Homework 11 Solutions

Problem 1 *(Unilateral Laplace Transform.)*

(a) Labeling the voltage across the inductor as $v_L(t)$ and the voltage across the resistor as $v_R(t)$, we use Kirchhoff's voltage law to find that

$$v_i(t) = v_R(t) + v_L(t) + v_o(t)$$

Using the fundamental equation for a capacitor, the current in the circuit is given by $i(t) = C\frac{dv_o(t)}{dt}$. Furthermore, the voltage across the resistor is $v_R(t) = R \cdot i(t) = RC\frac{dv_o(t)}{dt}$ and the voltage across the inductor is given by $v_L(t) = L\frac{di(t)}{dt} = LC\frac{d^2v_o(t)}{dt^2}$. Combining these equations, we obtain

$$v_i(t) = LC\frac{d^2v_o(t)}{dt^2} + RC\frac{dv_o(t)}{dt} + v_o(t)$$

which can be rewritten as

$$\frac{d^2v_o(t)}{dt^2} + \frac{R}{L}\frac{dv_o(t)}{dt} + \frac{1}{LC}v_o(t) = \frac{1}{LC}v_i(t)$$

Substituting in the values of R, L, and C,

$$\frac{d^2v_o(t)}{dt^2} + 3\frac{dv_o(t)}{dt} + 2v_o(t) = 2v_i(t)$$

(b) In order to take the unilateral Laplace transform of this differential equation, we need to have initial conditions at $t = 0^-$. Because the voltage across a capacitor cannot change instantaneously, the initial condition $v_o(0^-) = v_o(0^+)$. Similarly, since the current through an inductor cannot change instantaneously, $i(0^-) = i(0^+)$. Using the fact that $i(t) = C\frac{dv_o(t)}{dt}$, it follows that the initial condition

$$\left.\frac{dv_o(t)}{dt}\right|_{t=0^-} = \left.\frac{dv_o(t)}{dt}\right|_{t=0^+}$$

Now, we can take the unilateral Laplace transform of the differential equation in part (a).

$$s^2\mathcal{V}_o(s) - sv_o(0^-) - v_o'(0^-) + 3s\mathcal{V}_o(s) - 3v_o(0^-) + 2\mathcal{V}_o(s) = 2\mathcal{V}_i(S)$$

Because $v_i(t) = e^{-3t}u(t)$ is equal to 0 for $t < 0^-$, the unilateral Laplace transform of $v_i(t)$ is identical to the bilateral Laplace transform

$$\mathcal{V}_i(s) = \frac{1}{s+3} \qquad\qquad Re\{s\} > -3$$

Substituting the expression for $\mathcal{V}_i(s)$ and the initial conditions

$$\begin{aligned}
\mathcal{V}_o(s)[s^2+3s+2] &= s+2+3+\frac{2}{s+3} && (1)\\
\mathcal{V}_o(s)[(s+2)(s+1)] &= s+5+\frac{2}{s+3} && (2)\\
\mathcal{V}_o(s)[(s+2)(s+1)] &= \frac{(s+5)(s+3)+2}{s+3} && (3)\\
\mathcal{V}_o(s) &= \frac{s^2+8s+17}{(s+1)(s+2)(s+3)} && (4)
\end{aligned}$$

By taking a partial fraction expansion of this expression, we obtain

$$\mathcal{V}_o(s) = \frac{s^2+8s+17}{(s+1)(s+2)(s+3)} = \frac{A}{s+1}+\frac{B}{s+2}+\frac{C}{s+3} \qquad (5)$$

$$(6)$$

$$\begin{aligned}
A &= \mathcal{V}_o(s)(s+1)\Big|_{s=-1} = \frac{s^2+8s+17}{(s+2)(s+3)}\Big|_{s=-1} = 5 && (7)\\
B &= \mathcal{V}_o(s)(s+2)\Big|_{s=-2} = \frac{s^2+8s+17}{(s+1)(s+3)}\Big|_{s=-2} = -5 && (8)\\
C &= \mathcal{V}_o(s)(s+3)\Big|_{s=-3} = \frac{s^2+8s+17}{(s+1)(s+2)}\Big|_{s=-3} = 1 && (9)
\end{aligned}$$

Taking the inverse unilateral Laplace transform (and knowing that ROC of a unilateral Laplace transform must be a right half plane) we find that

$$v_o(t) = 5e^{-t}u(t) - 5e^{-2t}u(t) + e^{-3t}u(t)$$

Problem 2 *(Pole/Zero Plots.)*

(a) (3). Note that

$$\frac{|j\omega - a|}{|j\omega + a|} = \frac{\sqrt{\omega^2+(-a)^2}}{\sqrt{\omega^2+a^2}} = 1$$

(b) (4). $|H(j\omega)|$ must be zero at $\omega = 0$. Since the number of poles and the number of zeros are equal, the limit of $|H(j\omega)|$ as ω approaches ∞ is non-zero and finite.

