EECS 120 Signals & Systems — University of California, Berkeley: Fall 2006
Ramchandran — November 2, 2006

Homework 8 Solutions

Problem 1 OWN 8.21 *(AM Communication Systems.)*

(a)

$$\begin{aligned} w(t) &= y(t)\cos(\omega_c t + \theta_c) \\ &= x(t)(\cos(\omega_c t + \theta_c))^2 \\ &= x(t)(\frac{1}{2} + \frac{1}{2}\cos(2(\omega_c t + \theta_c)) \\ &= x(t)(\frac{1}{2} + \frac{1}{2}\cos(2\omega_c t + 2\theta_c) \end{aligned}$$

(b)

$$\begin{aligned} w(t) &= x(t)\cdot(\frac{1}{2} + \frac{1}{4}(e^{j(2\omega_c t + 2\theta_c t)} + e^{-j(2\omega_c t + 2\theta_c t)})) \\ W(j\omega) &= \frac{1}{2}X(j\omega) + \frac{1}{4}(e^{j2\theta_c}X(j(\omega - 2\omega_c)) + e^{-j2\theta_c}X(j(\omega + 2\omega_c))) \end{aligned}$$

The terms involving θ_c do not affect the magnitude of the spectra of $W(j\omega)$ if $\omega_M < \omega_c$. Hence, for the output of the lowpass filter to be proportional to $x(t)$, we require (1) $\omega_c > \omega_M$ (2) $\omega_{co} > \omega_M$.

If these conditions are satisfied, the value of θ_c doesn't matter.

Problem 2 OWN Problem 8.26. *(Phase synchronization in communication systems.)*

The Fourier transform of $y(t)$ is as sketched in Figure S8.26. We also sketch the Fourier transforms of $y(t)\cos(\omega_c t)$ and $y(t)\sin(\omega_c t)$ in Figure S8.26.

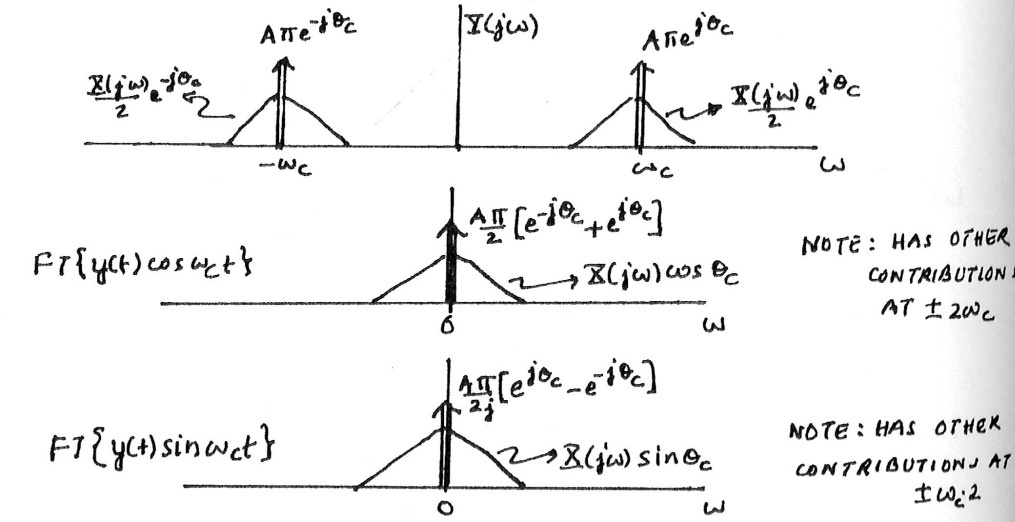


Figure S8.26

From these figures, it is clear that the outputs of the lowpass filters are $[x(t)+A]\cos(\theta_c)$ and $[x(t)+A]\sin(\theta_c)$. Upon squaring and adding, we obtain the signal $[x(t)+A]^2\{\cos^2\theta_c + \sin^2\theta_c\} = [x(t)+A]^2$. Therefore, $r(t) = x(t) + A$.

