EECS 120 Signals & Systems | University of California, Berkeley: Fall 2007
Ramchandran | Nov 1, 2007

Homework 9
Due: Thursday, November 8, 2007, at 5pm
GSI: Pulkit Grover

Reading OWN Sections 8.7, 9.1, 9.2, 9.3.

Practice Problems *(Suggestions.)* OWN 9.1, 9.2, 9.3, 9.4, 9.5.

Problem 1 *(Angle Modulations.)*

An angle-modulated signal has the form

$$u(t) = 100\cos(2\pi f_c t + 4\sin 2\pi f_m t)$$

where $f_c = 10MHz$ and $f_m = 1KHz$.

(a) Assuming that this is an FM signal, determine the modulation index and the bandwidth (in Hz) of the transmitted signal.

(b) Repeat part (a) assuming a PM signal.

(c) Compare the signal bandwidths in parts (a) and (b) with SSB AM.

(d) Repeat parts (a) and (b) if f_m is doubled.

Problem 2 *(Nyquist Pulses.)*

One of the design parameters in a pulse amplitude modulation (PAM) system is the choice of pulse shape. As we have seen in class, a PAM signal has the form

$$s(t) = \sum_{k=-\infty}^{\infty} x[k]p(t - kT_s) \qquad (1)$$

where $x[k]$ is the data sequence, $p(t)$ is the pulse, and T_s is the pulse period.

We use the following system architecture at the receiver:[1] The received signal, which in the absence of noise and other adverse effects is merely $s(t)$, is sampled with period T_s. Hence, the receiver obtains the following discrete-time signal:

$$r[n] \stackrel{def}{=} s(nT_s) = \sum_{k=-\infty}^{\infty} x[k]p((n-k)T_s) \qquad (2)$$

$$= x[n]p(0) + x[n-1]p(1) + x[n-2]p(2) + \ldots + x[n+1]p(-1) + x[n+2]p(-2) + \ldots \qquad (3)$$

Further processing at the receiver is greatly simplified if for each time instant n, $r[n]$ depends only on $x[n]$, but not on the other data samples $\{x[k]\}_{k\neq n}$. Which pulses $p(t)$ satisfy this condition?

This is immediately obvious from the above equation: $p(t)$ must be such that

$$p(nT_s) = \begin{cases} 1, & \text{if } n = 0 \\ 0, & \text{if } n \neq 0 \end{cases} \qquad (4)$$

[1] In EE121, you will learn that under some assumptions, this is actually the optimum receiver architecture.

It is important to note that for all other t, the pulse shape $p(t)$ can be whatever you'd like it to be! This freedom can be exploited in various ways, but most importantly to obtain a pulse with *small spectral support.* (Notice that $p(0)$ is selected to be 1 merely for normalization purposes; it could be any non-zero constant.)

(a) Show that if $p(nT_s) = \delta[n]$, then

$$\sum_{k=-\infty}^{\infty} P(j(\omega - k\frac{2\pi}{T_s})) \quad = \quad T_s \tag{5}$$

If a signal satisfies Equation (5), it is referred to as a *Nyquist pulse* (with pulse period T_s).

(b) A pulse that satisfies Equation (5) is the sinc function:

$$\begin{aligned} p(t) &= \frac{T_s \sin(\frac{\pi}{T_s}t)}{\pi t} \\ &= sinc(\frac{t}{T_s}) \\ P(j\omega) &= \begin{cases} T_s, & |\omega| < \frac{\pi}{T_s} \\ 0, & |\omega| > \frac{\pi}{T_s} \end{cases} \end{aligned}$$

Convince yourself that the sinc is the time signal of lowest bandwidth that satisfies Equation (5).

Problem 3 *(Narrowband FM.)*

OWN Problem 8.45.

Problem 4 *(Laplace Transforms.)*

(a) OWN Problem 9.21 (c).

(b) OWN Problem 9.21 (e).

Problem 5 *(Laplace Transforms: ROC.)*

OWN Problem 9.23 (all parts).

Problem 6 *(Inverting Laplace Transforms.)*

(a) OWN Problem 9.22 (a).

(b) OWN Problem 9.22 (b).