EECS 120 Signals & Systems University of California, Berkeley: Fall 2007
Ramchandran September 20, 2007

Homework 4

Due: Thursday, September 27, 2007, at 5pm
Homework 4 GSI: Mark Johnson

(*Submit your grades to ee120.gsi@gmail.com*)

Reading OWN Chapter 3-4 (Fourier Representations.)

Practice Problems *(Suggestions.)* OWN 4.5, 4.11, 4.19

Problem 1 *(Properties of the CTFS.)*

OWN Problem 3.46, all parts (details below). For Part (a), start out by showing (in 3 lines at most!) that

$$c_k = \frac{1}{T}\int_T z(t)e^{-jk\omega_0 t}dt \tag{1}$$

$$= \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} a_n b_m \left(\frac{1}{T}\int_T e^{j(n+m-k)\omega_0 t}dt\right). \tag{2}$$

In this case, do not worry about shifting around infinite sums. Then, evaluate the integral to obtain

$$c_k = \sum_{n=-\infty}^{\infty} a_n b_{k-n}. \tag{3}$$

Then, do Part (b), but **only** for Figure P3.46(a)

Finally, do Part (c).

Problem 2 *(Properties of the CTFS.)*

Recall that if a signal is periodic with period T, then it is also periodic with period mT, for any integer m.

Assume $x(t)$ is a periodic signal with fundamental period T, whose Fourier series coefficients a_k are given by Equation 3.39 in OWN. Let b_k be the Fourier series coefficients that are found by considering $x(t)$ to be periodic with period $2T$. Prove that

$$b_k = \begin{cases} a_{k/2} & k \text{ even} \\ 0 & k \text{ odd} \end{cases}$$

Problem 3 *(Properties of the DTFS.)*

(a) OWN Problem 3.48, Part (a)

(b) OWN Problem 3.48, Part (e)

Problem 4 *(Properties of the DTFS.)*

(a) OWN Problem 3.48, Part (f)

(b) OWN Problem 3.48, Part (h)

Problem 5 *(FT.)*

(a) OWN Problem 4.22, Part (e)

(b) OWN Problem 4.23, Parts (a) and (b)

Problem 6 *(FT.)*

(a) OWN Problem 4.29, **only** for the signal $x_a(t)$ and $x_c(t)$

(b) OWN Problem 4.41

Problem 7 *(FT.)*

OWN 4.36 (all parts)

Problem 8 *(Frequency response of linear time-invariant systems.)*

Let a system be specified by a differential equation. Then, its frequency response can be found easily, as you will establish in this homework problem.

(a) Suppose that the LTI system is specified by

$$\sum_{k=0}^{N} a_k \frac{d^k}{dt^k} y(t) = \sum_{m=0}^{M} b_m \frac{d^m}{dt^m} x(t). \tag{4}$$

Solve the differential equation for the input $x(t) = e^{j\omega t}$. As we have seen in class, if the input to a continuous-time LTI system is $x(t) = e^{j\omega t}$, then the output can be expressed as $y(t) = H(j\omega)e^{j\omega t}$. Plug this into the above differential equation to determine $H(j\omega)$. You will obtain an expression for $H(j\omega)$ in terms of the coefficients $a_k, k = 0, \ldots, N$ and $b_m, m = 0, \ldots, M$.

(b) For the system specified by the differential equation

$$2\frac{dy(t)}{dt} + 6y(t) = x(t), \tag{5}$$

determine the frequency response $H(j\omega)$, and the corresponding impulse response $h(t)$. Then, find the output when the input is $x(t) = \sin(t/4)$. *Hint:* Write $\sin(t/4)$ in terms of functions of the form $e^{j\omega_0 t}$, and recall from class that for such inputs, the output is imply given by $y(t) = H(j\omega_0)x(t)$.