Problem 3 OWN Problem 8.29. *(Single-sideband amplitude modulation.)*

(a) The sketches in the Figure 1 show $S(j\omega)$ and $R(j\omega)$.

(b) In Figure 1 we show how $P(j\omega)$ may be obtained by considering the outputs of the various stages of Figure P8.28(c). From the sketch for $P(j\omega)$, it is clear that $P(j\omega) = 2S(j\omega)$.

(c) In Figure 1 we show the results of demodulation on both $s(t)$ and $r(t)$. It is clear that $x(t)$ is recovered in both cases.

Problem 4 OWN Problem 8.40 *(Quadrature Multiplexing.)*

Let $X_1(j\omega)$ and $X_2(j\omega)$ be as shown in Figure 2. Then $R(j\omega)$ is as shown in Figure 2. The overlapping regions in the figure need to be summed.

When $r(t)$ is multiplied by $\cos\omega_c t$, in the vicinity of $\omega = 0$ we get

$$\frac{1}{2}\{\frac{1}{2}X_1(j\omega) + \frac{1}{2}jX_2(j\omega) + \frac{1}{2}X_1(j\omega) - \frac{1}{2}jX_2(j\omega)\} = \frac{1}{2}X_1(j\omega).$$

Therefore the first lowpass filter output is equal to $x_1(t)$.

When $r(t)$ is multiplied by $\sin\omega_c t$, in the vicinity of $\omega = 0$ we get

$$\frac{1}{2}\{-j[\frac{1}{2}jX_2(j\omega) + \frac{1}{2}X_1(j\omega)] + j[-j\frac{1}{2}X_2(j\omega) + \frac{1}{2}X_1(j\omega)]\} = \frac{1}{2}X_2(j\omega).$$

Therefore the second lowpass filter output is equal to $x_2(t)$.