(c) (5). $|H(j\omega)|$ must have two symmetric peaks because of the two poles.

(d) (1). $|H(j\omega)|$ must be zero at $\omega = 0$. Since there are two poles and only one zero, the limit of $|H(j\omega)|$ as ω approaches ∞ is equal to 0.

(e) (2). $|H(j\omega)|$ should approach 0 at $\omega = 0$. It will not equal 0, because the zeros are not on the $j\omega$ axis. Also, because there are two zeros and no poles, $|H(j\omega)|$ should increase with $|\omega|$.

Problem 3 *(A simple feedback control system.)*

(a)

$$Y(s) = \frac{s+2}{s-1}F(s)E(s) \tag{10}$$
$$E(s) = X(s) - Y(s) \tag{11}$$

Substituting the second equation into the first, we see that

$$Y(s) = \frac{s+2}{s-1}F(s)\left(X(s) - Y(s)\right)$$
$$T(s) = \frac{Y(s)}{X(s)} = \frac{\frac{s+2}{s-1}F(s)}{1 + \frac{s+2}{s-1}F(s)}$$

Further, we observe that

$$E(s) = X(s) - Y(s) = X(s) - T(s)X(s) = (1 - T(s))\,X(s)$$

(b) The overall transfer function of the feedback system is

$$T(s) = \frac{\frac{s+2}{s-1}K}{1 + \frac{s+2}{s-1}K} = \frac{K(s+2)}{(K+1)s + (2K-1)}$$

where K is a real number which represents an adjustable gain in the system. There are multiple ways to solve this problem. Here we will describe two methods.

Method 1

From the above equation we see that this system has one pole. We are given that the system is causal. Thus in order for the system to be stable the pole should be to the left of the $j\omega$-axis. First, let's solve for the pole.

$$(K+1)s + (2K-1) = 0 \tag{12}$$
$$s = \frac{1-2K}{K+1} \tag{13}$$

For the pole to occur on the left side of the $j\omega$-axis, either $(1-2K) < 0 \cap (K+1) > 0$ or $(1-2K) > 0 \cap (K+1) < 0$. Solving those equations we get that $K > \frac{1}{2}$ or $K < -1$.

Method 2

The root locus is the path in the complex plane of the poles of $T(s)$ as K is varied. $T(s)$ has a zero at $s = -2$, and a pole at $s = -\frac{2K-1}{K+1}$. When $K = 0$, the pole is located at $s = 1$. For K positive, as $K \to \infty$, the pole moves left to $s \to -2$. For K negative, as $K \to -1$, the pole moves right to $s \to \infty$. As K is varied from -1 to $-\infty$, the pole moves right from $-\infty$ to $s \to -2$.

Since the system is known to be causal, the ROC is a right-half plane, to the right of the rightmost pole. The system is stable iff the ROC includes the $j\omega$-axis. Therefore the system is stable iff all the poles of $T(s)$ lie in the left-half plane, which is true when $K < -1$ or $K > \frac{1}{2}$.

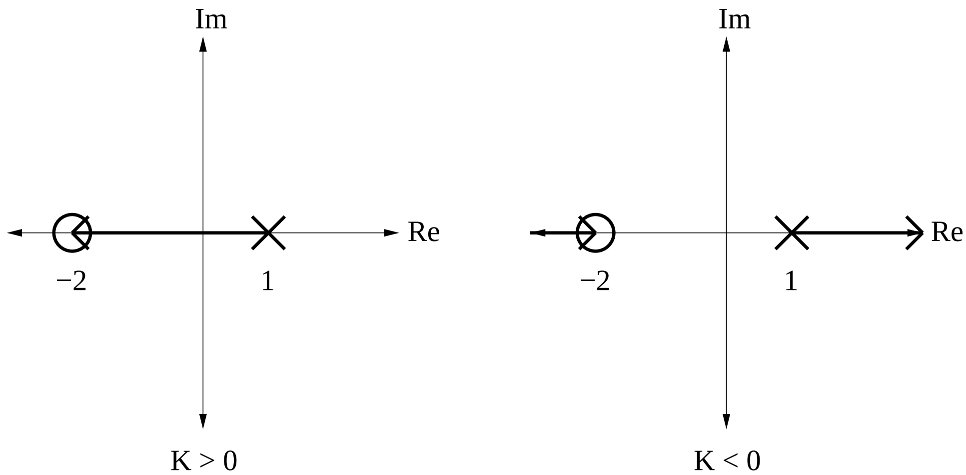


Figure 1: Root Locus for Problem 3

Problem 4 *(Bode Plots.)*

(a) The Bode plot of the magnitude frequency response of system $H(s)$ is defined as

$$\begin{aligned} H(s) &= \frac{1}{1+s/10} \\ |H(j\omega)|_{dB} &\stackrel{def}{=} 20\log_{10}|H(j\omega)| \\ &= -20\log_{10}\left|1+\frac{j\omega}{10}\right| \end{aligned}$$

