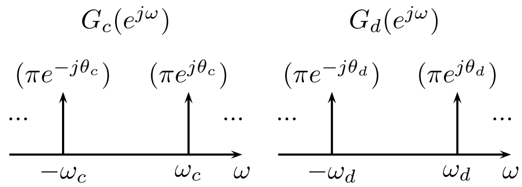
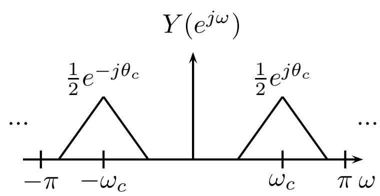
Homework 8 Solutions

Problem 1 OWN 8.47 *(Effects from loss of synchronization.)*

In this problem we assume that $\omega_c > \omega_M$ and $\pi > \omega_c + \omega_M$. Let $G_c(e^{j\omega})$ represent the Fourier transform of $\cos(\omega_c n + \theta_c)$ and $G_d(e^{j\omega})$ represent the Fourier transform of $\cos(\omega_d n + \theta_d)$, which are shown below.

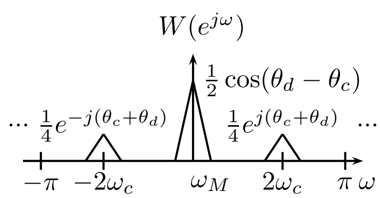


$$Y(e^{j\omega}) = \frac{1}{2\pi} \int_{2\pi} X(e^{j(\omega-\theta)}) G_c(e^{j\theta}) d\theta$$



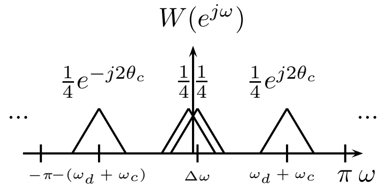
$$W(e^{j\omega}) = \frac{1}{2\pi} \int_{2\pi} Y(e^{j(\omega-\theta)}) G_d(e^{j\theta}) d\theta$$

(a) If $\Delta\omega = 0$, then $\omega_d = \omega_c$. Therefore, $W(e^{j\omega})$ is as shown below.



(b) If we pass $W(e^{j\omega})$ from Figure through the LPF $R(e^{j\omega}) = \cos(\theta_d - \theta_c)X(e^{j\omega}) = \cos(\Delta\theta)X(e^{j\omega})$ and $r[n] = \cos(\Delta\theta)x[n]$. If $\Delta\theta = \pi/2$, then $r[n] = 0$.

(c) In this case, $W(e^{j\omega})$ is as shown below. If w $> \omega_M + \Delta\omega$, then $R(e^{j\omega}) = \frac{1}{2}X(e^{j(\omega-\Delta\omega)}) + \frac{1}{2}X(e^{j(\omega+\Delta\omega)})$ for $-\pi \le \omega < \pi$. Therefore, $r[n] = x[n]\cos(\Delta\omega n)$.



Problem 2 OWN Problem 8.26. *(Asynchronous demodulation.)*

First let's solve for the Fourier transform of $y(t)$.

$$Y(j\omega) = \frac{1}{2}e^{j\theta_c}X(j(\omega-\omega_c)) + \frac{1}{2}e^{-j\theta_c}X(j(\omega+\omega_c)) + A\pi\left(e^{j\theta_c}\delta(\omega-\omega_c) + e^{-j\theta_c}\delta(\omega+\omega_c)\right)$$

Let's define $w_1(t)$ to be the output after $y(t)$ is multiplied by $\cos(\omega_c t)$ and $w_2(t)$ to be the output after $y(t)$ is multiplied by $\sin(\omega_c t)$. Also, let $z_1(t)$ be the output after $w_1(t)$ is passed through the low-pass filter (LPF) and $z_2(t)$ be the output after $w_2(t)$ is passed through the LPF.

$$\begin{aligned}
W_1(j\omega) &= \frac{1}{2}Y(j(\omega-\omega_c)) + \frac{1}{2}Y(j(\omega+\omega_c)) \\
&= \frac{1}{4}e^{j\theta_c}\big(X(j(\omega-2\omega_c)) + 2A\pi\delta(\omega-2\omega_c)\big) + \frac{1}{4}(e^{-j\theta_c}+e^{j\theta_c})\big(X(j\omega)+2A\pi\delta(\omega)\big) \\
&\quad + \frac{1}{4}e^{-j\theta_c}\big(X(j(\omega+2\omega_c)) + 2A\pi\delta(\omega+2\omega_c)\big)
\end{aligned}$$

$$\begin{aligned}
W_2(j\omega) &= \frac{1}{2j}Y(j(\omega-\omega_c)) - \frac{1}{2j}Y(j(\omega+\omega_c)) \\
&= \frac{1}{4j}e^{j\theta_c}\big(X(j(\omega-2\omega_c)) + 2A\pi\delta(\omega-2\omega_c)\big) + \frac{1}{4j}(e^{-j\theta_c}-e^{j\theta_c})\big(X(j\omega)+2A\pi\delta(\omega)\big) \\
&\quad - \frac{1}{4j}e^{-j\theta_c}\big(X(j(\omega+2\omega_c)) + 2A\pi\delta(\omega+2\omega_c)\big)
\end{aligned}$$