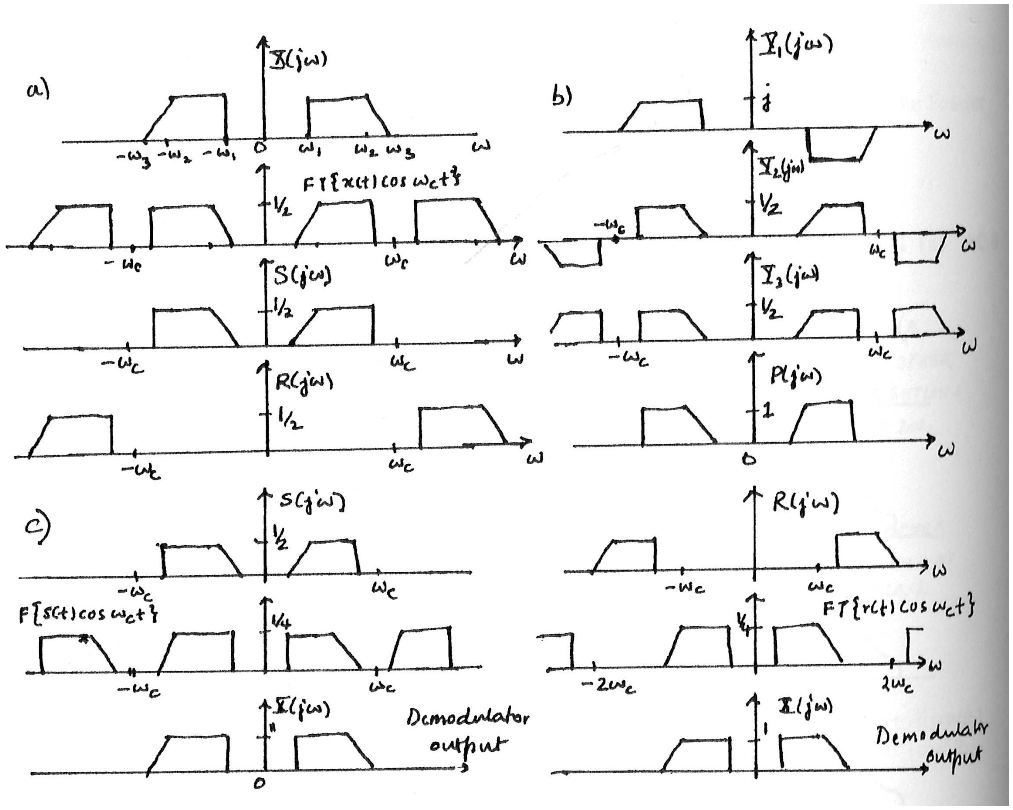


Figure 1: OWN Problem 8.29

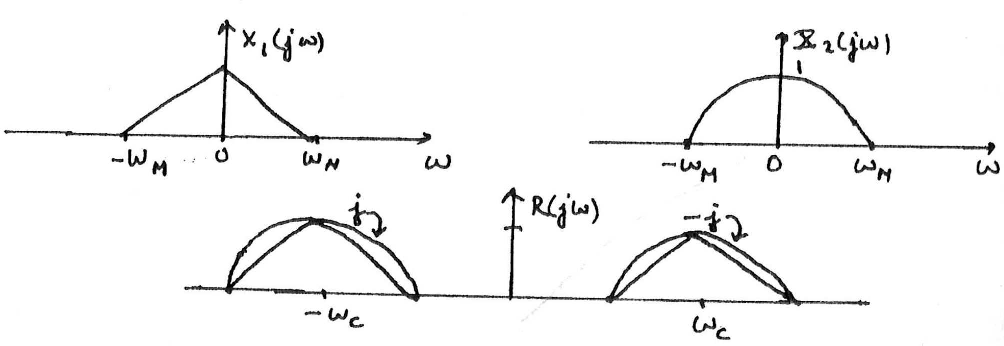


Figure 2: OWN Problem 8.40

Problem 5 OWN Problem 8.42 *(PAM.)*

(a)

According to the description, $P_1(j\omega)$ is band-limited to $[-\frac{\pi}{T_1}, \frac{\pi}{T_1}]$. $\tilde{P}_1(j\omega)$ is a periodic version of $P_1(j\omega)$ with $\tilde{P}_1(j\omega) = P_1(j\omega)$ for $\omega \in [-\frac{\pi}{T_1}, \frac{\pi}{T_1}]$ and period $\frac{4\pi}{T_1}$.

Since $P_1(j\omega)$ is even, we have

$$P_1(j\omega - j\frac{\pi}{T_1}) = P_1(-j\omega + j\frac{\pi}{T_1}) = -P_1(j\omega + j\frac{\pi}{T_1}),\ 0 \le \omega \le \frac{\pi}{T_1}.$$

Therefore

$$P_1(j\omega - j\frac{\pi}{T_1}) = -P_1(j\omega + j\frac{\pi}{T_1}),\ 0 \le \omega \le \frac{\pi}{T_1}.$$

Since $\tilde{P}_1(j\omega)$ has period $\frac{4\pi}{T_1}$,

$$\tilde{P}_1(j\omega - j\frac{\pi}{T_1}) = -\tilde{P}_1(j\omega + j\frac{\pi}{T_1}) \text{ for all } \omega.$$

This is equivalent to

$$\tilde{P}_1(j\omega) = -\tilde{P}_1(j\omega \pm j\frac{2\pi}{T_1}).$$

Therefore

$$\tilde{P}_1(j\omega) = -\tilde{P}_1(j\omega - j\frac{2\pi}{T_1}).$$

(b)

Since $\tilde{P}_1(j\omega) = -\tilde{P}_1(j\omega - j\frac{2\pi}{T_1})$, $\tilde{p}_1(t) = -e^{j\frac{2\pi}{T_1}t}\tilde{p}_1(t)$. For $t = kT_1$, $k = 0, \pm1, \pm2, \ldots$, this becomes

$$\tilde{p}_1(kT_1) = -e^{j\frac{2\pi}{T_1}kT_1}\tilde{p}_1(t) = -\tilde{p}_1(kT_1).$$

Therefore $\tilde{p}_1(kT_1) = 0$, $k = 0, \pm1, \pm2, \ldots.$ $\Rightarrow T = \frac{T_1}{2}$.

