Homework 7 Solutions

(*Submit your grades to ee120.gsi@gmail.com*)

Problem 1 *(Interpolation and Decimation)*

OWN Problem 7.19

Label the output of the Zero insertion box $x_z[n]$, as shown in Figure 1.

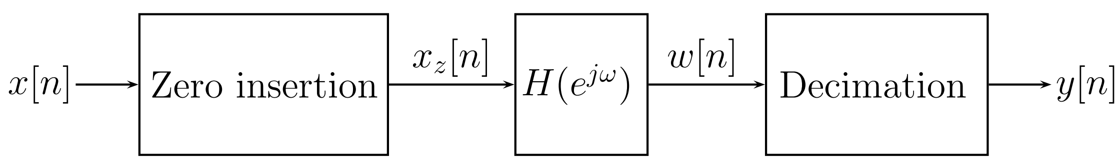

Figure 1: Block Diagram for Problem 1.

Since we are given $x[n]$ we can determine $X(e^{j\omega})$.

$$X(e^{j\omega}) = \begin{cases} 1, & \text{if } 0 \leq |\omega| \leq \omega_1 \\ 0, & \text{if } \omega_1 < |\omega| \leq \pi \end{cases}$$

Since we are inserting 2 zeros between the points in the original sequence $x[n]$,

$$X_z(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x_z[n]e^{-j\omega n} = \sum_{n \text{ if } n \bmod 3=0} x[n/3]e^{-j\omega n} = \sum_{k=-\infty}^{\infty} x[k]e^{-j\omega 3k} = X(e^{j3\omega})$$

Let's examine decimation in the frequency domain:

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} y[n]e^{-j\omega n} = \sum_{n=-\infty}^{\infty} w[5n]e^{-j\omega n} = \sum_{k \text{ if } k \bmod 5=0} w[k]e^{-j\omega k/5}$$

$$= \sum_{k=-\infty}^{\infty} w[k]\Big(\frac{1 + e^{j2\pi k/5} + e^{j4\pi k/5} + e^{j6\pi k/5} + e^{j8\pi k/5}}{5}\Big)e^{-j\omega k/5}$$

$$= \frac{1}{5}\Big(W(e^{j\omega/5}) + W(e^{j(\omega-2\pi)/5}) + W(e^{j(\omega-4\pi)/5}) + W(e^{j(\omega-6\pi)/5}) + W(e^{j(\omega-8\pi)/5})\Big)$$

In other words, $Y(e^{j\omega})$ is an expanded or stretched out version of $W(e^{j\omega})$.

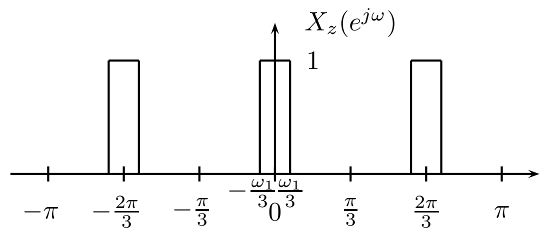

Figure 2: Problem 1 (a).

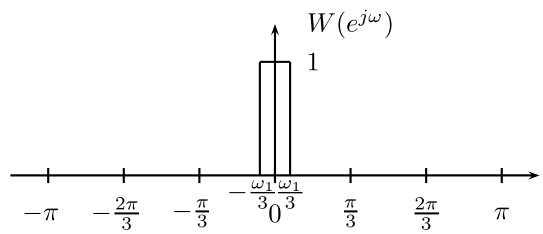

Figure 3: Problem 1 (a).

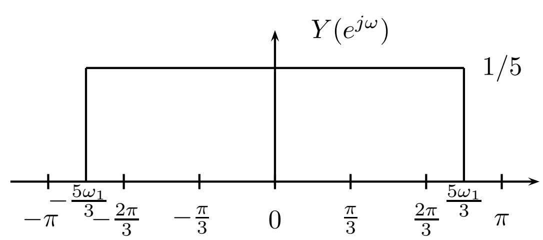

Figure 4: Problem 1 (a).

