(93)

EECS 120 Signals & Systems
Ramchandran

University of California, Berkeley: Fall 2007
December 6, 2007

Midterm II Exam

Last name	First name	SID

Rules.

- You have two hours to complete this exam.
- There are 100 points for this exam.
- The exam is closed-book and closed-notes; calculators, computing and communication devices are *not* permitted.
- Two handwritten and *not photocopied* double-sided sheets of notes are allowed.
- Moreover, you receive, together with the exam paper, copies of Tables 4.2, 5.2, 9.2 and 10.2 of the course textbook.
- No form of collaboration between the students is allowed. If you are caught cheating, you may fail the course and face disciplinary consequences.
- Write down your name on all sheets. DO IT NOW!

Tips.

- Show all work to get any partial credit.
- Take into account the points that may be earned for each problem when splitting your time between the problems.

Problem	Points earned	out of	Problem	Points earned	out of
Problem 1		20	Problem 2		30
Problem 3		20	Problem 4		30
Total					100

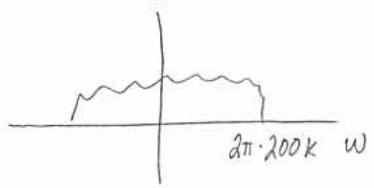
Problem 1 *(20 points)*

(a) (7 points) A television signal $z(t)$ occupies the frequency band from $0 - 200\ kHz$. The following FM signal $y(t)$ is simultaneously transmitted:

$$y(t) = \cos(\omega_c t + 0.01\sin(2\pi \times 10^4 t)). \tag{1}$$

What is the minimum value of the carrier frequency ω_c of the FM signal, such that $y(t)$ and $z(t)$ do not interfere with each other?

$Z(jw)$



$$y(t) = \cos(w_c t)\cos(0.01\sin(2\pi \cdot 10^4 t)) - \sin(w_c t)\, 0.01 \sin(2\pi \cdot 10^4 t)$$

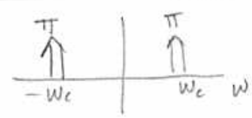
$$\approx \cos(w_c t) - \sin(w_c t)\, 0.01 \sin(2\pi \cdot 10^4 t)$$

$$w_c > 2\pi \cdot 200k + 2\pi \cdot 10k = 2\pi(210k)$$

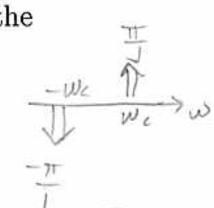
minimum $\omega_c =$ $2\pi \cdot 210K$

Plot the magnitude of the spectrum of $y(t)$ for this value of ω_c. Clearly label both of the axes.

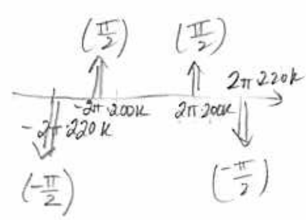
$\cos(w_c t) \longleftrightarrow$



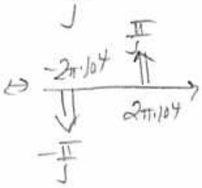
$\sin w_c t \leftrightarrow$



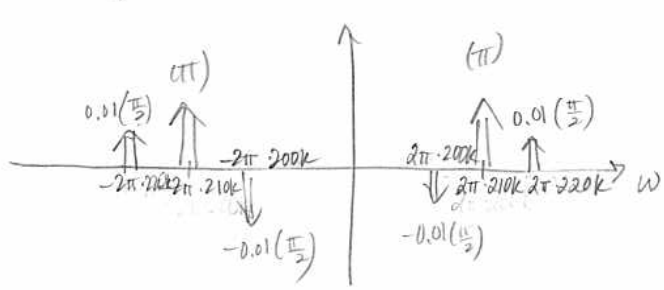
$\sin(w_c t)\sin(2\pi \cdot 10^4 t) \longleftrightarrow$



$\sin(2\pi \cdot 10^4 t) \leftrightarrow$



$Y(jw)$



(b) (8 points) The spectra of two analog signals $x(t)$ and $y(t)$ are shown in Fig. 1. Draw a block diagram of a continuous-time system that inputs $x(t)$ and outputs $y(t)$.

(Note: You may **not** use A/D or D/A converters.)

