EECS 120 Signals & Systems
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University of California, Berkeley: Fall 2006
October 17, 2006

Homework 7

Due: Thursday, October 26, 2006, at 5pm
Homework 7 GSI: Mark Johnson

Reading OWN Chapter 7.

Problem 1 *(Downsampling.)*

OWN Problem 7.35

Problem 2 *(Upsampling.)*

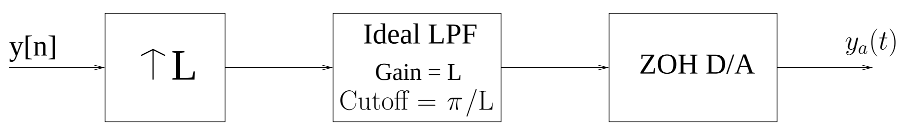
OWN Problem 7.49

Problem 3 *(Zero-Order and First-Order Holds.)*

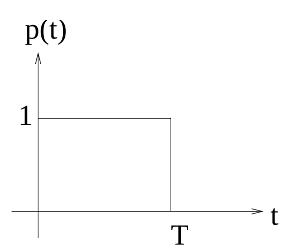
OWN Problem 7.50

Problem 4 *(Oversampled D/A.)*

An oversampled D/A is implemented using a digital interpolator and a zero-order hold, as shown below.



The interpolator inserts $L-1$ zeros between each sample of $y[n]$. The zero-order hold has period $T = 1/L$, i.e., the duration of the ZOH is dependent on the upsampling factor. As usual, the impulse response of the ZOH is given by



The discrete time signal $y[n]$ has DTFT $Y(e^{j\Omega}) = \begin{cases} 1 & |\Omega| < 3\pi/4 \\ 0 & 3\pi/4 < |\Omega| < \pi \end{cases}$

(a) Plot the magnitude spectrum $|Y_a(j\omega)|$ for three cases: $L = 1$, $L = 2$, and $L = 4$.

Plot each spectrum over the interval $-5\pi/T \leq \omega \leq 5\pi/T$ (remember that T is dependent on L). You may either use Matlab or sketch the plots by hand. Be sure to accurately label all important points on the ω-axis.

(b) Plot the magnitude spectrum $|Y_a(j\omega)|$ when the ZOH is replaced with an ideal D/A converter.

(c) For the four plots in parts (a) and (b), calculate the magnitude of the largest component of $Y_a(j\omega)$ *outside* the band $|\omega| \leq \pi/T$

Problem 5 *(Aliasing.)*

The signal $x(t)$ has a real-valued spectrum

$$X(j\omega) \quad = \quad e^{-|\omega|}, \ -\infty \leq \omega \leq \infty. \tag{1}$$

(a) To avoid aliasing, we could pass the signal $x(t)$ through an ideal low-pass filter of width $W = \frac{\pi}{T}$ before sampling with a sampling interval T. Denote the resulting samples by $x_T[n]$. As we have seen, the spectrum $X_T(e^{j\Omega})$ of the discrete-time signal $x_T[n]$ can be expressed as

$$X_T(e^{j\Omega}) \quad = \quad \frac{1}{T}e^{-\left|\frac{\Omega}{T}\right|}, \quad \text{for } -\pi \leq \Omega \leq \pi. \tag{2}$$

Sketch the spectrum $X_T(e^{j\Omega})$ for $-2\pi \leq \Omega \leq 2\pi$. Carefully label the frequency axis.

(b) Suppose we directly sample $x(t)$ (without prior low-pass filtering) with a sampling interval T. Denote the samples by $\tilde{x}_T[n] = x(nT)$. Since $x(t)$ is not bandlimited, aliasing occurs. Draw the shifted copies of $X(j\omega)$ that make up the spectrum $\tilde{X}_T(e^{j\Omega})$ of the discrete-time signal $\tilde{x}_T[n]$, without summing them up. Identify the parts which cause aliasing.

(c) Based on your sketch from Part *(b)*, the spectrum $\tilde{X}_T(e^{j\Omega})$ of the discrete-time signal $\tilde{x}_T[n]$ can be written as

$$\tilde{X}_T(e^{j\Omega}) \quad = \quad \frac{1}{T}\sum_{n=-\infty}^{\infty} e^{-\left|\frac{\Omega - n2\pi}{T}\right|}. \tag{3}$$

Decompose the sum into three parts: one term for $n = 0$, an infinite sum for negative n, and an infinite sum for positive n. Assuming $-\pi \leq \Omega \leq \pi$, evaluate the two infinite sums. *Hint:* In each of the two infinite sums *individually*, carefully get rid of the magnitude $|\cdot|$ in the exponent.

Problem 6 *(Aliasing - continued.)*

This problem refers to the signals $x_T[n]$ and $\tilde{x}_T[n]$ from Problem 5. A good measure of the error due to aliasing is the mean-squared error between $x_T[n]$ and $\tilde{x}_T[n]$, defined as

$$E \quad = \quad \sum_{n=-\infty}^{\infty} |\tilde{x}_T[n] - x_T[n]|^2. \tag{4}$$

(a) Calculate E.

(b) Determine the limiting value of E as T tends to zero. Explain why your result makes sense.

Problem 7 *(AM Communication Systems.)*

OWN Problem 8.22