Assuming the LPF has a gain of 2,

$$\begin{aligned}
Z_1(j\omega) &= \frac{1}{2}(e^{-j\theta_c}+e^{j\theta_c})\big(X(j\omega)+2A\pi\delta(\omega)\big) \\
&= \cos(\theta_c)\big(X(j\omega)+2A\pi\delta(\omega)\big)
\end{aligned}$$

$$\begin{aligned}
Z_2(j\omega) &= \frac{1}{2j}(e^{-j\theta_c}-e^{j\theta_c})\big(X(j\omega)+2A\pi\delta(\omega)\big) \\
&= -\sin(\theta_c)\big(X(j\omega)+2A\pi\delta(\omega)\big)
\end{aligned}$$

Thus,

$$z_1(t) = \cos(\theta_c)\big(x(t)+A\big)$$
$$z_2(t) = -\sin(\theta_c)\big(x(t)+A\big)$$

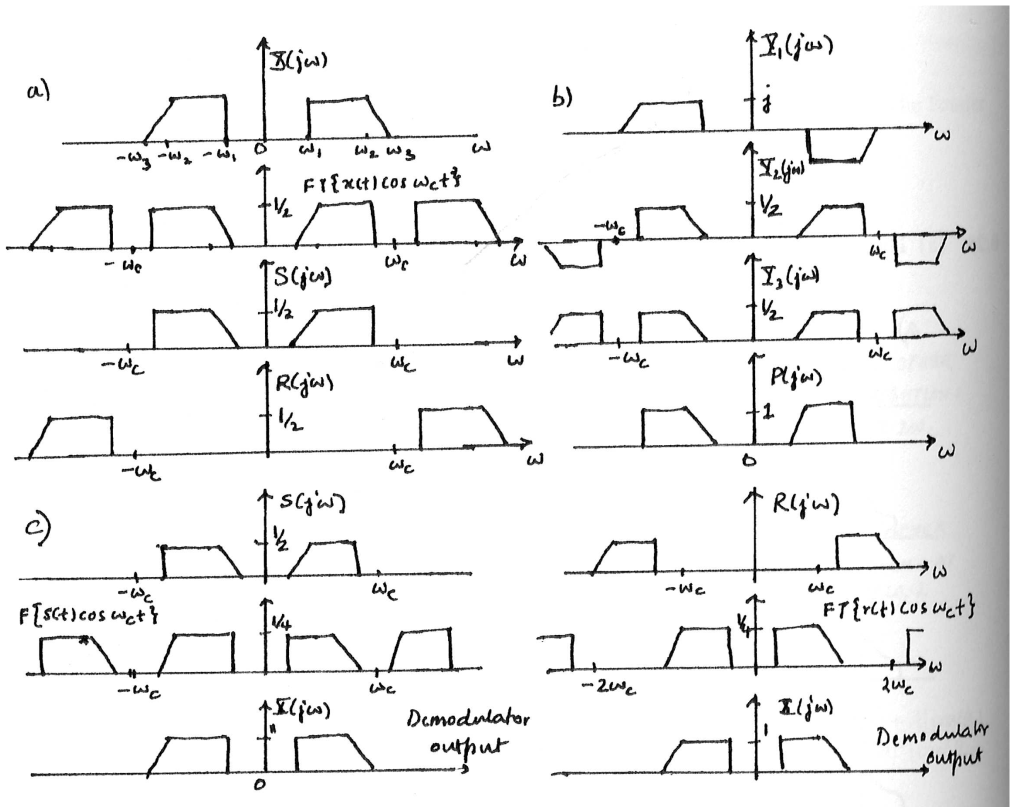


Figure 1: OWN Problem 8.29

$$
\begin{aligned}
r(t) &= \sqrt{z_1^2(t) + z_2^2(t)} \\
&= \sqrt{(x(t)+A)^2 \cos^2\theta_c + (x(t)+A)^2 \sin^2\theta_c} \\
&= x(t) + A
\end{aligned}
$$

Problem 3 OWN Problem 8.29. *(Single-sideband amplitude modulation.)*

(a) The sketches in the Figure 1 show $S(j\omega)$ and $R(j\omega)$.