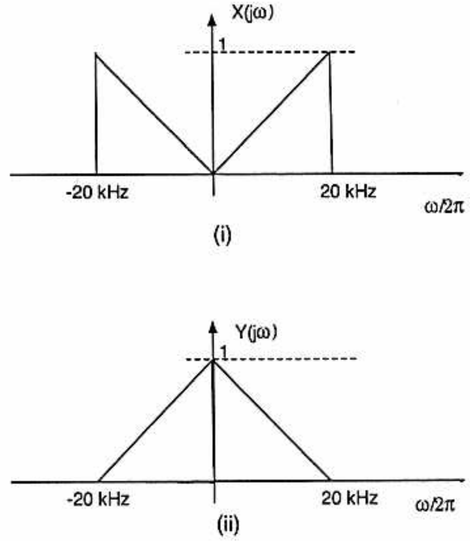
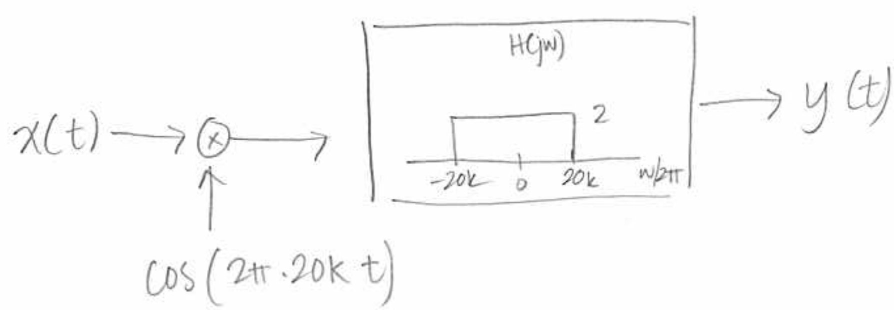


Figure 1: Problem 1(b)



(c) *(5 points)* Match the z-domain pole-zero plots on the next page to the magnitude plots of their corresponding Discrete-Time Fourier transforms. In each box below, write the number of the DTFT magnitude plot that corresponds with the pole-zero plot with the given letter.

Note that a double pole or a double zero is indicated by a 2 by its side. Also note that there are five pole-zero plots, and seven possible Fourier plots from which to choose.