(c)

$$\tilde{P}_1(j\omega) = P_1(j\omega) * \sum_{m=-\infty}^{+\infty} \delta(j\omega - jm\frac{4\pi}{T_1})$$

Therefore

$$\tilde{p}_1(t) = p_1(t) \cdot \frac{T_1}{2} \sum_{m=-\infty}^{+\infty} \delta(t - n\frac{T_1}{2})$$

.

Since $\tilde{p}_1(kT_1) = 0$, $k = 0, \pm1, \pm2, \ldots$, and $\frac{T_1}{2}\sum_{m=-\infty}^{+\infty} \delta(kT_1 - n\frac{T_1}{2}) = \frac{T_1}{2} \ne 0$, we have to have $p_1(kT_1) = 0$, $k = 0, \pm1, \pm2, \ldots.$

(d)

Note that $P(j\omega) = P_1(j\omega) + P_2(j\omega)$, where

$$P_2(j\omega) = \begin{cases} 1, & |\omega| \le \pi/T_1 \\ 0, & \text{otherwise} \end{cases}$$

Therefore, $p(t) = p_1(t) + \frac{\sin(\pi t/T_1)}{\pi t}$. We know that $\frac{\sin(\pi t/T_1)}{\pi t} = 0$ for $t = 0, \pm T_1, \pm 2T_1, \ldots.$ We have also shown in (c) that $p_1(t) = 0$, $t = 0, \pm T_1, \pm 2T_1, \ldots.$ Therefore $p(t) = 0$ for $t = 0, \pm T_1, \pm 2T_1, \ldots.$

Problem 6 OWN Problem 8.44 *(PAM.)*

(a) We may write $y(t)$ as

$$y(t) = x(t) * \sum_{l=-N}^{N} a_l \delta(t - lT_1).$$

Therefore, $y(t)$ is obtained by passing $x(t)$ through a filter with impulse response $h(t) = \sum_{l=-N}^{N} a_l \delta(t - lT_1)$.

(b) Using eq. (P8.44-1), we obtain the following three simultaneous equations

$$y(0) = a_{-1}x(T_1) + a_0 x(0) + a_1 x(-T_1),$$

$$y(T_1) = a_{-1}x(2T_1) + a_0 x(T_1) + a_1 x(0),$$

and

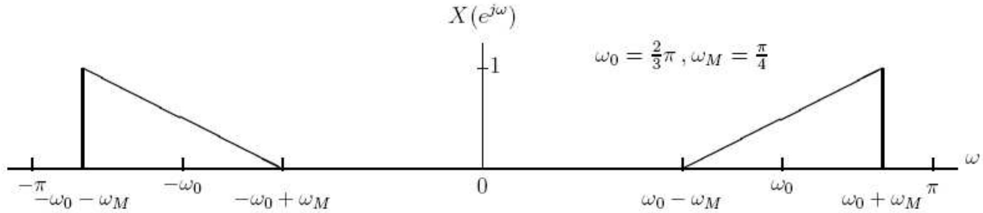
$$y(-T_1) = a_{-1}x(0) + a_0 x(-T_1) + a_1 x(-2T_1).$$

Substituting the given values for $x(t)$ and $y(t)$ and solving, we obtain

$$a_0 = 0, \qquad a_1 = a_{-1}.$$

Problem 7

(a) $x[n]$ is a real-valued DT signal whose DTFT for $-\pi < \omega < \pi$ is given by