- *7.19 (a)*

  When $\omega_1 \leq \frac{3\pi}{5}$, $H(e^{j\omega})$ merely eliminates the copies created by the zero insertion. From the plot shown in Figure 4, we see that $y[n] = \frac{sin(5\omega_1 n/3)}{5\pi n}$.

- *7.19 (b)*

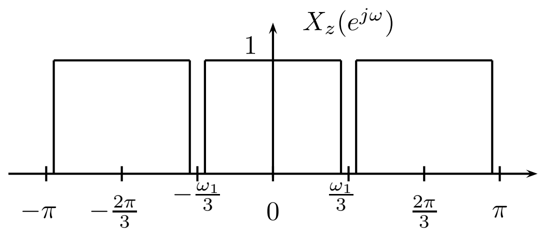

Figure 5: Problem 1 (b).

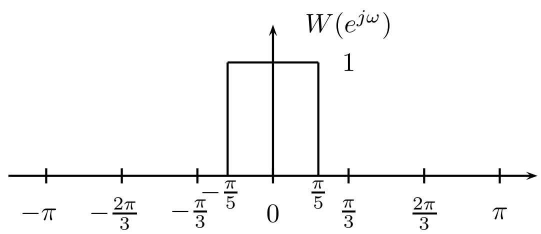

Figure 6: Problem 1 (b).

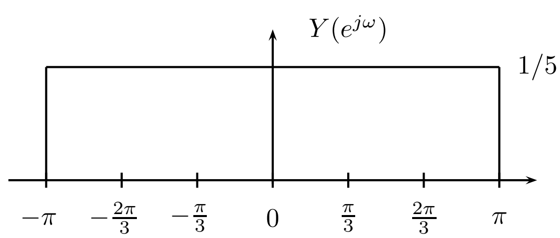

Figure 7: Problem 1 (b).

  When $\omega_1 > \frac{3\pi}{5}$, $H(e^{j\omega})$ not only eliminates the copies created by the zero insertion but also cuts off part of the central copy of the spectrum (limiting it to be bandlimited with maximum frequency $\pi/5$). Then, when the signal is decimated, $Y(e^{j\omega}) = \frac{1}{5}$ for all values of ω. Thus, $y[n] = \frac{1}{5}\delta[n]$.

Problem 2 *(Upsampling.)*

OWN Problem 7.49

- *7.49 (a)*

  Let $x_{d_1}[n]$ and $x_{d_2}[n]$ be two inputs to system A, with corresponding outputs $x_{p_1}[n]$ and $x_{p_2}[n]$. Now, consider an input of the form $x_{d_3}[n] = \alpha_1 x_{d_1}[n] + \alpha_2 x_{d_2}[n]$. With this input, the output of the system will be

$$x_{p_3}[n] = \begin{cases} \alpha_1 x_{d_1}[n/N] + \alpha_2 x_{d_2}[n/N] & n = 0, \pm N, \pm 2N, \ldots \\ 0 & else \end{cases}$$

  Thus, $x_{p_3}[n] = \alpha_1 x_{p_1}[n] + \alpha_2 x_{p_2}[n]$. This means that the system is **linear**.

- *7.49 (b)*

  The system is **not time invariant**. For example, an input $x_d[n] = \delta[n]$ gives an output $x_p[n] = \delta[n]$, while the shifted input $x_d[n-1] = \delta[n-1]$ gives an output $\delta[n-N] \neq x_p[n-1]$.

- *7.49 (c)*

$$\begin{aligned} X_p(e^{j\omega}) &= \sum_{n=-\infty}^{\infty} x_p[n] e^{j\omega n} \\ &= \sum_{k=-\infty}^{\infty} x_p[kN] e^{j\omega kN} \\ &= \sum_{k=-\infty}^{\infty} x_d[k] e^{j\omega kN} \\ &= X_d(e^{j\omega N}) \end{aligned}$$

  A plot of $X_p(e^{j\omega})$ is shown in Figure 8.