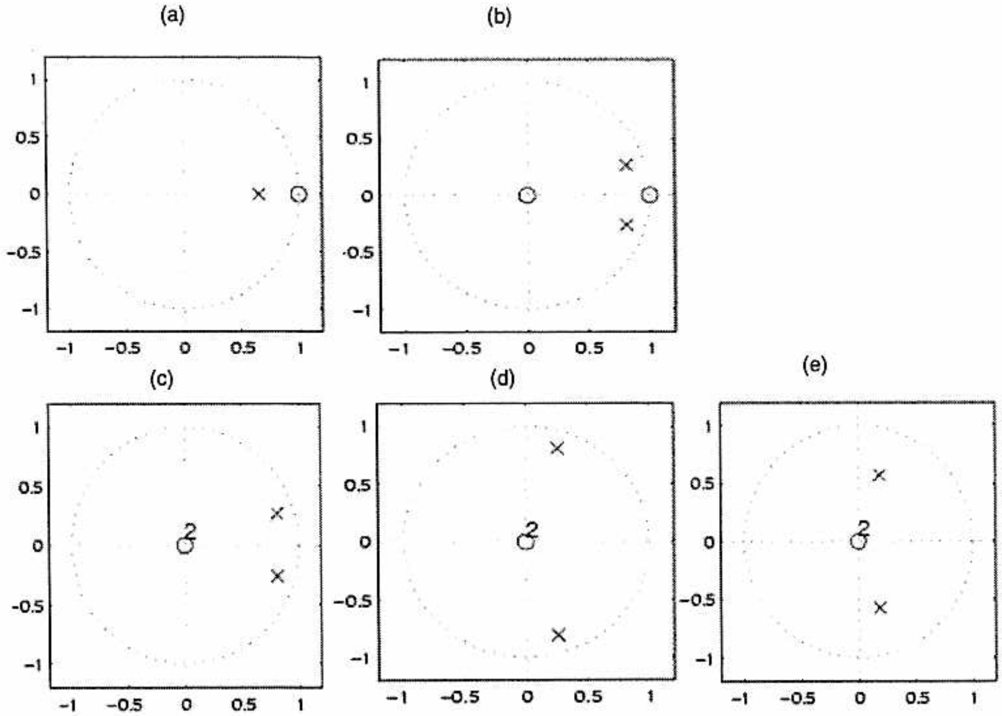
$(a) \rightarrow$ $\boxed{5}$

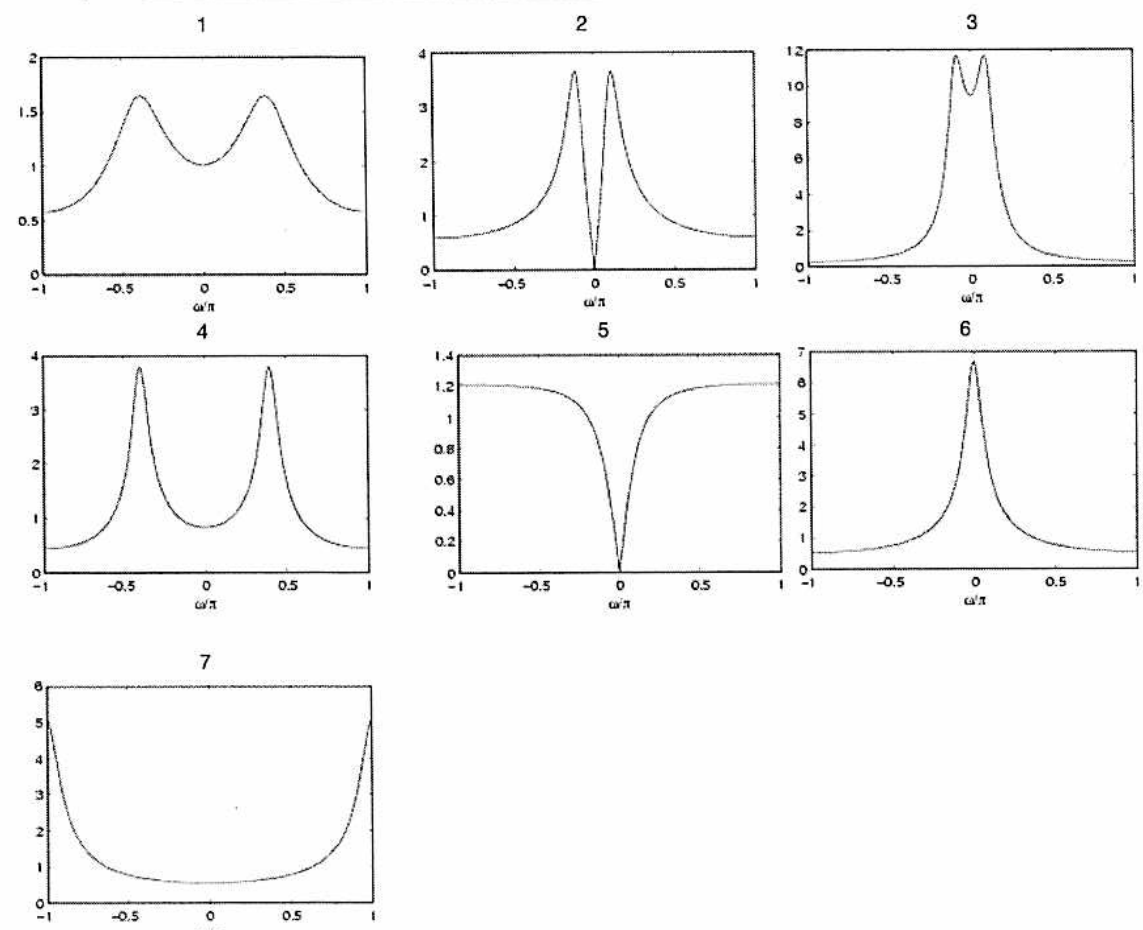
$(b) \rightarrow$ $\boxed{2}$

$(c) \rightarrow$ $\boxed{3}$

$(d) \rightarrow$ $\boxed{4}$

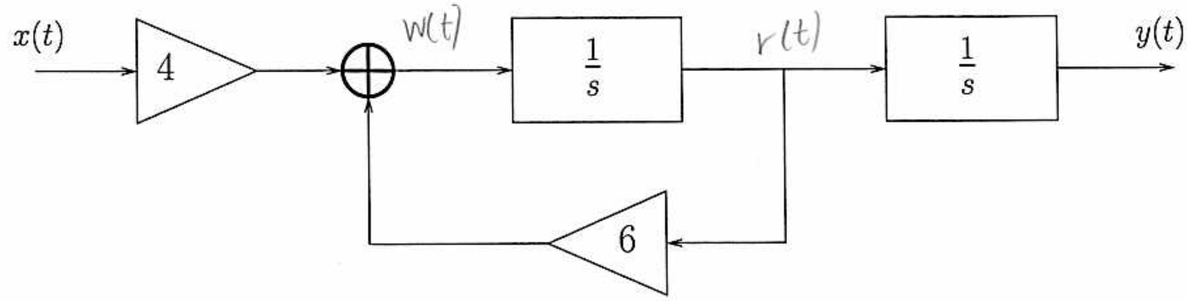
$(e) \rightarrow$ $\boxed{1}$


(a)
(b)
(c)
(d)
(e)
2


1
2
3
4
5
6
7
ω/π

Problem 2 *(30 Points)*

Consider the following causal LTI system with input $x(t)$ and output $y(t)$.



(a) (5 points) Find the transfer function $H(s) = \frac{Y(s)}{X(s)}$. Express your answer as a rational function.

$$W(s) = 4X(s) + 6R(s)$$

$$\frac{W(s)}{s} = R(s)$$

$$\frac{4X(s)+6R(s)}{s} = R(s)$$