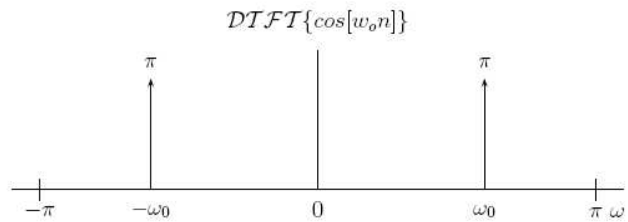


Let

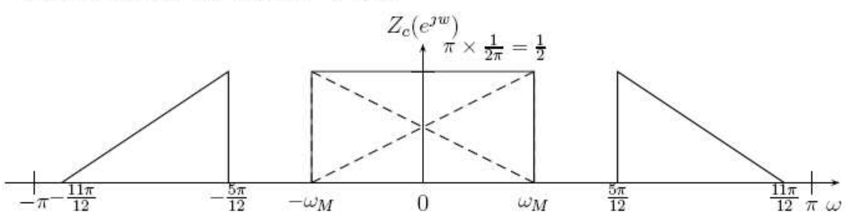
$$z_c[n] = x[n]\cos[w_o n]$$

Using table 5.2 and taking Fourier transform of $\cos[w_o n]$,

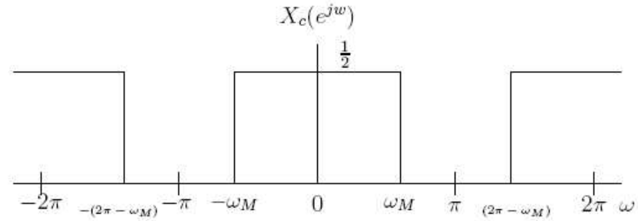
$$\mathcal{DTFT}\{\cos[w_o n]\} = \pi \sum_{l=-\infty}^{+\infty} \{\delta(w - w_o - 2\pi l) + \delta(w + w_o - 2\pi l)\}$$



Using the multiplication property from table 5.1, $Z_c(e^{jw})$ is the periodic convolution of $X(e^{jw})$ and $\mathcal{DTFT}\{\cos[w_o n]\}$ over period 2π and then scaled by $\frac{1}{2\pi}$. We take one period, from $-\pi$ to π, of $\mathcal{DTFT}\{\cos[w_o n]\}$ and do regular convolution with $X(e^{jw})$. Centered at $w = 0$, we get the superposition of two $X(e^{jw})$ scaled by $\frac{1}{2}$. $Z_c(e^{jw})$ is shown below for the interval $-\pi$ to π.



$Z_c(e^{jw})$ is then passed through a low-pass filter with cut-off frequency w_M and gain of 1. DTFT of $x_c[n]$ is shown below.



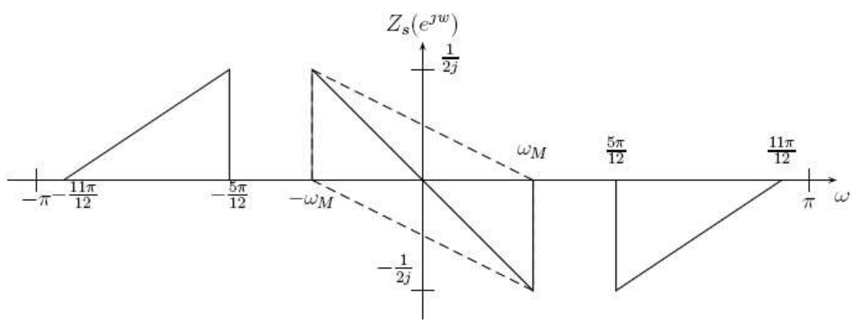
Let

$$z_s[n] = x[n]\sin[w_o n]$$

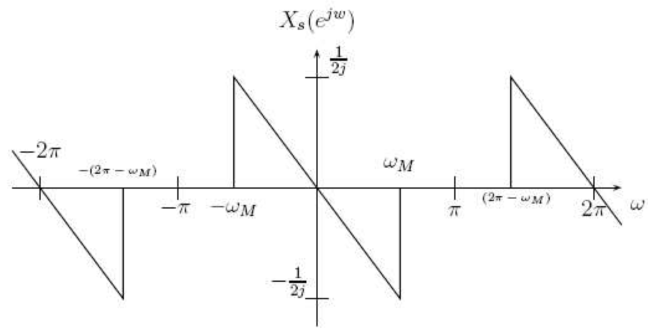
Using table 5.2 and taking Fourier transform of $\sin[w_o n]$,

$$\mathcal{DTFT}\{\sin[w_o n]\} = \frac{\pi}{j}\sum_{l=-\infty}^{+\infty}\{\delta(w-w_o-2\pi l)-\delta(w+w_o-2\pi l)\}$$