- *7.49 (d)*

  A plot of $X(e^{j\omega})$ is shown in Figure 8.

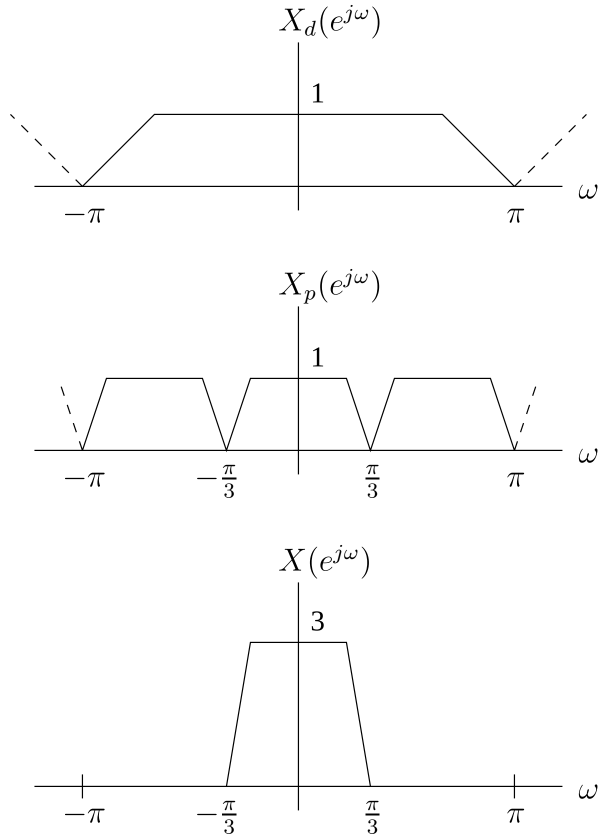

Figure 8: Problem 2 (c) and (d)

Problem 3 *(Downsampling.)*

OWN Problem 7.35

- *7.35 (a)*

  The sequences $x_p[n]$ and $x_d[n]$ are sketched in Figure 9.

- *7.35 (b)*

  Discrete-time impulse-train sampling of $x[n]$ generates the signal $x_p[n]$.