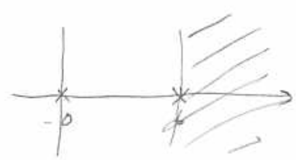
$$4X(s)+6R(s) = sR(s)$$

$$\frac{R(s)}{s} = Y(s)$$

$$4X(s) + 6sY(s) = s^2Y(s)$$

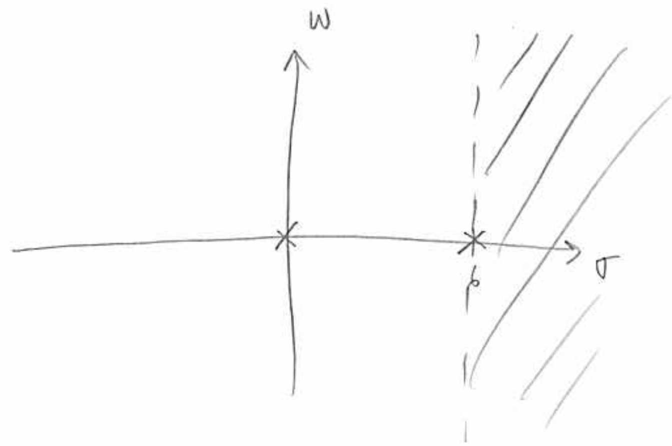
$$4X(s) = Y(s)(s^2-6s)$$

$$\frac{Y(s)}{X(s)} = \frac{4}{s(s-6)}$$



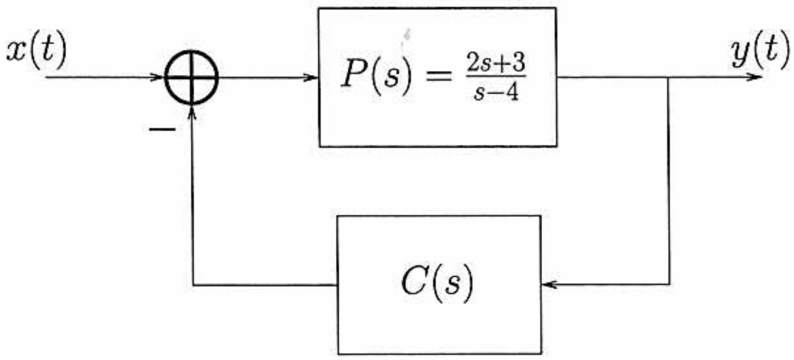
$$H(s) = \frac{4}{s(s-6)} \qquad Re\{s\} > 6$$

(b) (7 points) Sketch the pole-zero plot of $H(s)$, and give its ROC.



ROC : Re{s} > 6

(c) (10 points) An unstable, causal plant system has transfer function $P(s) = \frac{2s+3}{s-4}$. You decide to stabilize this plant by placing it in the following feedback loop with controller $C(s) = K$, where K is a real number.



