# Homework 11
## Due: Thursday, November 29, 2007, at 5pm
## Homework GSI: Mary Knox

**Reading** OWN Chapter 9.8-9.9, 11.1-11.2, 10.

**Problem 1** *(Unilateral Laplace Transform.)*

OWN Problem 9.65

**Problem 2** *(Pole/Zero Plots)*

Match the pole/zero plots (a)-(e) with the corresponding magnitude responses (1)-(5). In each case, provide a brief justification. (For example: "must have two symmetric peaks, therefore can only be plot (x)".)

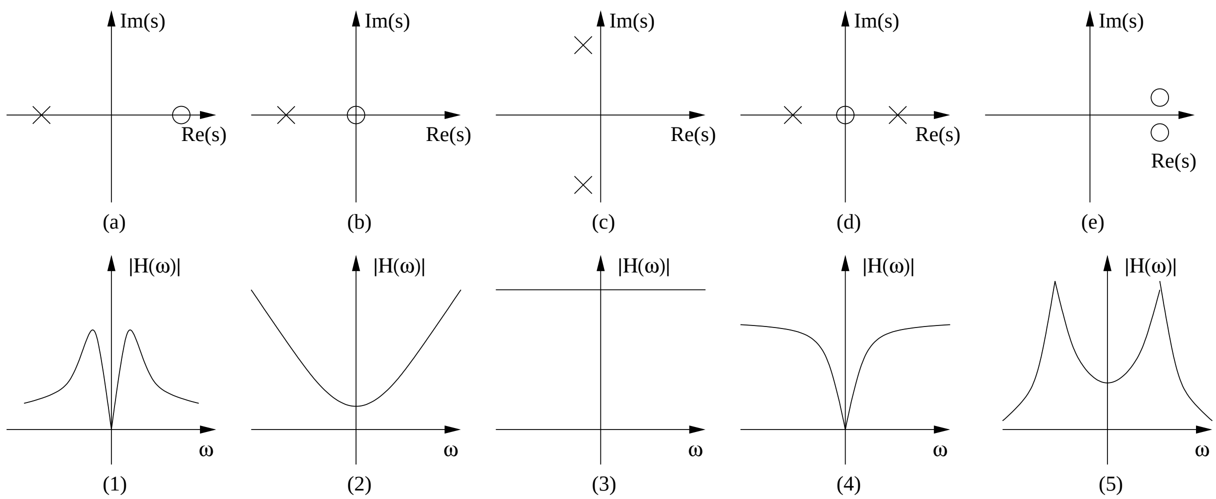


Figure 1: Matching of Pole/Zero Plots and Frequency response.

**Problem 3** *(A simple feedback control system)*

One of the key applications of the Laplace transform is the control of feedback systems. Consider the following simple *causal* feedback system.

The second box, $\frac{s+2}{s-1}$, models an industrial plant. This system is unstable. The task of the engineer is to design the controller $F(s)$ in a clever way. The overall goal of the control is to make the error signal $e(t) = 0$.

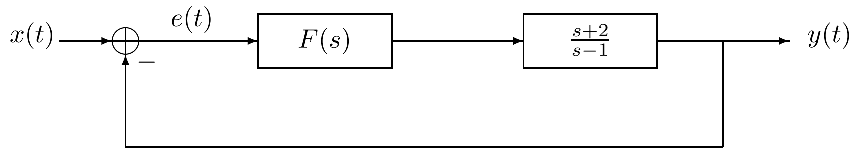


Figure 2: A simple feedback system

*(a)* Determine the overall transfer function

$$T(s) \quad = \quad \frac{Y(s)}{X(s)} \tag{1}$$

and show that the Laplace transform of the error signal satisfies

$$E(s) \quad = \quad (1 - T(s))X(s). \tag{2}$$

*(b)* Suppose that $F(s) = K$, where $K$ is a real number. Determine the range of $K$ such that the overall system $T(s)$ is stable. (Recall that we assume the system to be *casual.*)

**Problem 4** *(Bode Plots)*

As we have seen in class, the Bode plot of a frequency response is simply the plot

$$|H(j\omega)|_{dB} \quad \stackrel{def}{=} \quad 20 \log_{10} |H(j\omega)|. \tag{3}$$

*(a)* In this problem, we use Matlab to confirm Bode's approximation. Consider the system with transfer function

$$H(s) \quad = \quad \frac{1}{1 + s/10}. \tag{4}$$

You may use the following Matlab code:

```
w = [ 0.1:0.1:1000 ];
semilogx(w, 20*log10(abs(1./(1 + j*w/10))));
grid;
```

Print out the resulting plot, and add (with a color pen) the approximation that we have seen in class.