For low frequencies $\omega << 10$, the magnitude frequency response can be approximated as

$$-20\log_{10}\left|1+\frac{j\omega}{10}\right| \approx 0$$

For high frequencies $\omega >> 10$, the magnitude frequency response can be approximated as

$$-20\log_{10}\left|1+\frac{j\omega}{10}\right| \approx -20\log_{10}\left|\frac{j\omega}{10}\right| = -20\log_{10}\left|\frac{\omega}{10}\right|$$

The phase response of $H(s)$

$$\arg(H(j\omega)) = -\arg(1+\frac{j\omega}{10})$$

can also be approximated for low frequencies $\omega << 10$ as

$$-\arg(1+\frac{j\omega}{10}) \approx 0$$

and for high frequencies $\omega >> 10$ as

$$-\arg(1+\frac{j\omega}{10}) \approx -\arg(\frac{j\omega}{10}) = -\frac{\pi}{2}$$

The Bode plots of the magnitude frequency response and phase response are shown in Figure 2.

(b) The Bode plot of the magnitude frequency response of system $H(s)$ is

$$\begin{aligned}
H(s) &= \frac{1}{1 + s/20 + (s/10)^2} \\
|H(j\omega)|_{dB} &= -20\log_{10}\left|1 + \frac{1}{2}\frac{j\omega}{10} + \left(\frac{j\omega}{10}\right)^2\right|
\end{aligned}$$

For low frequencies $\omega << 10$, (ie $|\omega/10| << 1$), this can be approximated by

$$-20\log_{10}\left|1 + \frac{1}{2}\frac{j\omega}{10} + \left(\frac{j\omega}{10}\right)^2\right| \approx 0$$

For high frequencies $\omega >> 10$, (ie $|\omega/10| >> 1$), this can approximated by

$$-20\log_{10}\left|1 + \frac{1}{2}\frac{j\omega}{10} + \left(\frac{j\omega}{10}\right)^2\right| \approx -20\log_{10}\left|\left(\frac{j\omega}{10}\right)^2\right| = -40\log_{10}\left|\frac{\omega}{10}\right|$$

The phase response of $H(s)$ is

$$-\arg\left(1 + \frac{1}{2}\frac{j\omega}{10} + \left(\frac{j\omega}{10}\right)^2\right)$$

For low frequencies $\omega << 10$, (ie $|\omega/10| << 1$), the phase response can be approximated as

$$-\arg\left(1 + \frac{1}{2}\frac{j\omega}{10} + \left(\frac{j\omega}{10}\right)^2\right) \approx 0$$

For high frequencies $\omega >> 10$, (ie $|\omega/10| >> 1$), the phase response can be approximated as

$$\begin{aligned}
-\arg\left(1 + \frac{1}{2}\frac{j\omega}{10} + \left(\frac{j\omega}{10}\right)^2\right) &\approx -\arg\left(\left(\frac{j\omega}{10}\right)^2\right) \\
&= -\arg\left(-\left(\frac{\omega}{10}\right)^2\right) \\
&= -\pi
\end{aligned}$$

The Bode plots of the magnitude frequency response and phase response are shown in Figure 3.

(c) The Bode plot of the magnitude frequency response of the system $H(s)$ is

$$\begin{aligned}
H(s) &= \frac{(s+1)(s+1000)}{(s+10)(s+100)} \\
&= \frac{(1+s)(1+\frac{s}{1000})}{(1+\frac{s}{10})(1+\frac{s}{100})} \\
|H(j\omega)|_{dB} &= 20\log_{10}|H(j\omega)| \\
&= 20\log_{10}|1+j\omega| + 20\log_{10}\left|1 + \frac{j\omega}{1000}\right| - 20\log_{10}\left|1 + \frac{j\omega}{10}\right| - 20\log_{10}\left|1 + \frac{j\omega}{100}\right|
\end{aligned}$$

