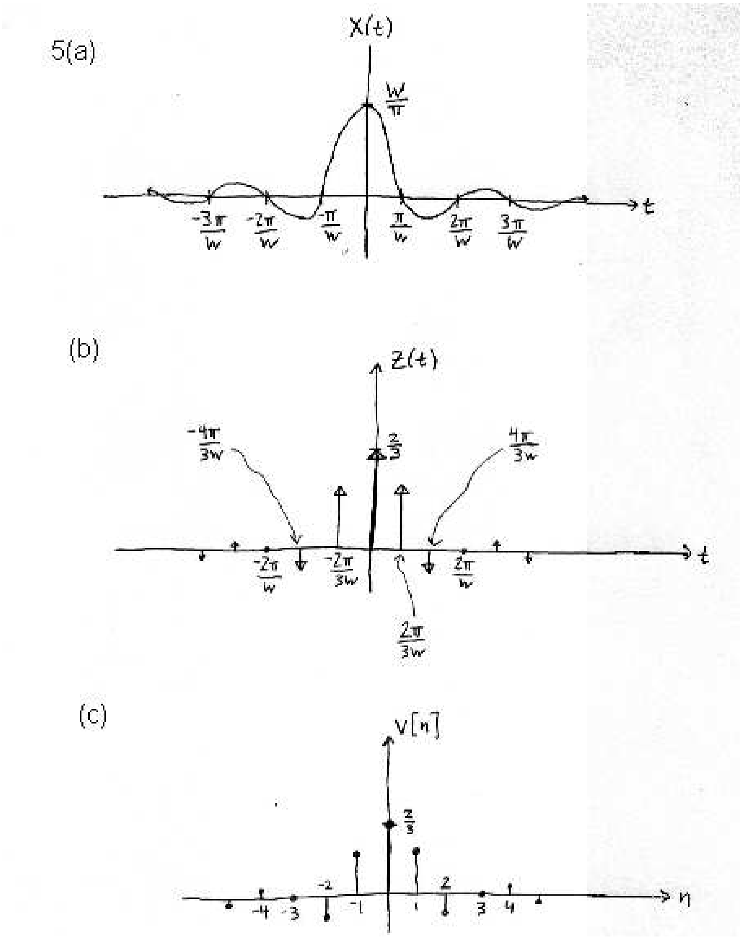
Hence, we see that $z(t)$ is just $x(t)$ sampled with a sampling period of $T = \frac{2\pi}{3W}$ . The plot is shown at the end of this problem.

*(c)* From Table 5.2, we see that

$$v[n] = \frac{\sin(n\cdot 2\pi/3)}{\pi n}$$

Hence, $v[n]$ is the impulse train $z(t)$ converted to a discrete time sequence.

The plots of $x(t)$, $z(t)$, and $v[n]$ are shown here.



**Problem 6** *(Fourier via Matlab.)*

*(a)*

```
function [F_OWN,F_M,F_u] = Fourier_matrix(N)

for m=1:N,
for n=1:N,
F_OWN(m,n) = 1/N*exp(-j*2*pi/N*(m-1)*(n-1));
F_M(m,n) = exp(-j*2*pi/N*(m-1)*(n-1));
F_u(m,n) = 1/sqrt(N)*exp(-j*2*pi/N*(m-1)*(n-1));
end
end
```

*(b)*

For this problem, we will use the OWN indexing (the columns are numbered $0$ through $N-1$, and the rows likewise) instead of the Matlab indexing in order to simplify the computations. Using the the OWN indexing convention, the entry in row $m$, column $n$ of $F_u$ is equal to

$$\frac{1}{\sqrt{N}}e^{-j2\pi mn/N}$$

If we take the dot product of column $n$ and column $k$ of $F_u$, we get the following (don't forget to take the conjugate of the first vector):

$$\sum_{m=0}^{N-1}\left(\frac{1}{\sqrt{N}}e^{-j2\pi mn/N}\right)^*\left(\frac{1}{\sqrt{N}}e^{-j2\pi mk/N}\right)=\frac{1}{N}\sum_{m=0}^{N-1}e^{-j2\pi m(k-n)/N}$$