Find all values of K such that the causal closed-loop system with transfer function $T(s) = \frac{Y(s)}{X(s)}$ is stable.

$$Y(s) = X(s)\,P(s) - Y(s)\,C(s)\,P(s)$$

$$\frac{Y(s)}{X(s)} = \frac{P(s)}{1 + C(s)P(s)} = \frac{\frac{2s+3}{s-4}}{1 + K\frac{(2s+3)}{s-4}}$$

$$= \frac{2s+3}{s-4+K(2s+3)}$$

pole

$$s - 4 + 2ks + 3k = 0$$
$$s[1 + 2k] = 4 - 3k$$
$$s = \frac{4-3k}{1+2k}$$

$4 - 3k > 0$ & $1 + 2k < 0$

$4 > 3k$ $\quad 2k < -1$

$4/3 > k$ $\quad k < -1/2$

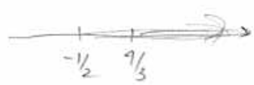
$k < 4/3$

or

$4 - 3k < 0$ & $1 + 2k > 0$

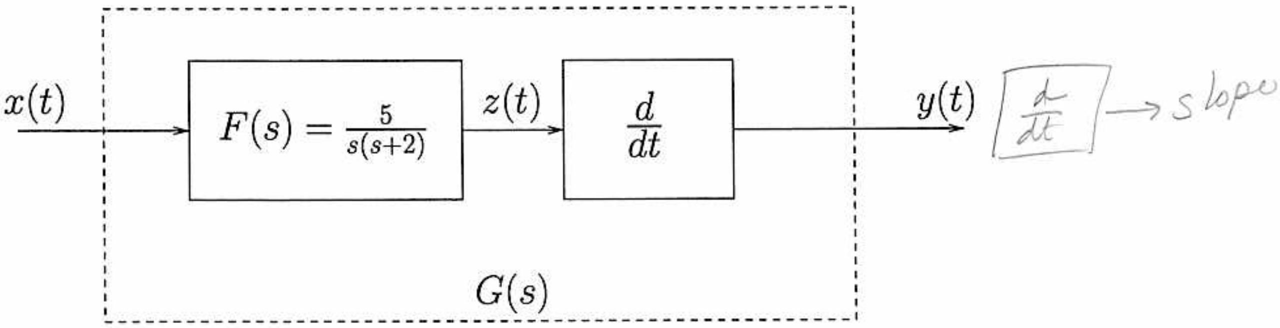
$4 < 3k$ $\quad 2k > -1$

$k > 4/3$ & $k > -1/2$



$$\boxed{k < -1/2 \quad \text{or} \quad k > 4/3}$$

(d) (8 points) A causal LTI system has transfer function $F(s) = \frac{5}{s(s+2)}$. Another causal system $G(s)$ is constructed by taking the first derivative of the output of $F(s)$, as shown in the figure below. Observe that $y(t) = \frac{dz(t)}{dt}$.



When the input to the system $G(s)$ is chosen to be the unit step function $u(t)$, the corresponding output is labelled $w(t)$. Evaluate the slope of $w(t)$ at time $t = 0^+$.

$$G(s) = \frac{5}{s(s+2)} \cdot s = \frac{5}{s+2}$$

$$u(s) = \frac{1}{s}$$

$$w(s) = \frac{5}{s(s+2)}$$

$$\text{slope} = \frac{5}{s+2}$$

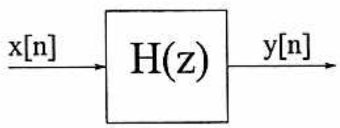
$$\underset{\lim t \to 0}{\text{slope}} = \lim_{s \to \infty} \left(\frac{5s}{s+2} \right) = 5$$

$$\left. \frac{dw(t)}{dt} \right|_{t=0^+} = 5$$

Problem 3 *(20 points)*

Consider an LTI system with input $x[n]$ and output $y[n]$ shown below, where

$$H(z) = \frac{3(z-2)}{(z-4)(z-1/2)}, \quad \frac{1}{2} < |z| < 4.$$



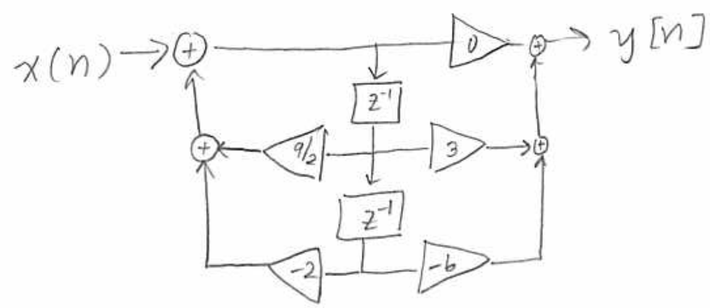
(a) (5 points) Write the difference equation relating $x[n]$ and $y[n]$.