We find $Z_s(e^{jw})$ using the periodic convolution as before. The superposition terms centered at $w=0$ from $X(e^{jw})$ (in dashed lines) are shown below. Adding the superposition terms, resulting $Z_s(e^{jw})$ is shown for interval $-\pi$ to π.



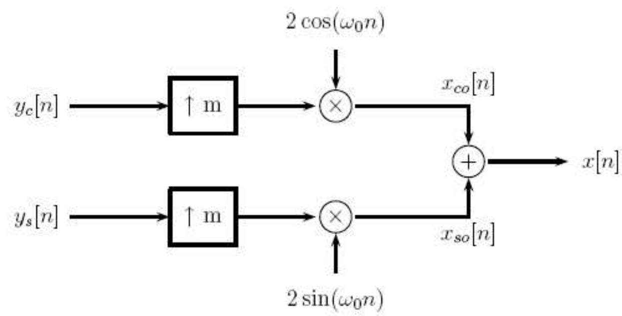
$Z_s(e^{jw})$ goes through the low-pass filter with cut-off frequency w_M and gain of 1, we find DTFT of $x_s[n]$ as shown below.



(b) Maximum possible downsampling is achieved once the non-zero portion of one period of the discrete-time spectrum has expanded to fill the entire band from $-\pi$ to π. Therefore,

$$m = \frac{\pi}{w_M} = \frac{\pi}{\frac{\pi}{4}} = 4$$

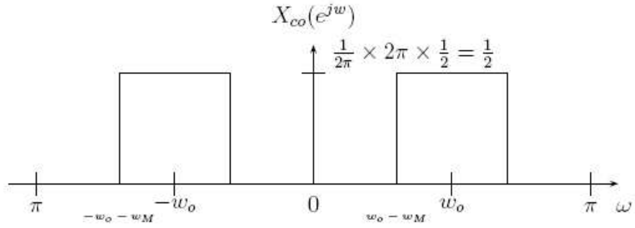
(c) Following is the system diagram to recover $x[n]$.

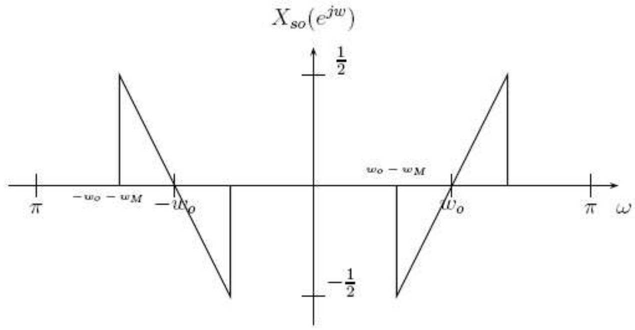


After upsampling by m, we get back $x_c[n]$ and $x_s[n]$ from $y_c[n]$ and $y_s[n]$ respectively. Note that upsampling by m has zero-insertion block (up-arrow m) and a low-pass filter for time-domain interpolation. DTFT of $x_c[n]$ and $x_s[n]$ are derived in part a. According to the system diagram,

$$x_{co}[n] = x_c[n] \times 2\cos[w_o n]$$

Using the multiplication property and doing periodic convolution, we get $X_{co}(e^{jw})$ as shown below.





Similarly, $x_{so}[n] = x_s[n] \times 2\sin[w_o n]$, and we get $X_{so}(e^{jw})$ as shown in the figure.

Adding $X_{co}(e^{jw})$ and $X_{so}(e^{jw})$, we get back the spectrum of $X(e^{jw})$. Thus, we recover $x[n]$.