For low frequencies, we can approximate each term of the magnitude frequency response

$$\begin{aligned}
\omega << 1 &\Rightarrow 20\log_{10}|1+j\omega| \approx 0 \\
\omega << 1000 &\Rightarrow 20\log_{10}\left|1 + \frac{j\omega}{1000}\right| \approx 0 \\
\omega << 10 &\Rightarrow -20\log_{10}\left|1 + \frac{j\omega}{10}\right| \approx 0 \\
\omega << 100 &\Rightarrow -20\log_{10}\left|1 + \frac{j\omega}{100}\right| \approx 0
\end{aligned}$$

For high frequencies, we can approximate each term of the magnitude frequency response

$$\begin{aligned}
\omega >> 1 \quad &\Rightarrow \quad 20\log_{10}|1+j\omega| \approx 20\log_{10}|\omega| \\
\omega >> 1000 \quad &\Rightarrow \quad 20\log_{10}\left|1+\frac{j\omega}{1000}\right| \approx 20\log_{10}\left|\frac{\omega}{1000}\right| \\
\omega >> 10 \quad &\Rightarrow \quad -20\log_{10}\left|1+\frac{j\omega}{10}\right| \approx -20\log_{10}\left|\frac{\omega}{10}\right| \\
\omega >> 100 \quad &\Rightarrow \quad -20\log_{10}\left|1+\frac{j\omega}{100}\right| \approx -20\log_{10}\left|\frac{\omega}{100}\right|
\end{aligned}$$

The overall magnitude frequency response Bode plot is found by summing these terms. Thus the system $H(s)$ has the approximate frequency response of a bandpass filter. The Bode plot of the magnitude frequency response is shown in Figure 4.

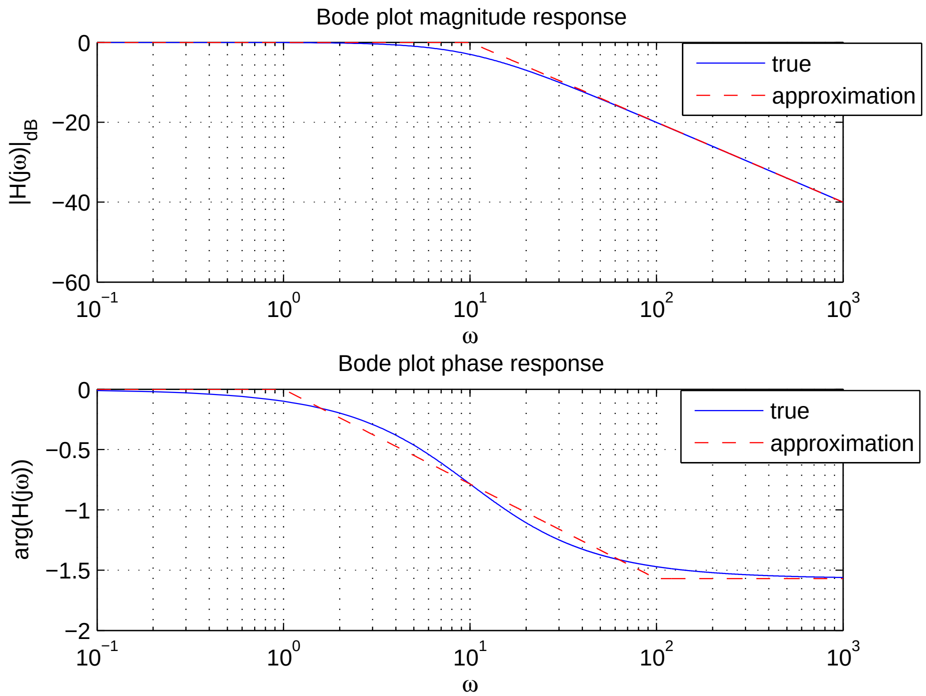


Figure 2: Problem 4 (a)