If $k=n$ (which means that we are taking the dot product of a column with itself), then

$$\sum_{m=0}^{N-1}\frac{1}{N}e^{j0}=\sum_{m=0}^{N-1}\frac{1}{N}=1$$

Since the norm of a vector is equal to the squareroot of the dot product of the vector with itself (see Handout 2), the columns of $F_u$ have unit length. If $\ell=k-n\neq 0$, then

$$\sum_{m=0}^{N-1}\frac{1}{N}e^{-j2\pi\ell/N\cdot m}=\frac{1}{N}\frac{1-e^{-j2\pi\ell}}{1-e^{-j2\pi\ell/N}}=0$$

Hence, the dot product of two unique columns of $F_u$ is equal to $0$. Combining these two facts, we see that the columns of $F_u$ are orthonormal.

For the matrix $F_M$, again using the OWN indexing convention, the entry in row $m$, column $n$ is

$$e^{-j2\pi mn/N}$$

Now, repeating the same steps as above, we find that the dot product of column $k$ and column $n$ of $F_M$ is

$$\sum_{m=0}^{N-1}e^{-j2\pi/N\cdot(k-n)m}=N\cdot\delta(k-n)$$

So, the columns of $F_M$ are orthogonal, but have length $\sqrt{N}$. For the matrix $F_{OWN}$, the entry in row $m$, column $n$ is

$$\frac{1}{N}e^{-j2\pi mn/N}$$

So the dot product of column $k$ and column $n$ of $F_{OWN}$ is

$$\sum_{m=0}^{N-1}\frac{1}{N^2}e^{-j2\pi/N\cdot(k-n)m} = \frac{1}{N}\cdot\delta(k-n)$$

The columns of $F_{OWN}$ are orthogonal, but have length $1/\sqrt{N}$.

Because $F_u$ is a symmetric matrix, the rows of $F_u$ are also orthonormal. This means that when we multiply $F_u x$, we are projecting the vector $x$ onto a new orthonormal basis. The DFT is expressing the signal in a new basis, the Fourier basis. We can think of the DFT as a change of basis operation. This notion extends to the other Fourier transforms, which can all be viewed as expressing the signal in terms of the Fourier basis.

*(c)*

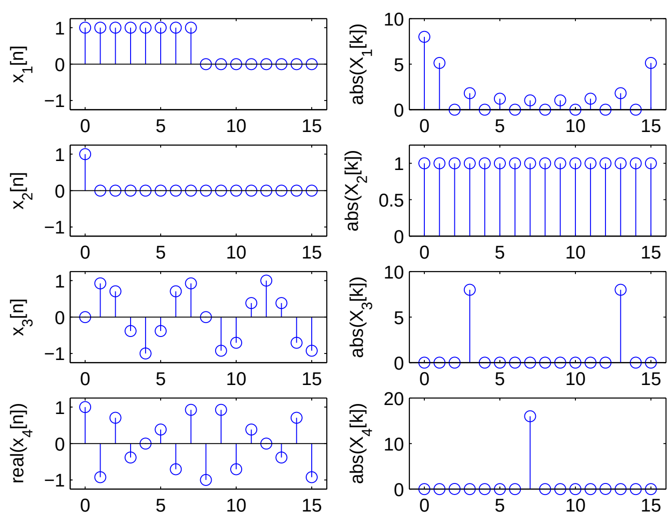
```
x1 = [1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0];
x2 = [1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0];
x3 = sin(3*pi*(0:15)/8);
x4 = exp(j*14*pi*(0:15)/16);
[ F_OWN,F_M,F_u ] = Fourier_matrix(16);
fftx1 = fft(x1.');
fft2x1 = F_M*(x1.');
fftx2 = fft(x2.');
fft2x2 = F_M*(x2.');
fftx3 = fft(x3.');
fft2x3 = F_M*(x3.');
fftx4 = fft(x4.');
fft2x4 = F_M*(x4.');
e1 = sum(abs(fftx1 - fft2x1))
e2 = sum(abs(fftx2 - fft2x2))
e3 = sum(abs(fftx3 - fft2x3))
e4 = sum(abs(fftx4 - fft2x4))
subplot(4,2,1), stem((0:15),x1)
axis([-1 16 -1.25 1.25])
ylabel('x_1[n]')
subplot(4,2,3), stem((0:15),x2)
axis([-1 16 -1.25 1.25])
ylabel('x_2[n]')
subplot(4,2,5), stem((0:15),x3)
axis([-1 16 -1.25 1.25])
ylabel('x_3[n]')
subplot(4,2,7), stem((0:15),real(x4))
axis([-1 16 -1.25 1.25])
ylabel('real(x_4[n])')
subplot(4,2,2), stem((0:15),abs(fftx1))
axis([-1 16 0 10])
ylabel('abs(X_1[k])')
subplot(4,2,4), stem((0:15),abs(fftx2))
```

```
axis([-1 16 0 1.25])
ylabel('abs(X_2[k])')
subplot(4,2,6), stem((0:15),abs(fftx3))
axis([-1 16 0 10])
ylabel('abs(X_3[k])')
subplot(4,2,8), stem((0:15),abs(fftx4))
axis([-1 16 0 20])
ylabel('abs(X_4[k])')
```