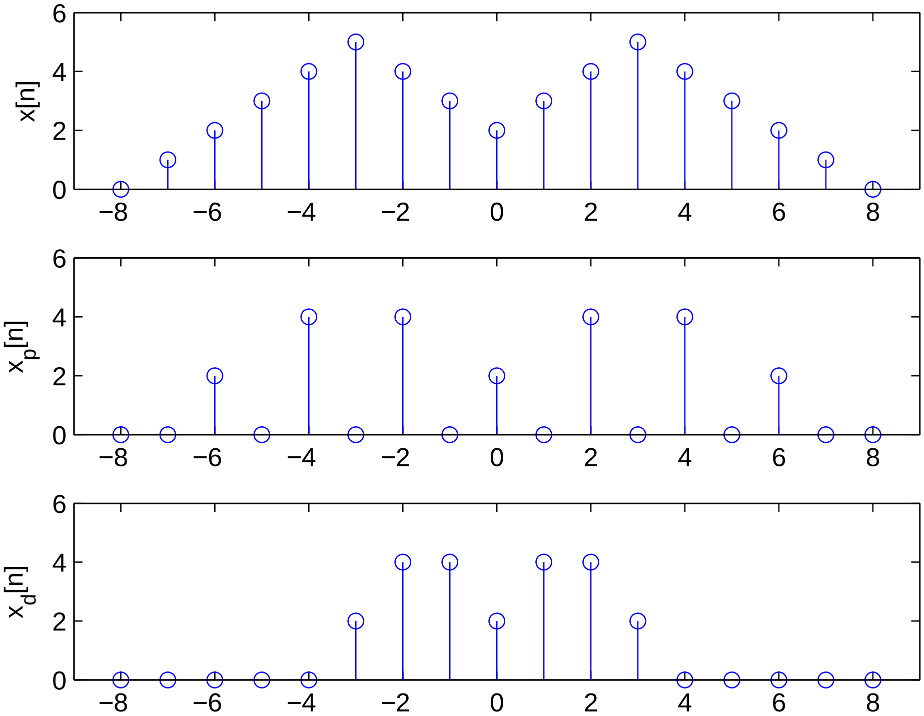

Figure 9: Problem 3 (a)

$$
\begin{aligned}
x_p[n] &= \begin{cases} x[n], & n = 0, \pm 2, \pm 4, \ldots \\ 0, & n = \pm 1, \pm 3, \ldots \end{cases} \\
&= \sum_{k=-\infty}^{\infty} x[2k]\delta[n-2k] \\
&= x[n] \cdot p[n] \\
P(e^{j\omega}) &= \pi \sum_{k=-\infty}^{\infty} \delta(\omega - \pi k) \\
X_p(e^{j\omega}) &= \frac{1}{2\pi} \int_{2\pi} P(e^{j\theta}) X(e^{j(\omega-\theta)}) d\theta \\
&= \frac{1}{2} \sum_{k=0}^{1} X(e^{j(\omega - \pi k)})
\end{aligned}
$$

Decimation of $x[n]$ generates the signal $x_d[n]$.

$$
\begin{aligned}
x_d[n] &= x[2n] = x_p[2n] \\
X_d(e^{j\omega}) &= X_p(e^{j\frac{\omega}{2}})
\end{aligned}
$$

$X_p(e^{j\omega})$ and $X_d(e^{j\omega})$ are sketched in Figure 10.

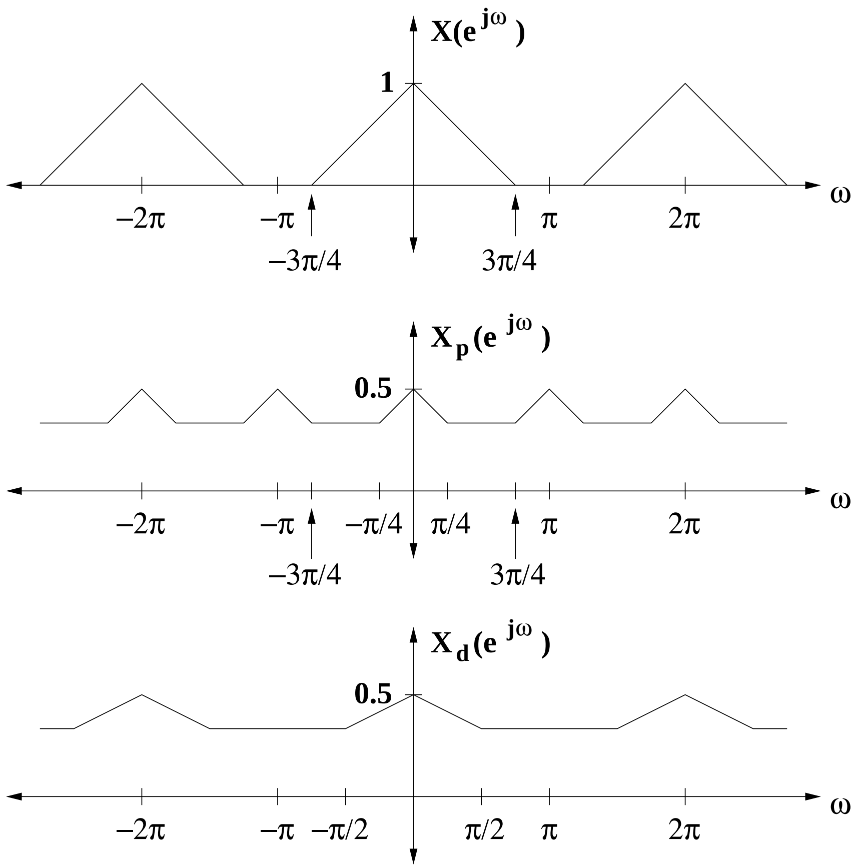

Figure 10: Problem 3 (b)