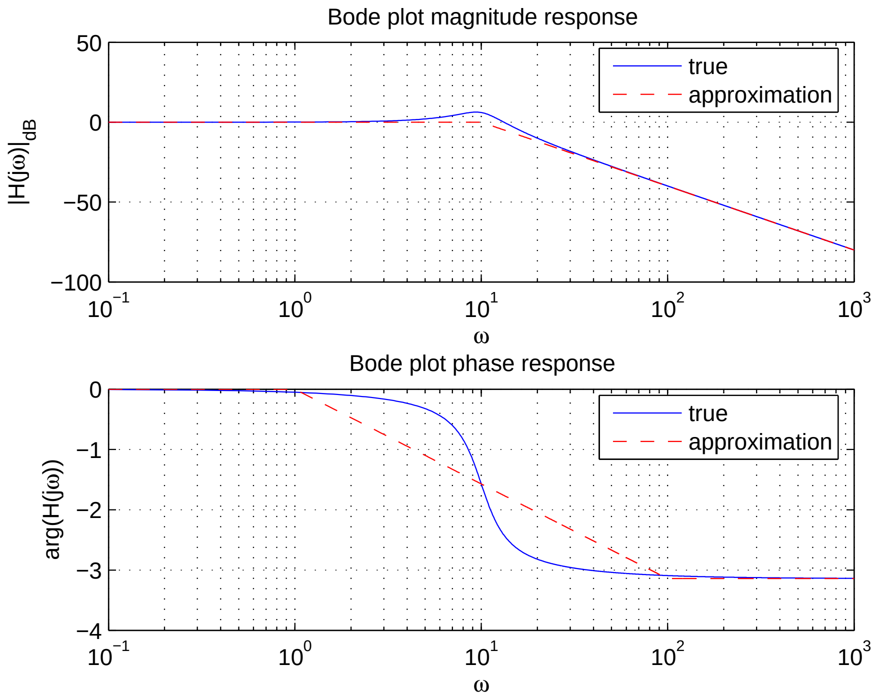


Figure 3: Problem 4 (b)

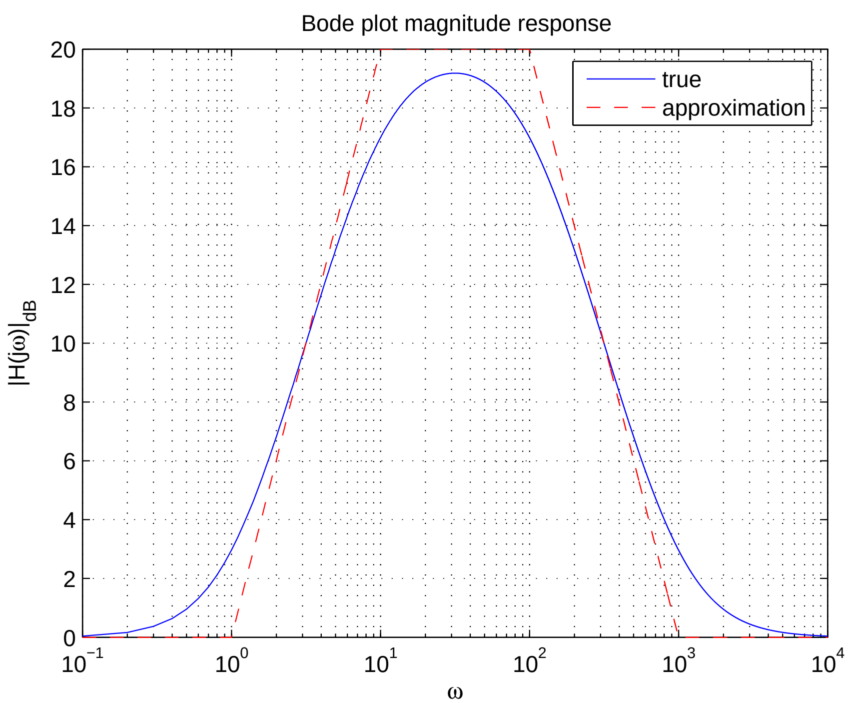


Figure 4: Problem 4 (c)

Problem 5 *(z-Transform Basics.)*

(a) **OWN 10.22 (b)**

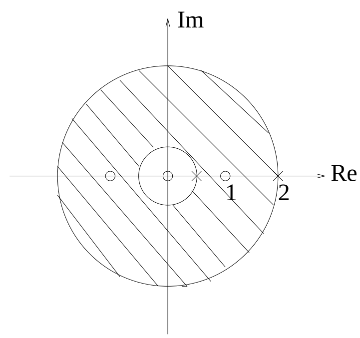
We can rewrite $x[n]$ as

$$\begin{aligned} x[n] &= n\left(\frac{1}{2}\right)^{|n|} \\ &= n\left(\frac{1}{2}\right)^{n} u[n] + n\left(\frac{1}{2}\right)^{-n} u[-n-1] \\ &= n\left(\frac{1}{2}\right)^{n} u[n] + n\,(2)^{n}\, u[-n-1]. \end{aligned}$$

In OWN Table 10.2, we find the z-transform of $n\left(\frac{1}{2}\right)^n u[n]$ is $\frac{\frac{1}{2}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)^2}$, with ROC $|z| > \frac{1}{2}$. Also in Table 10.2, we find the z-transform of $n2^n u[-n-1]$ is $-\frac{2z^{-1}}{(1-2z^{-1})^2}$, with ROC $|z| < 2$. Thus by linearity (see OWN Table 10.1),

$$\begin{aligned} X(z) &= \frac{\frac{1}{2}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)^2} - \frac{2z^{-1}}{\left(1-2z^{-1}\right)^2} \\ &= -\frac{3}{2}\frac{z(z+1)(z-1)}{(z-\frac{1}{2})^2(z-2)^2} \end{aligned}$$

with ROC $\frac{1}{2} < |z| < 2$. Since the ROC includes the unit circle, the Fourier transform of $x[n]$ exists.