$$\frac{Y(z)}{X(z)} = \frac{3z-6}{z^2 - \frac{9}{2}z + 2}$$

$$y[n+2] - \tfrac{9}{2}\, y[n+1] + 2y[n] = 3x[n+1] - 6x[n]$$

$$\boxed{y[n] = 3x[n-1] - 6x[n-2] - 2y[n-2] + \tfrac{9}{2}\, y[n-1]}$$

(b) *(5 points)* Draw the most economical block diagram which implements the difference equation found in part (a).



(c) *(5 points)* If $x[n] = 5^n$ for all values of n, determine the corresponding output signal $y[n]$.

$$y[n] = H(5) \cdot 5^n$$

$$= \frac{3(3)}{(9/2)} 5^n = 2 \cdot 5^n$$

actually H(5) is not in the ROC!

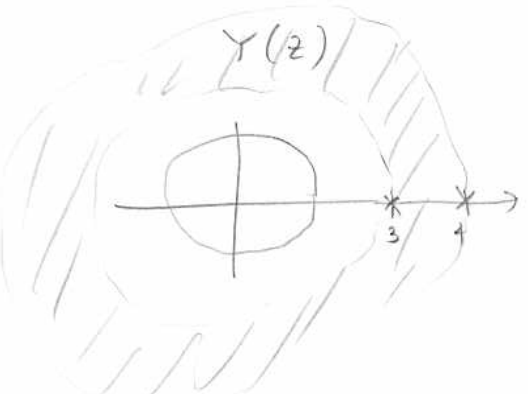
$y[n] = ?$

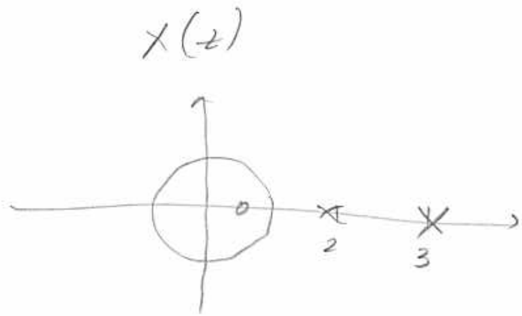
$y[n] = 2 \cdot 5^n$

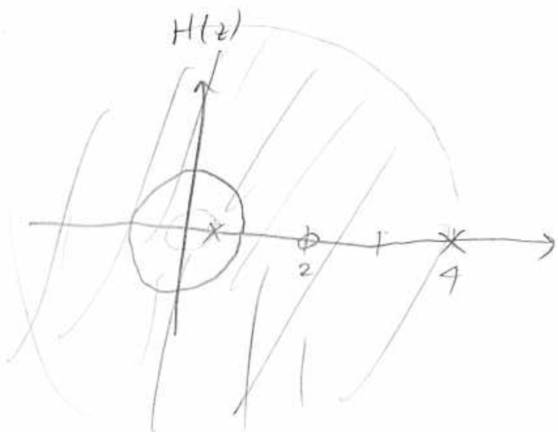
(d) (5 points) For $Y(z) = \frac{3}{(z-4)(z-3)}$ $\;3 < |z| < 4$, find $X(z)$ including all possible ROCs.

$$X(z) = \frac{Y(z)}{H(z)} = \frac{\cancel{3}\,(\cancel{z-4})(z-1/2)}{(\cancel{z-4})(z-3)\,\cancel{3}\,(z-2)}$$

$$= \frac{z-1/2}{(z-3)(z-2)}$$







$$\boxed{X(z) = \frac{z-1/2}{(z-3)(z-2)} \qquad |z| > 3}$$

Problem 4 *(30 points)*

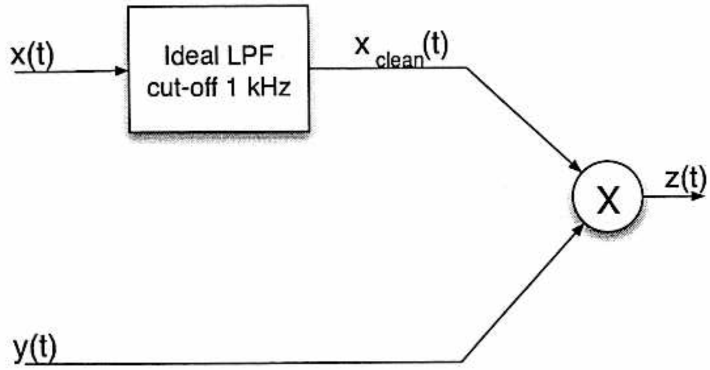


Figure 2: Analog Circuit

Consider the analog circuit in Fig. 2 that implements the system:

$$z(t) = x_{\text{clean}}(t)y(t) \tag{2}$$

where $y(t)$ is an input signal bandlimited to $2\ kHz$ as shown below, and $x_{\text{clean}}(t)$ is produced by filtering a "noisy" input $x(t)$ with an ideal low-pass filter that has cut-off frequency $1\ kHz$ and gain 1. Note that the useful portion of the signal $x(t)$ is bandlimited to $1\ kHz$ and noise exists from $1-2\ kHz$, as shown below.

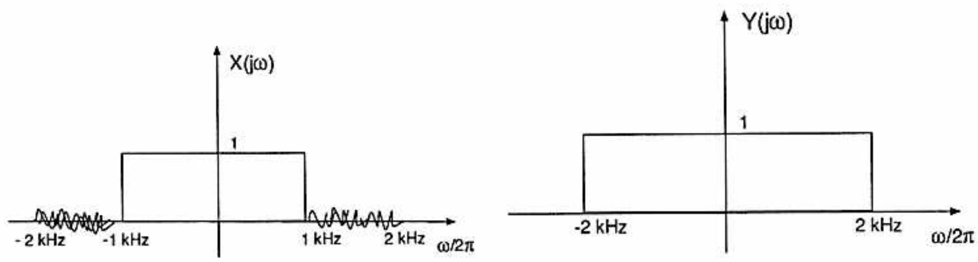
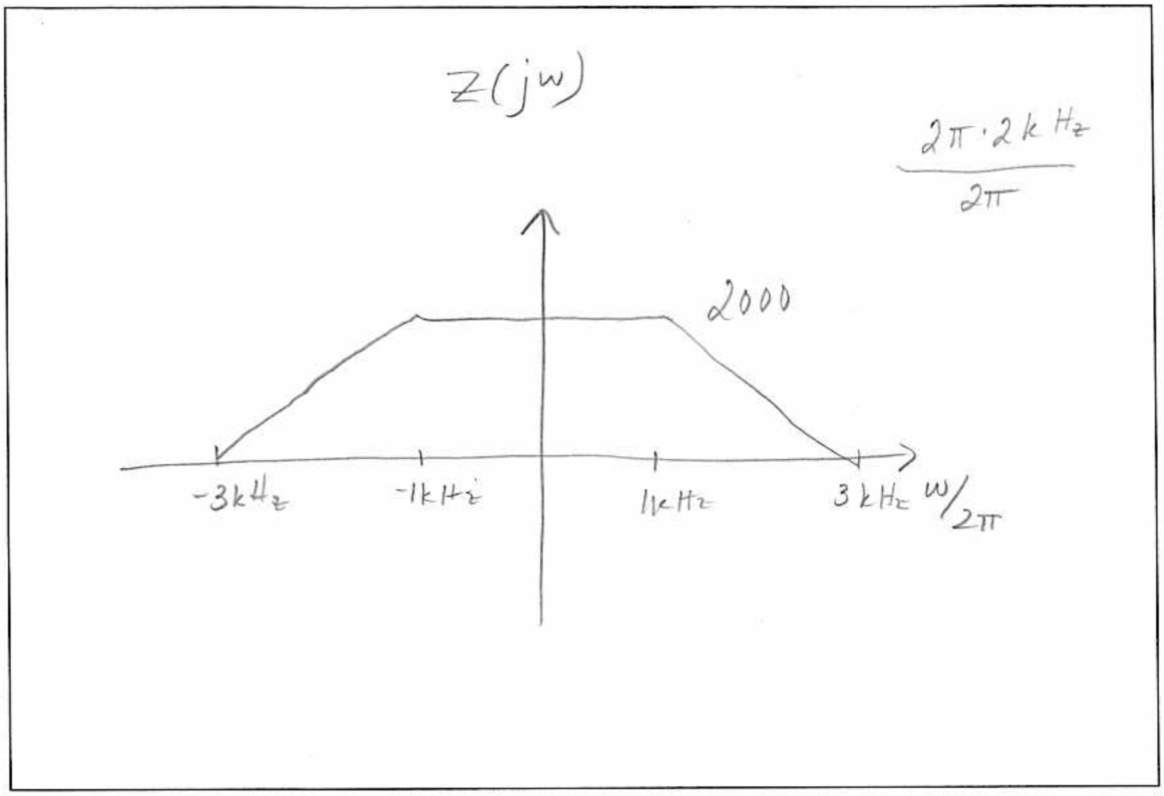


Figure 3: $X(j\omega)$ and $Y(j\omega)$

(a) (4 points) What is the bandwidth of $z(t)$.