(b) In Figure 1 we show how $P(j\omega)$ may be obtained by considering the outputs of the various stages of Figure P8.28(c). From the sketch for $P(j\omega)$, it is clear that $P(j\omega) = 2S(j\omega)$.

(c) In Figure 1 we show the results of demodulation on both $s(t)$ and $r(t)$. It is clear that $x(t)$ is recovered in both cases.

Problem 4 OWN Problem 8.40 *(Quadrature multiplexing.)*

If we approach this problem analytically in the time domain, we see that:

$$r(t) = x_1(t)\cos(\omega_c t) + x_2(t)\sin(\omega_c t)$$

$$R(j\omega) = \frac{1}{2}X_1(j(\omega-\omega_c)) + \frac{1}{2}X_1(j(\omega+\omega_c)) + \frac{1}{2j}X_2(j(\omega-\omega_c)) - \frac{1}{2j}X_2(j(\omega+\omega_c))$$

Let $z_1(t) = r(t)\cos(\omega_c t)$ and $z_2(t) = r(t)\sin(\omega_c t)$.

$$\begin{aligned} z_1(t) &= r(t)\cos(\omega_c t) \\ &= x_1(t)\cos^2(\omega_c t) + x_2(t)\sin(\omega_c t)\cos(\omega_c t) \\ &= x_1(t)\left(\frac{1+\cos(2\omega_c t)}{2}\right) + x_2(t)\left(\frac{\sin(2\omega_c t)+\sin(0)}{2}\right) \\ &= x_1(t)\left(\frac{1+\cos(2\omega_c t)}{2}\right) + x_2(t)\left(\frac{\sin(2\omega_c t}{2})\right) \end{aligned}$$

$$\begin{aligned} z_2(t) &= r(t)\sin(\omega_c t) \\ &= x_1(t)\cos(\omega_c t) sin(\omega_c t) + x_2(t)\sin^2(\omega_c t) \\ &= x_1(t)\left(\frac{\sin(2\omega_c t)+\sin(0)}{2}\right) + x_2(t)\left(\frac{1-cos(2\omega_c t)}{2}\right) \\ &= x_1(t)\left(\frac{\sin(2\omega_c t)}{2}\right) + x_2(t)\left(\frac{1-\cos(2\omega_c t}{2})\right) \end{aligned}$$

Thus, after the LPF with gain 2.

$$\begin{aligned} y_1(t) = x_1(t) \\ y_2(t) = x_2(t) \end{aligned}$$

Let's approach this problem graphically now and in the frequency domain. Let $X_1(j\omega)$ and $X_2(j\omega)$ be as shown in Figure 2. Then $R(j\omega)$ is as shown in Figure 2. The overlapping regions in the figure need to be summed.

When $r(t)$ is multiplied by $\cos\omega_c t$, in the vicinity of $\omega = 0$ we get

$$\frac{1}{2}\left(\frac{1}{2}X_1(j\omega) + \frac{j}{2}X_2(j\omega) + \frac{1}{2}X_1(j\omega) - \frac{j}{2}X_2(j\omega)\right) = \frac{1}{2}X_1(j\omega).$$

Therefore the first lowpass filter output is equal to $x_1(t)$.

When $r(t)$ is multiplied by $\sin\omega_c t$, in the vicinity of $\omega = 0$ we get

$$\frac{1}{2j}\left(\frac{1}{2}X_1(j\omega) + \frac{j}{2}X_2(j\omega) - \left(\frac{1}{2}X_1(j\omega) - \frac{j}{2}X_2(j\omega)\right)\right) = \frac{1}{2}X_2(j\ \omega).$$

Therefore the second lowpass filter output is equal to $x_2(t)$.

Problem 5 OWN Problem 8.13 *(Intersymbol spacing.)*

(a)

$$\begin{aligned} p(0) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} P(j\omega)d\omega \\ &= \frac{1}{2\pi}\int_{-\frac{2\pi}{T_1}}^{\frac{2\pi}{T_1}} \frac{1}{2}\left(1+\cos(\frac{\omega T_1}{2})\right)d\omega \\ &= \frac{1}{2\pi}\left[\frac{\omega}{2} + \frac{\sin\left(\frac{\omega T_1}{2}\right)}{T_1}\right]\Bigg|_{-\frac{2\pi}{T_1}}^{\frac{2\pi}{T_1}} \\ &= \frac{1}{T_1} \end{aligned}$$

