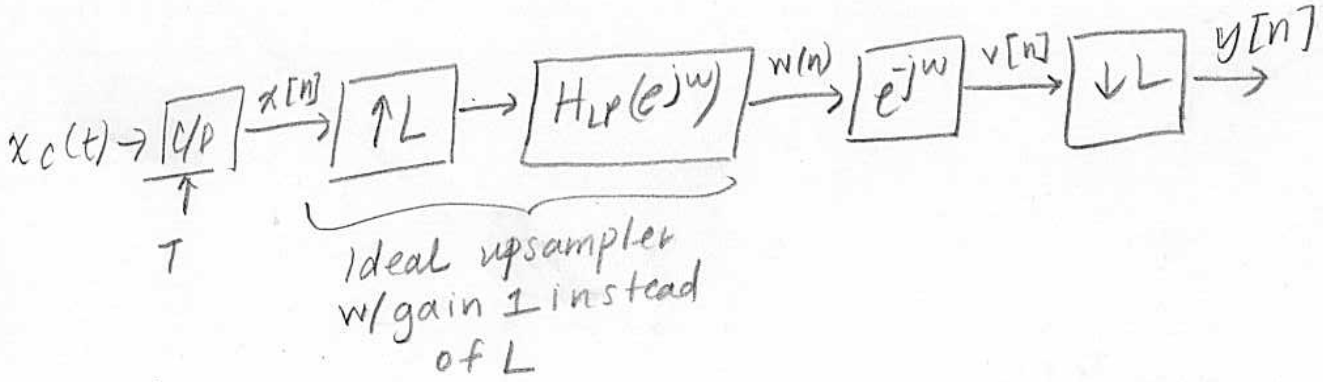
Midterm 1 Review Problems

1. Split $H(e^{jw})$ into a LPF and a delay

$$H(e^{jw}) = H_{LP}(e^{jw})\, e^{-jw}$$

$$H_{LP}(e^{jw}) = \begin{cases} 1 & |w| < \frac{\pi}{L} \\ 0 & \frac{\pi}{L} < |w| < \pi \end{cases}$$



$$x[n] = x_c(nT)$$

$$w[n] = \frac{1}{L} x[n/L] = \frac{1}{L} x_c\left(\frac{nT}{L}\right)$$

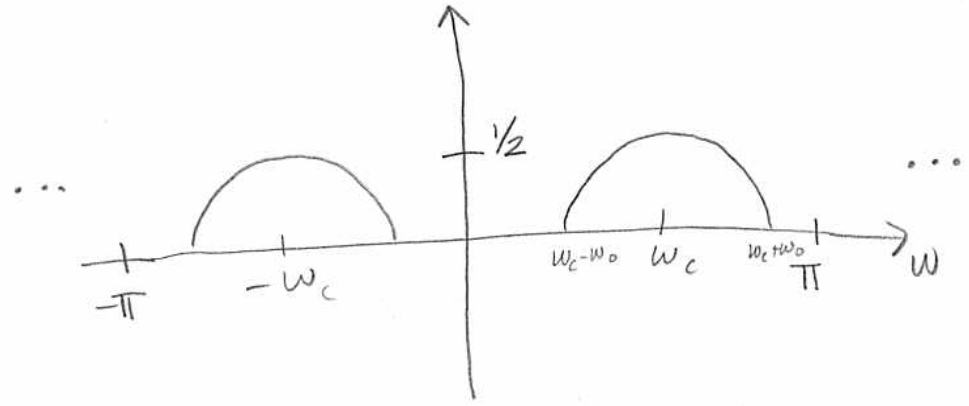
$$v[n] = w[n-1] = \frac{1}{L} x_c\left(\frac{nT}{L} - \frac{T}{L}\right)$$ delay at higher rate

$$y[n] = v[nL] = \frac{1}{L} x_c\left(nT - \frac{T}{L}\right)$$

2. OWN 8.32

(a)

$Y(e^{jw})$



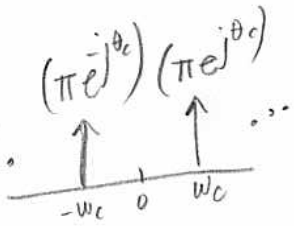
$w_c + w_0 < \pi$

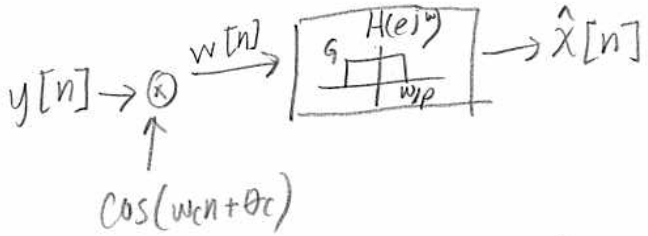
$\Rightarrow w_c < \pi - w_0$

$w_c - w_0 > 0$

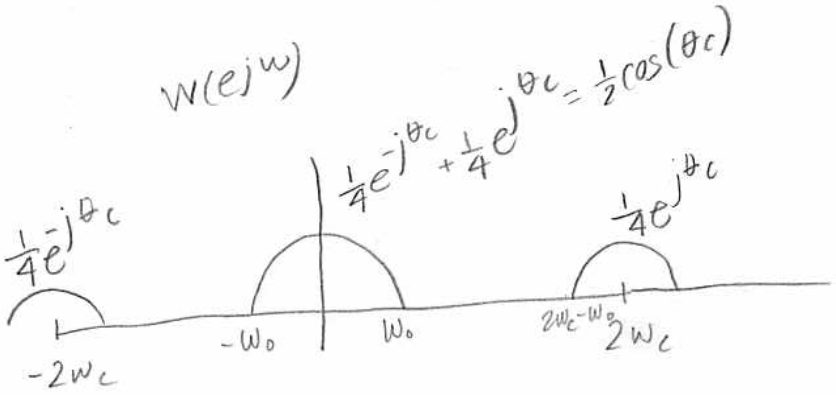
$\Rightarrow w_c > w_0$

(b) $\cos(w_c n + \theta_c) = \dfrac{e^{jw_c n}e^{j\theta_c} + e^{-jw_c n}e^{-j\theta_c}}{2}$ $\xleftrightarrow{\mathcal{F}}$





$W(e^{jw})$



$\hat{x}[n] = x[n]$ if $G = \dfrac{2}{\cos\theta_c}$ thus $\cos\theta_c \neq 0$

or $\theta_c \neq \frac{\pi}{2} + \pi k$

$w_0 < w_{LP} < 2w_c - w_0$