3kHz

(b) (8 points) Plot the spectrum of $z(t)$.



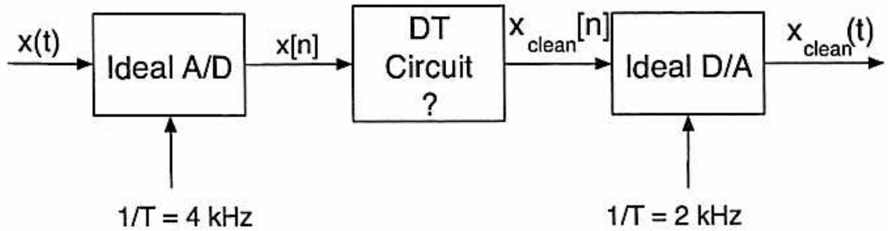
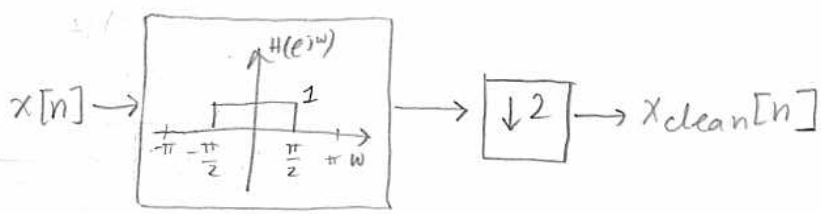


Figure 4: Discrete Time circuit

(c) (8 points) It is desired to replace the analog ideal LPF in Fig. 2 with a discrete-time implementation, as shown in Fig. 4. Note that the ideal A/D operates at 4 kHz while the ideal D/A operates at 2 kHz. Design the Discrete-Time circuit below, i.e., the system that inputs $x[n]$ and outputs $x_{\text{clean}}[n]$.

Remark: You can use ideal digital filters, delay elements, upsamplers, downsamplers, Discrete-Time adders, Discrete-Time multipliers, etc.



(d) (10 points)

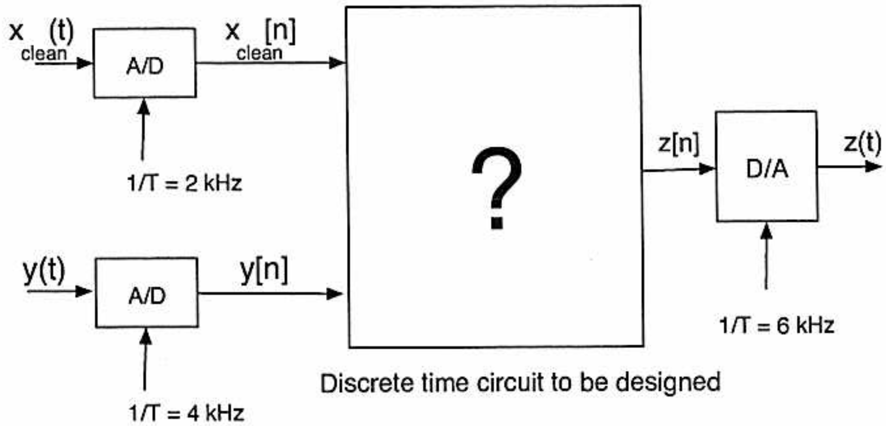


Figure 5: Multiplier circuit

Recall that the specturm of $x_{\text{clean}}(t)$ should be equal to the spectrum of $x(t)$ for $|\omega| < 1\ kHz$, and zero elsewhere. It is desired to implement the analog multiplier circuit for $z(t) = x_{\text{clean}}(t)\ y(t)$ using a Discrete-Time processor, as shown in Fig. 5. Fill in the Discrete-Time design needed to realize this. Note that the D/A is operating at $6\ kHz$.