Homework 9
Due: Thursday, November 9, 2006, at 5pm
GSI: Omar Bakr

Reading OWN Chapters 8 and 9.

Practice Problems *(Suggestions.)* OWN 8.8, 8.13, 8.19, 8.20, 9.1, 9.2, 9.3.

Problem 1 *(Image Suppression.)*

Consider the system shown in Figure 1 (also known as the Hilbert Architecture), which is found in many RF receivers. Let the input to the system $x(t) = m_r(t)cos(\omega_{LO} + \omega_{IF})t + m_I(t)cos(\omega_{LO} - \omega_{IF})t$. $m_r(t)$ is sometimes refered to as the desired signal, and $m_I(t)$ as the image. The goal of the system is to provide as much image rejection as possible.

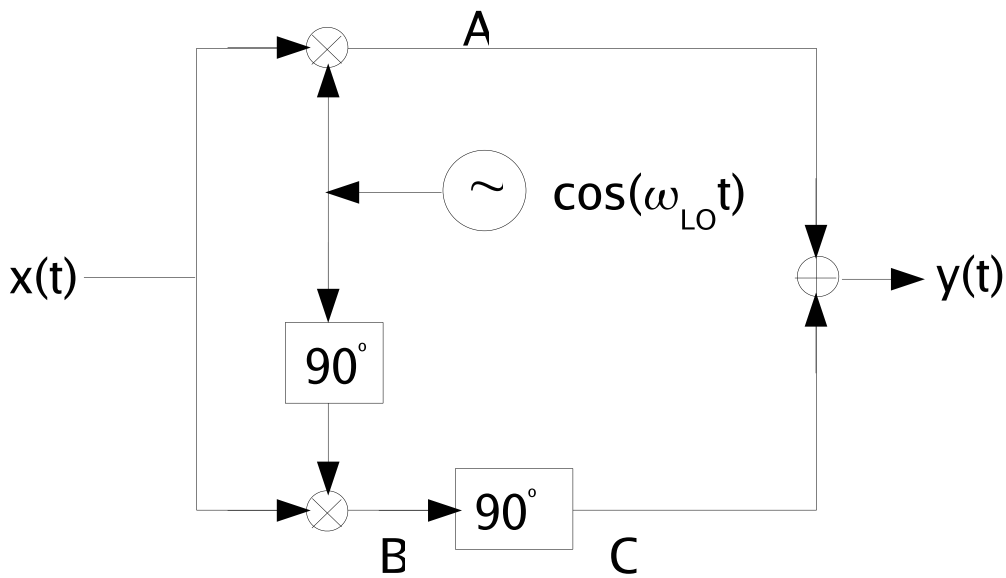

Figure 1: Problem 1

(a) Find the output $y(t)$. *(hint: write down the output after each stage.)*.

(b) In practice, it is hard to achieve perfect symmetry between the top and bottom branches of the circuit, and there will be some gain and phase mismatch. For example, if we have a gain mismatch α, and phase mismatch ϕ, then instead of generating $\cos(\omega_{LO}t)$ and $\sin(\omega_{LO}t)$ at the top and bottom branches of the circuit, we get $(1+\alpha)\cos(\omega_{LO}t + \frac{\phi}{2})$ and $(1-\alpha)\sin(\omega_{LO}t - \frac{\phi}{2})$ respectively. What is the output $y(t)$ in the presence of gain and phase imbalance?

(c) A common metric for measuring the quality of the system is image rejection (IR). IR is the ratio of the power of desired signal to the power of the image at the output (assuming they were equal in power at the input). What is IR as a function of ϕ and α. *(hint: you can use Parseval's Relation.)*

Problem 2 *(FSK.)*

OWN Problem 8.39.

Problem 3 *(Angle Modulations.)*

An angle-modulated signal has the form

$$u(t) = 100\cos(2\pi f_c t + 4\sin 2\pi f_m t)$$

where $f_c = 10MHz$ and $f_m = 1KHz$.

(a) Assuming that this is an FM signal, determine the modulation index and the bandwidth (in Hz) of the transmitted signal.

(b) Repeat part (a) assuming a PM signal.

(c) Compare the signal bandwidths in parts (a) and (b) with SSB AM.

(d) Repeat parts (a) and (b) if f_m is doubled.

Problem 4 *(Laplace Transforms.)*

(a) OWN Problem 9.21 (c).

(b) OWN Problem 9.21 (e).

Problem 5 *(LT/LT Properties.)*

(a) OWN Problem 9.21 (g).

(b) OWN Problem 9.26.

Problem 6 *(Inverse Laplace.)*

(a) OWN Problem 9.22 (a).

(b) OWN Problem 9.22 (b).

Problem 7 *(Matlab - Synchronization in PAM.)*

We have seen in class that care has to be taken when designing pulses for PAM systems in order to avoid intersymbol interference (ISI). In particular, we saw that the pulse $p(t)$ must satisfy the following condition:

$$p(nT_s) = \begin{cases} 1, & \text{if } n = 0 \\ 0, & \text{if } n \neq 0 \end{cases} \qquad (1)$$

However, this may not be sufficient (especially when the transmitter and receiver are not perfectly synchronized). Even if the decoder samples exactly at intervals of T_s, those samples may not be aligned with the way the transmitter generated the PAM signal. In this matlab problem, we want to study the effects of this lack of synchronization.

More precisely, let's denote the non-synchronized received signal as

$$\tilde{r}[n] \quad = \quad \sum_{k=-\infty}^{\infty} x[k]p((n-k)T_s + \Delta t). \tag{2}$$

In this formula, the main problem with a lack of synchronization is the fact that neither the transmitter nor the receiver knows the value of Δt.