Problem 4 *(AM Communication Systems.)*

OWN Problem 8.21

- *8.21(a)*

  We have $y(t) = x(t)\cos(\omega_c t + \theta_c)$. Therefore,

$$\begin{aligned} w(t) &= y(t)\cos(\omega_c t + \theta_c) \\ &= x(t)\cos^2(\omega_c t + \theta_c) \\ &= x(t)\left[\frac{1 + \cos[2(\omega_c t + \theta_c)]}{2}\right] \\ &= \frac{1}{2}x(t) + \frac{1}{2}x(t)\cos(2\omega_c t + 2\theta_c) \end{aligned}$$

- *8.21(b)*

  The Fourier Transforms $X(j\omega)$ and $Y(j\omega)$ of $x(t)$ and $y(t)$ are sketched in Figure 11. From the sketch of $Y(j\omega)$, it is clear that in order to avoid aliasing, we require that $\omega_c > \omega_M$

  We also sketch $W(j\omega)$ and the LPF in the same figure. In order to have the output proportional to $x(t)$, the cutoff frequency ω_{co} must satisfy the inequalities

$$\omega_M \leq \omega_{co} \leq 2\omega_c - \omega_M$$

If these two conditions are satisfied, the value of θ_c does not matter.

8.21 (b)

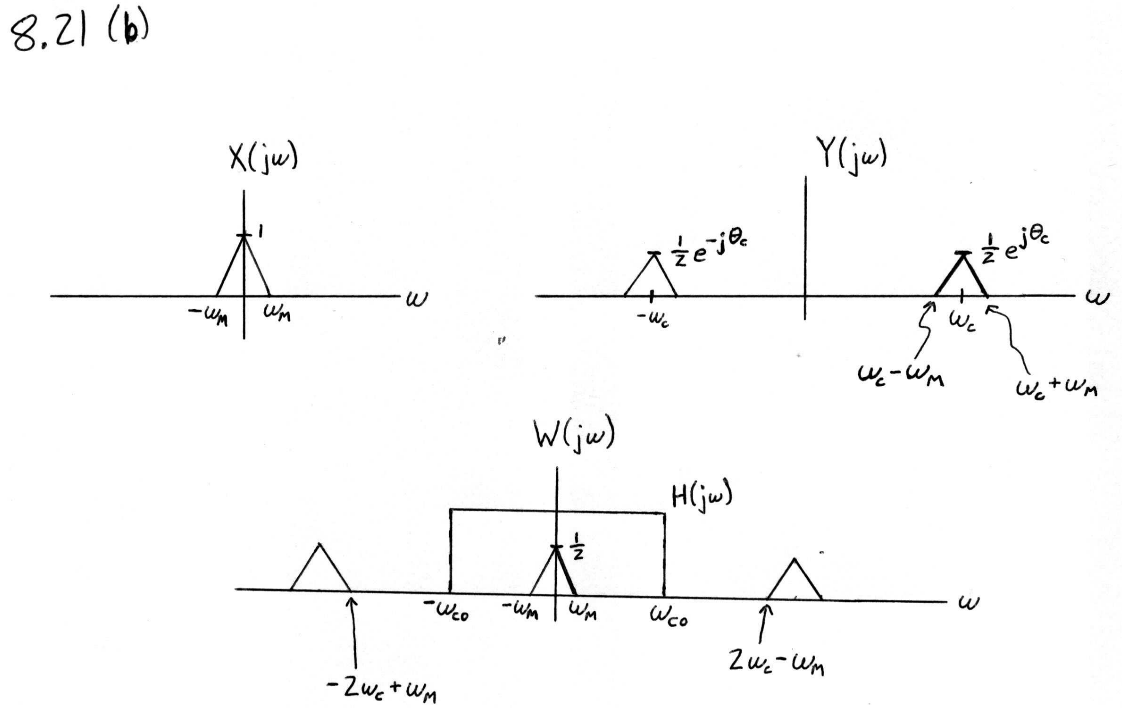

Figure 11: Problem 4 (b)