OWN 10.22 (d)

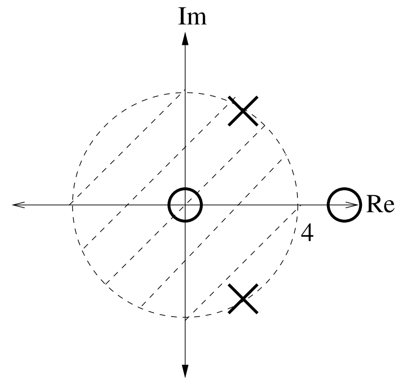
We can rewrite $x[n]$ as

$$\begin{aligned} x[n] &= 4^n \cos(\frac{2\pi}{6}n + \frac{\pi}{4})u[-n-1] \\ &= \frac{1}{2}\left(e^{j\left(\frac{2\pi}{6}n+\frac{\pi}{4}\right)} + e^{-j\left(\frac{2\pi}{6}n+\frac{\pi}{4}\right)}\right) 4^n u[-n-1] \\ &= \frac{1}{2}e^{j\frac{\pi}{4}}e^{j\left(\frac{2\pi}{6}n\right)}4^n u[-n-1] + \frac{1}{2}e^{-j\frac{\pi}{4}}e^{-j\left(\frac{2\pi}{6}n\right)}4^n u[-n-1]. \end{aligned}$$

In OWN Table 10.2, we find the z-transform of $y[n] = 4^n u[-n-1]$ is $Y(z) = -\frac{1}{1-4z^{-1}}$, with ROC $|z| < 4$. The scaling in the z-domain property, given in OWN Table 10.1, states that the z-transform of $e^{j\omega_0 n}y[n]$ is $Y(e^{-j\omega_0}z)$. Therefore, by linearity, the z-transform of $x[n]$ is

$$\begin{aligned} X(z) &= -\frac{\frac{1}{2}e^{j\pi/4}}{1-4e^{j2\pi/6}z^{-1}} - \frac{\frac{1}{2}e^{-j\pi/4}}{1-4e^{-j2\pi/6}z^{-1}} \\ &= -\frac{z(\cos(\frac{\pi}{4})z - 4\cos(\frac{\pi}{12}))}{(z-4e^{j2\pi/6})(z-4e^{-j2\pi/6})} \end{aligned}$$

with ROC $|z| < 4$. Since the ROC includes the unit circle, the Fourier transform of $x[n]$ exists.



(b) **OWN 10.23 (i)**

By partial fraction expansion, we rewrite $X(z)$ as

$$\begin{aligned} X(z) &= \frac{1 - z^{-1}}{1 - \frac{1}{4}z^{-2}} \\ &= \frac{1 - z^{-1}}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{2}z^{-1}\right)} \\ &= \frac{-\frac{1}{2}}{1 - \frac{1}{2}z^{-1}} + \frac{\frac{3}{2}}{1 + \frac{1}{2}z^{-1}} \end{aligned}$$

In OWN Table 10.2, we find the inverse z-transform of $\frac{1}{1-\alpha z^{-1}}$ with ROC $|z| > |\alpha|$ is $\alpha^n u[n]$. Thus by linearity,

$$x[n] = -\frac{1}{2}\left(\frac{1}{2}\right)^n u[n] + \frac{3}{2}\left(-\frac{1}{2}\right)^n u[n].$$

OWN 10.23 (ii)

Again, we rewrite $X(z)$ using partial fraction expansion.

$$\begin{aligned} X(z) &= \frac{1 - z^{-1}}{1 - \frac{1}{4}z^{-2}} \\ &= \frac{-\frac{1}{2}}{1 - \frac{1}{2}z^{-1}} + \frac{\frac{3}{2}}{1 + \frac{1}{2}z^{-1}} \end{aligned}$$

Now we find in OWN Table 10.2, that the inverse z-transform of $\frac{1}{1-\alpha z^{-1}}$ with ROC $|z| < |\alpha|$ is $-\alpha^n u[-n-1]$. Thus by linearity,

$$x[n] = \frac{1}{2}\left(\frac{1}{2}\right)^n u[-n-1] - \frac{3}{2}\left(-\frac{1}{2}\right)^n u[-n-1].$$

Problem 6 *(Properties of the z-Transform.)*

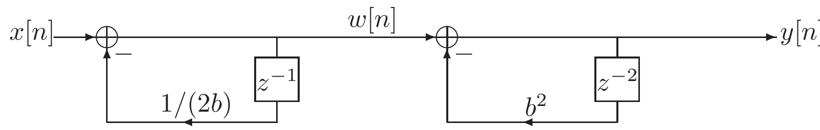
(a) Yes. The order of the numerator is equal to the order of the denominator in the given z-transform. Therefore, we can perfom long-division to expand the z-transform such that the highest power of z in the expansion is 0. This would make $x[n] = 0$ for $n < 0$.