We note that the four error values $e1$, $e2$, $e3$, and $e4$ are all equal to $0$ (or less than $10^{-13}$, which is within the machine precision), so the `fft` function in Matlab is giving the same answers as our matrix multiplication.

The FFTs of signals $3$ and $4$ make sense because if a signal can be expressed as the sum of complex exponentials of period $N$, then the DTFS coefficients can be easily read off from the time-domain signal.

The FFT of signal $2$ makes sense because we know that the DTFS of a (periodic) impulse is a constant function.



**Problem 7** *(Fourier Transform via Matlab.)*

- Plots (Figure 7)

```
T = 10;
dt = 0.01;
t = [ -T : dt :  T-dt];
x1 = cos(30*t).*sinc(t).^2;
Xnotquite = fft(x1);
x = fftshift(x1);
X = fft(x);
figure(1); plot(t,x1); ylabel('x1'); xlabel('t')
figure(2); plot(real(Xnotquite)); ylabel('Xnotquite')
figure(3); plot(t,x); ylabel('x'); xlabel('t')
figure(4); plot(real(X)); ylabel('X')
```

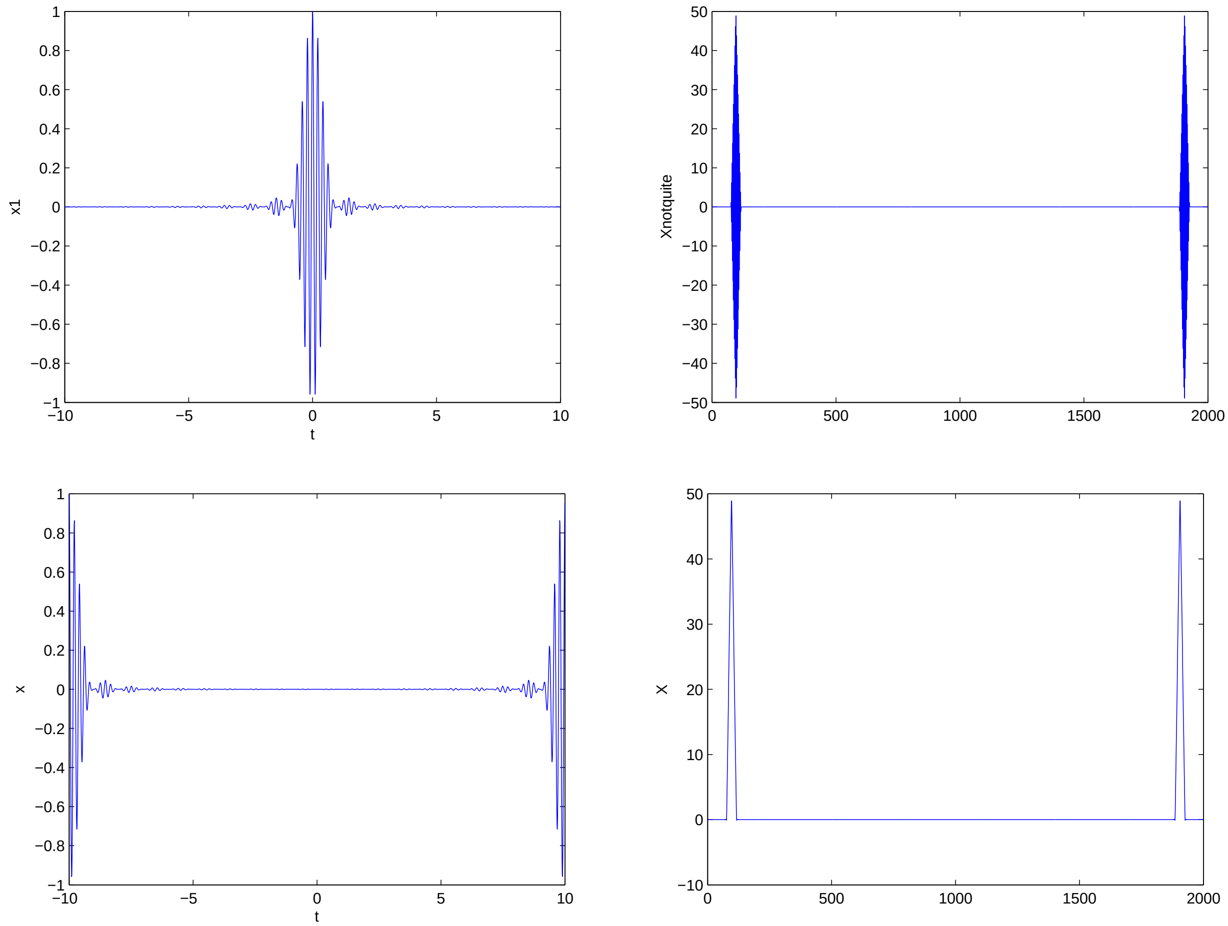



Figure 7: Problem 7 plots

- Explanation of `Xnotquite`

  We know that the Fourier transform of $(\sin(\pi t)/(\pi t))^2$ is a triangle. We also know that the Fourier transform of $\cos(30t)$ is two delta functions, and that multiplying in the time domain is equivalent to convolving in the frequency domain. Therefore, we would expect the spectrum of $x(t)$ to be two triangles.