In order to get a sense for the effect of Δt, we consider the amount of intersymbol interference that results. How should we measure this amount? We cannot give a very good answer to this question with the tools that we currently have (you'll need to take EE121 to understand this in more detail). To make matters simple, we will assume that $\Delta t \leq T_s/2$, i.e., a small synchronization error, and we will use the following "signal-to-interference" power ratio.

$$SIR(\Delta t) = \frac{|p(\Delta t)|^2}{\sum_{n=-\infty}^{\infty} |p(nT_s + \Delta t)|^2}. \tag{3}$$

(a) Fix $T_s = 1$. Download `PAM_SIR.m` from the course website. Go through the code and convince yourself that it is computing the SIR for the sinc pulse for values of Δt between 0 and 1/2. Now run the code.

What does the plot of the SIR tell you about the sensitivity of the sinc to synchronization offsets?

(b) The sinc pulse is relatively sensitive to synchronization offsets because it dies off as $\frac{1}{|t|}$. A generalization of the sinc pulse is the *Raised Cosine* pulse. The Raised Cosine pulse $p_{rc}(t)$ (with zero crossings at multiples of $T_s = 1$) is defined in time and frequency below.

$$p_{rc}(t) \quad = \quad sinc(t) \cdot \frac{\cos(\pi \alpha t)}{1 - 4\alpha^2 t^2} \tag{4}$$

$$P_{rc}(j\omega) \quad = \quad \begin{cases} 1, & |\omega| < (1-\alpha)\pi \\ \frac{1}{2}\left[1 - \sin\left(\frac{\pi(|\omega| - \pi)}{2\alpha\pi}\right)\right], & (1-\alpha)\pi \leq |\omega| < (1+\alpha)\pi \\ 0, & |\omega| \geq (1+\alpha)\pi \end{cases} \tag{5}$$

The parameter α is called the *rolloff factor*, and it can be in the interval $[0, 1]$. Note that $\alpha = 0$ gives the sinc pulse. For non-zero α, the raised cosine pulse dies off like $\frac{1}{|t|^2}$.

Download `rc.m` and `rcspectrum.m` from the course website. These functions return raised cosine pulses and spectra for various input parameters (type `help rc` for information on the parameters).

Using `trange = 20`, `tstep = 1` and `alpha = [0 0.25 0.5 0.75 1];`, redo the experiment in part (a) for the raised cosine pulse.

That is, for each of the α values, obtain a SIR curve for
`delta_t = [0:.01:1];`. Then plot all 5 curves in the same graph. Turn in this graph.

What can you say about the timing offset sensitivity of the raised cosine pulse as a function of α?

(c) Communications systems engineers are always dealing with tradeoffs when making choices for a system. What is the qualitative tradeoff, in terms of α, when choosing a raised cosine pulse to use in a PAM system? (Hint: Look at the spectra $P_{rc}(j\omega)$.)

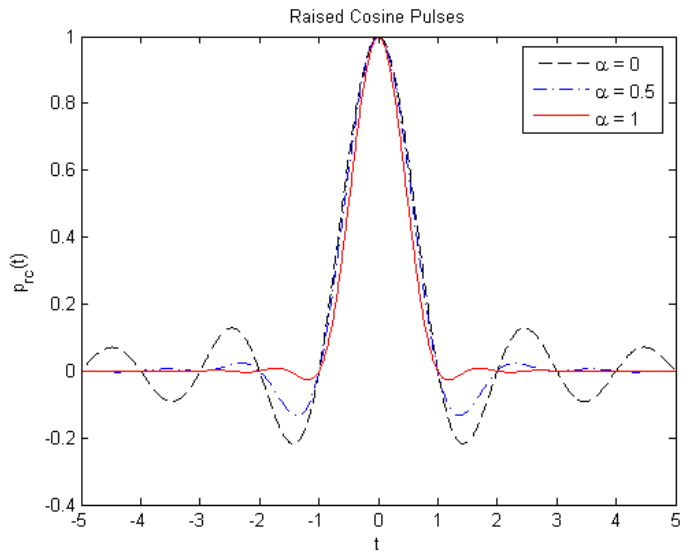

Figure 2: Problem 7

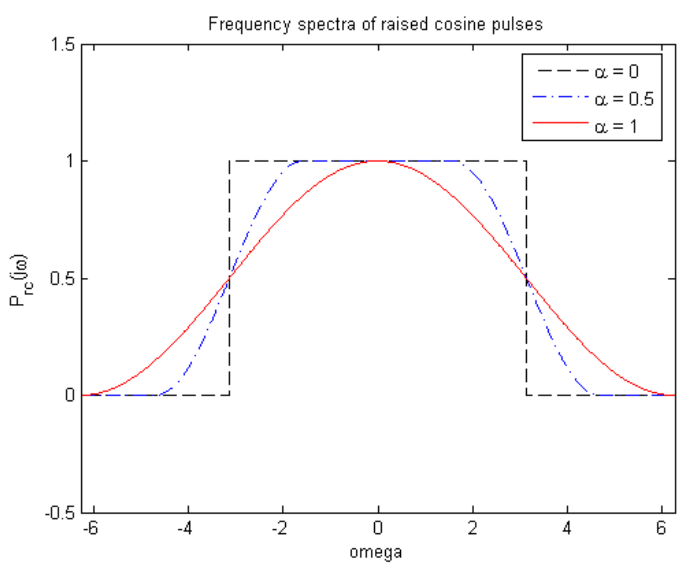

Figure 3: Problem 7