(b) No. This z-transform can be obtained by multiplying the z-transform of the previous part by z. Hence, its inverse is the inverse of the previous part shifted by 1 to the left. This implies that the resultant signal is not zero at $n = -1$.

(c) Yes. We can perform long-division to expand the z-transform such that the highest power of z in the expansion is -1. This would make $x[n] = 0$ for $n \leq 0$.

(d) No. When long-division is used to expand the z-transform, the highest power of z in the expansion is 1. This would make $x[-1] \neq 0$.

Problem 7 *(Discrete-time LTI system analysis.)*



To find the overall transfer function $H(z)$ of this causal LTI system, we examine the input-output relationships of the sub-systems.

$$
\begin{aligned}
W(z) &= X(z) - \frac{1}{2b} z^{-1} W(z) \\
W(z)\left(1 + \frac{1}{2b} z^{-1}\right) &= X(z) \\
\frac{W(z)}{X(z)} &= \frac{1}{1 + \frac{1}{2b} z^{-1}} \\
Y(z) &= W(z) - b^2 z^{-2} Y(z) \\
Y(z)\left(1 + b^2 z^{-2}\right) &= W(z) \\
\frac{Y(z)}{W(z)} &= \frac{1}{1 + b^2 z^{-2}} \\
H(z) &= \frac{Y(z)}{X(z)} = \frac{Y(z)}{W(z)} \frac{W(z)}{X(z)} \\
&= \frac{1}{\left(1 + \frac{1}{2b} z^{-1}\right)\left(1 + b^2 z^{-2}\right)} \\
&= \frac{z^3}{\left(z + \frac{1}{2b}\right)(z + jb)(z - jb)}
\end{aligned}
$$

$H(z)$ has three zeros located at $z = 0$, and three poles located at $z = -\frac{1}{2b}, -jb, jb$ respectively. We know the system is causal and rational, which implies the ROC must be outside of the pole with the largest magnitude. Since the system is stable iff the ROC includes the unit circle, all the poles of $H(z)$ must be inside the unit circle.

$$
\begin{aligned}
|jb| < 1 &\Leftrightarrow |b| < 1 \\
|-jb| < 1 &\Leftrightarrow |b| < 1 \\
\left|-\frac{1}{2b}\right| < 1 &\Leftrightarrow \frac{1}{2} < |b|
\end{aligned}
$$

Therefore $H(z)$ is stable iff

$$\frac{1}{2} < |b| < 1.$$

Problem 8 *(Discrete-time LTI system.)*

OWN Problem 10.34

$$y[n] = y[n-1] + y[n-2] + x[n-1]$$

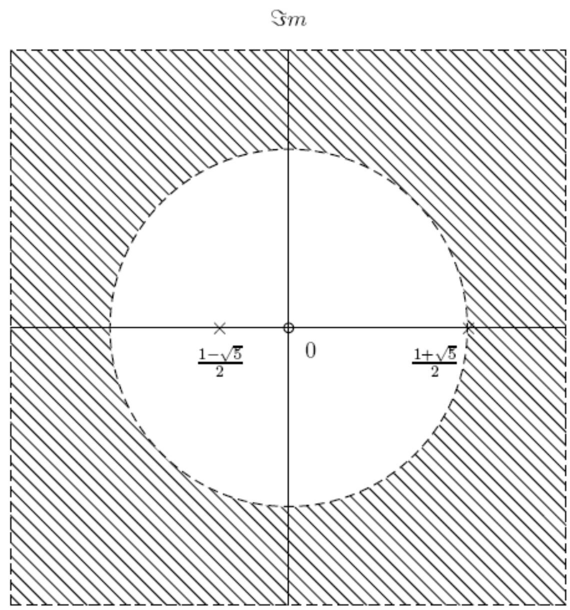
Taking the z-transform of this equation, we get

$$Y(z) = z^{-1}Y(z) + z^{-2}Y(z) + z^{-1}X(z)$$

$$\begin{aligned} H(z) &= \frac{Y(z)}{X(z)} = \frac{z^{-1}}{1 - z^{-1} - z^{-2}} = \frac{z}{z^2 - z - 1} \\ &= \frac{z}{\left(z - \frac{1+\sqrt{5}}{2}\right)\left(z - \frac{1-\sqrt{5}}{2}\right)} \end{aligned}$$

Therefore $H(z)$ has a zero at $z = 0$ and poles at $z = \frac{1 \pm \sqrt{5}}{2}$.