However, `Xnotquite` actually looks like two triangular shapes multiplied by a function that alternates between $1$ and $-1$. If you zoom in on one of the triangular pulses, you can see the oscillatory nature of `Xnotquite` more clearly. The reason for this is that Matlab does not understand that the time origin is in the middle of `x1`. In fact, Matlab assumes that $t = 0$ is at the left edge of `x1`, which means that Matlab has essentially delayed the signal $x(t)$ by $10$ time units. Time-shifting is equivalent to multiplying by a complex exponential in the frequency domain. This means that `Xnotquite` is the true spectrum multiplied by a complex exponential, which causes the oscillatory behavior.

- Explanation of `X`

  When we use the `fftshift` function to produce `x`, Matlab swaps the left and right halves of `x1`. This means that the peak of $x(t)$ is now centered at $t = 0$. (Remember that `fft` computes the DTFS, which assumes that the input signal is periodic.)

  Now, the spectrum `X` looks just like we expected, because Matlab has the signal properly aligned with the origin.

  We see that `x` is the original continuous-time signal $x(t)$ sampled, where $\tau = 0.01$ is the sampling period. We saw in Section 7.4 that the spectrum of the sampled signal is $\frac{1}{\tau}$ times the spectrum of the continuous-time signal (see Figure 7.22). Therefore, we should multiply $X$ by $0.01$ to get the correct amplitude values.

  In addition, the two triangles should be centered at $\omega = 30$ and $\omega = -30$, because the spectrum of the term $\cos(30t)$ in $x(t)$ has a spectrum of two impulses at $\omega = \pm 30$