Problem 5 *(AM Communication Systems.)*

OWN Problem 8.22

The spectrum $Y(j\omega)$ is sketched in Figure 12.

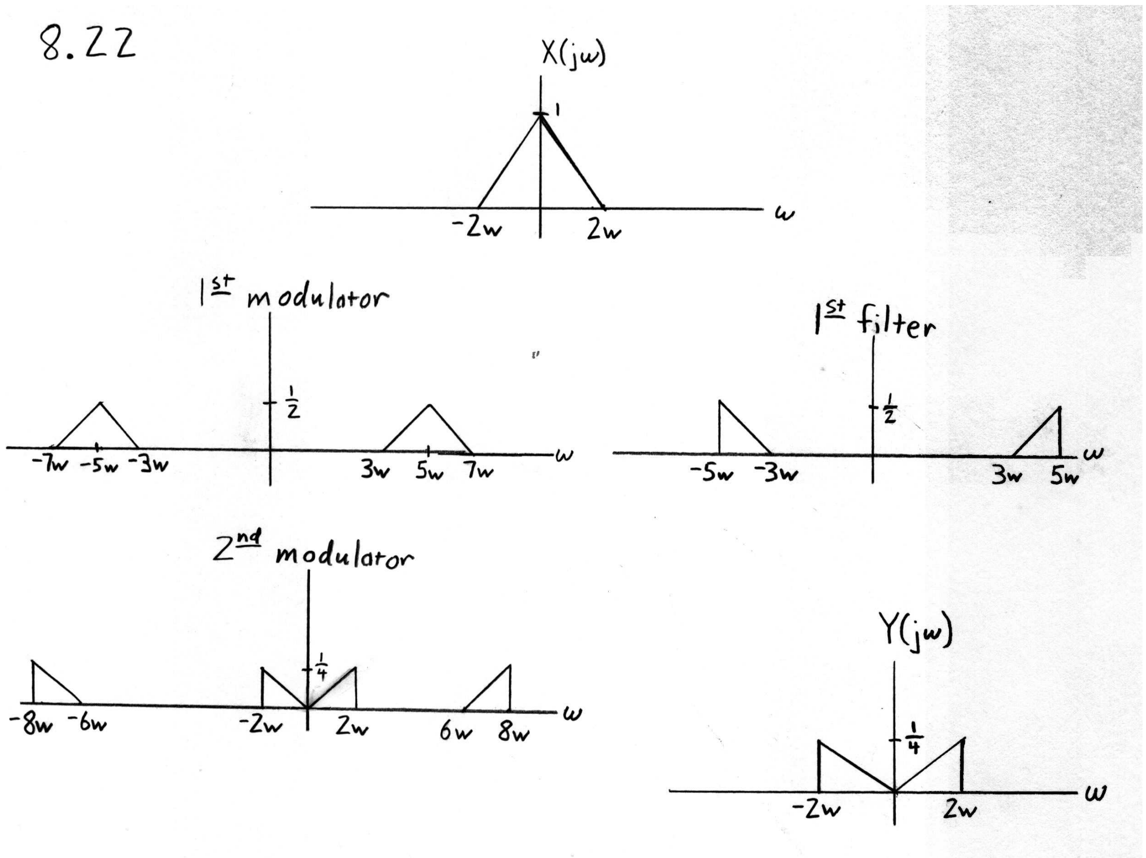

Figure 12: Problem 5