Since the system is causal, the ROC of $H(z)$ will be outside the circle containing its outermost pole. The pole-zero map and ROC are depicted below.



(b)

The partial fraction expansion of $H(z)$ is

$$H(z) = \frac{1/\sqrt{5}}{1 - \frac{1+\sqrt{5}}{2}z^{-1}} - \frac{1/\sqrt{5}}{1 - \frac{1-\sqrt{5}}{2}z^{-1}}.$$

Therefore

$$h[n] = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n u[n] - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n u[n].$$

(c)

The system is unstable, as its ROC does not contain the unit circle. The instability is also apparent in $h[n]$, as the $\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n u[n]$ term will grown indefinitely as $n \to \infty$.

To make the system stable, the ROC must contain the unit circle. The ROC should then be: $\frac{\sqrt{5}-1}{2} < |z| < \frac{\sqrt{5}+1}{2}$. In this case, we get

$$h[n] = -\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n u[-n-1] - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n u[n].$$

Problem 9 *(Discrete-time LTI system analysis.)*

(a)

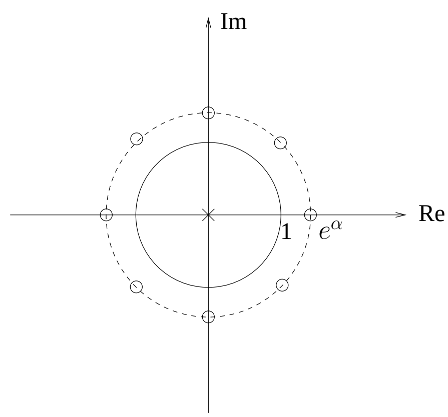
$$x[n] = s[n] - e^{8\alpha} s[n-8]$$

Taking the z-transform,

$$X(z) = S(z)(1 - e^{8\alpha} z^{-8}) \quad (14)$$

$$H_1(z) = \frac{X(z)}{S(z)} = 1 - e^{8\alpha} z^{-8} = \frac{z^8 - e^{8\alpha}}{z^8} \quad (15)$$

This system has an 8th order pole at $z = 0$ and 8 zeros distributed around a circle of radius e^{α}. The ROC is everywhere on the z-plane except at $z = 0$.



(b)

$$H_2(z) = \frac{z^8}{z^8 - e^{8\alpha}} = \frac{1}{1 - z^{-8} e^{8\alpha}}$$

There are two possible ROCs for $H_2(z)$: $|z| < e^{\alpha}$ or $|z| > e^{\alpha}$. If the ROC is $|z| < e^{\alpha}$, then the ROC includes the unit circle. This in turn implies that the system would be stable and anti-causal. If the ROC is $|z| > e^{\alpha}$, then the ROC would not include the unit circle. This in turn implies that the system would be unstable and causal.

(c) We need to choose the ROC to be $|z| < e^{\alpha}$ in order to get a stable system. Now consider

$$P(z) = \frac{1}{1 - z^{-1} e^{8\alpha}}$$

with ROC $|z| < e^{\alpha}$. Taking the inverse z-transform, we get

$$p[n] = -e^{8\alpha n} u[-n-1].$$

Now, note that

$$H_2(z) = P(z^8).$$

From Table 10.1, we know that

$$h_2[n] = \begin{cases} p[n/8] = -e^{\alpha n} u[-n-1], & \text{if } n \bmod 8 = 0 \\ 0, & \text{otherwise} \end{cases}$$

Problem 10 *(Pole/Zero plots.)*

(a) Pole-zero plot (a) $\Leftrightarrow$ magnitude response (5).

A pole at $z = a$ and a zero at $z = 1/a$ will cancel each other's effects on the magnitude response, resulting in a constant magnitude response.

(b) Pole-zero plot (b) $\Leftrightarrow$ magnitude response (1).

Magnitude response must have two symmetric peaks around $\omega = \pi/2$ and $\omega = -\pi/2$.

(c) Pole-zero plot (c) $\Leftrightarrow$ magnitude response (3).

Magnitude response must peak near $\omega = 0$. Although we might expect two peaks corresponding to two poles, if the poles are sufficiently close their peaks will merge.

(d) Pole-zero plot (d) $\Leftrightarrow$ magnitude response (4).

Magnitude response must approach but not reach 0 at $\omega = 0$.

(e) Pole-zero plot (e) $\Leftrightarrow$ magnitude response (2).

Magnitude response must hit 0 at $\omega = 0$.