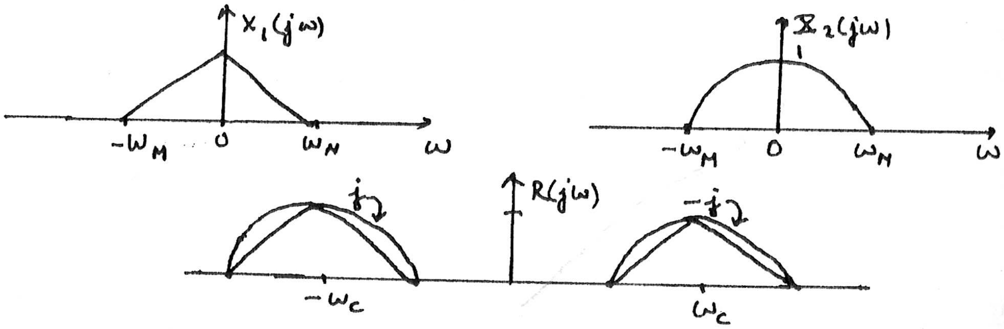


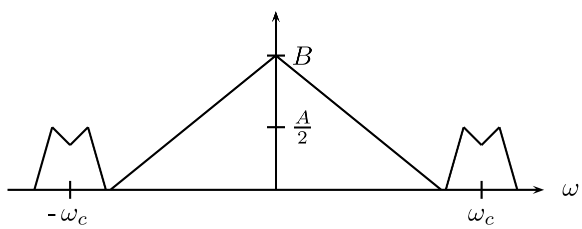
Figure 2: OWN Problem 8.40

(b) Since $P(j\omega)$ satisfies eq/ (8.28), we know that it must have zero-crossings every T_1. Therefore,

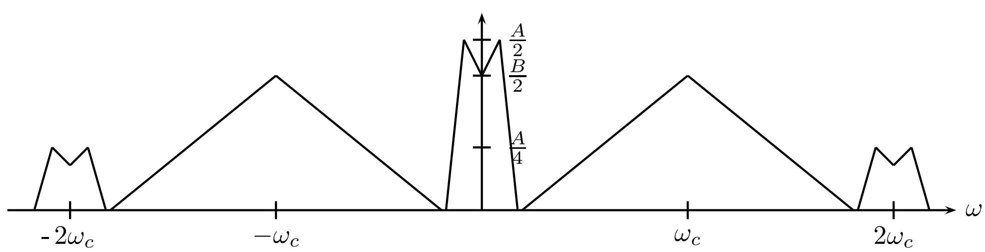
$$p(kT_1) = 0, \quad \text{for } k = \pm 1, \pm 2, \cdots$$

Problem 6 *(Signal transmission system.)*

(a) $\omega_c > \omega_z + \omega_x$. This is because we need to avoid aliasing.



(b) $C = 2$ and $\omega_c - \omega_z > \omega_f > \omega_x$.



Problem 7 OWN Problem 8.39 *(FSK.)*

(a) There are two possible cases.

Case 0: $b(t) = m_0(t)$.

$$D_0 = \int_0^T \cos^2(\omega_0 t)dt - \left| \int_0^T \cos(\omega_0 t)\cos(\omega_1 t)dt \right|$$

Case 1: $b(t) = m_1(t)$.

$$D_1 = \int_0^T \cos^2(\omega_1 t)dt - \left|\int_0^T \cos(\omega_0 t)\cos(\omega_1 t)dt\right|$$

Both D_0 and D_1 are maximum when $\left|\int_0^T \cos(\omega_0 t)\cos(\omega_1 t)dt\right| = 0$.

(b)

$$\begin{aligned}
\int_0^T \cos(\omega_0 t)\cos(\omega_1 t)dt &= \int_0^T \frac{1}{2}\left(\cos((\omega_0+\omega_1)t) + \cos((\omega_0-\omega_1)t)\right)dt \\
&= \left[\frac{\sin((\omega_0+\omega_1)t)}{2(\omega_0+\omega_1)} + \frac{\sin((\omega_0-\omega_1)t)}{2(\omega_0-\omega_1)}\right]_0^T \\
&= \frac{\sin((\omega_0+\omega_1)T)}{2(\omega_0+\omega_1)} + \frac{\sin((\omega_0-\omega_1)T)}{2(\omega_0-\omega_1)}
\end{aligned}$$

Thus for any choice of ω_0 and ω_1, $\omega_0 \neq \omega_1$, we can always find T so that $\int_0^T \cos(\omega_0 t)\cos(\omega_1 t)dt = 0$.