Then repeat this exercise for the phase, using the approximation in Handout 3 and the following Matlab code:

```
w = [ 0.1:0.1:1000 ];
semilogx(w, angle(1./(1 + j*w/10)));
grid;
```

*(b)* Repeat Part *(a)* for the second-order system

$$H(s) \quad = \quad \frac{1}{1 + s/20 + (s/10)^2}. \tag{5}$$

For the *phase response*, use the approximation for very small $\omega$ and for very large $\omega$.

*(c)* By hand (without using Matlab), provide the Bode plot of the *magnitude* of the frequency response of the system with transfer function

$$H(s) = \frac{(s+1)(s+1000)}{(s+10)(s+100)}. \qquad (6)$$

Describe the system behavior in words.

*Hint: Confirm your result using Matlab, but be sure you know how to do it by hand — after all, that's the main point of Bode plots.*

**Problem 5** *(z-Transform Basics.)*

*(a)* OWN 10.22 (b), (d)

*(b)* OWN 10.23, the first and second X(z). No need to do the Taylor series method.

**Problem 6** *(Properties of the z-Transform)*

OWN 10.45

**Problem 7** *(Discrete-time LTI system analysis.)*

A causal LTI system is described by the following system diagram, where $b$ is a real number:

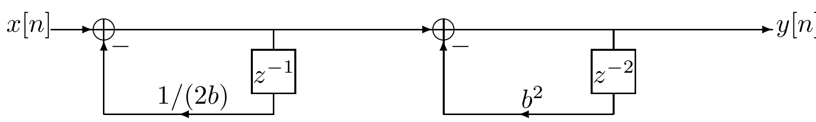


Figure 3: A causal discrete-time system.

Find the pole-zero plot and the range of $b$ such that the system is stable.

**Problem 8** *(Discrete-time LTI system.)*

OWN Problem 10.34

**Problem 9** *(Discrete-time LTI system analysis.)*

OWN Problem 10.46

**Problem 10** *(Pole/Zero plots.)*

Match the pole/zero plots (a)-(e) with the corresponding magnitude responses (1)-(5). Provide a brief justification for each case. (Example: "must have 2 symmetric peaks, therefore can only be plot $(x)$".)

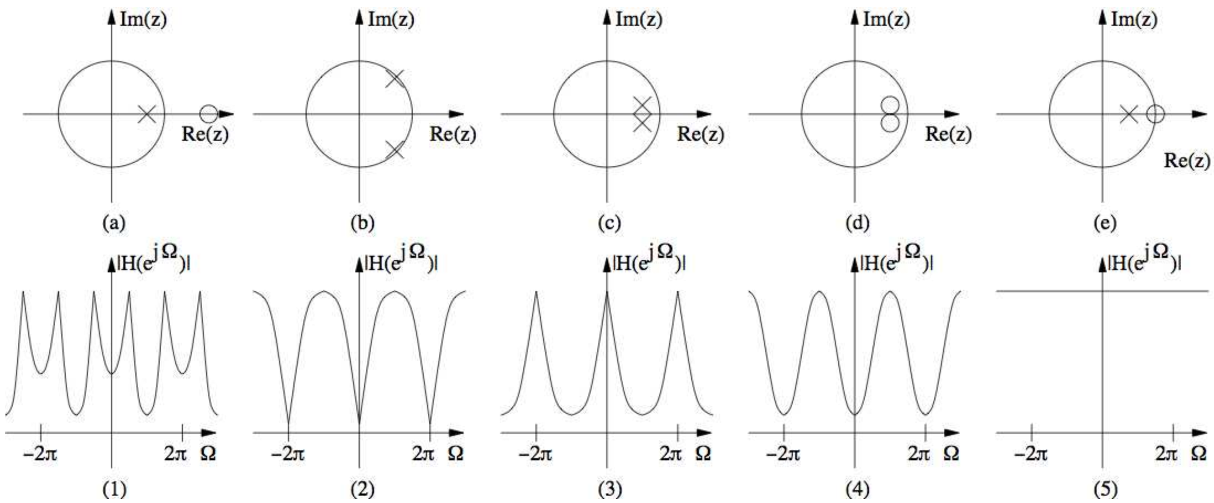


Figure 4: Matching of pole/zero plots and frequency responses.