
Homework 1

Due: **Friday**, September 7, 2007, at 5pm
Homework 1 GSI: Mark Johnson

Reading OWN Chapter 1 and Sections 2.1 and 2.2.

Practice Problems (*Suggestions.*) OWN 1.3, 1.11, 1.17, 2.4, 2.11(a)

Problem 1 (*Complex numbers.*)

(a) With two real numbers, a and α , a complex number can be specified in its *polar form* as

$$z = ae^{j\alpha} \quad (1)$$

where j is the imaginary unit (i.e., the root of -1). Express the following complex numbers both in Cartesian and polar coordinates (remember to consider the range of α):

$$e^{j5\pi}, \left(\frac{1}{3}e^{j\pi/4}\right)^*, j^{-j} \quad (2)$$

(b) Express $\frac{a+jb}{c+jd}$ in polar form.

(c) Consider the complex-valued function

$$H(j\omega) = \frac{1}{1-2j\omega} \quad (3)$$

where ω is a real number. Sketch $|H(j\omega)|$ and $\arg(H(j\omega))$ versus ω for $-5 < \omega < 5$. *Remark:* Writing $H(j\omega)$ is standard engineering practice, even though strictly speaking, it would be more logical to write $H(\omega)$.

Problem 2 (*Implementation of Complex Systems.*)

Consider a system described by the relation

$$y[n] = (2 - j3)x[n] + (3 + j)x[n - 1]$$

where $x[n] = x_R[n] + jx_I[n]$ and $y[n] = y_R[n] + jy_I[n]$ are **complex** valued signals. Draw a block diagram that implements this system, which consists only of **real** adders, multipliers, and delay registers.

Problem 3 (*Elementary functions and their graphs.*)

(a) Consider the complex-valued signal $y(t) = e^{j2\pi t}$

Express (as functions of t): $Re\{y(t)\}$, $Im\{y(t)\}$, $|y(t)|$, $\arg(y(t))$

Sketch by hand, in one figure, for $-1 \leq t \leq 1$, the following functions: $y_1(t) = Re\{y(t)\}$, $y_2(t) = Im\{y(3t)\}$, $y_3(t) = Re\{y(2t + 0.5)\}$, $y_4(t) = Re\{y(-2t + 0.5)\}$. Clearly label the sketches and their zero-crossings.

Problem 4 (*Periodic continuous-time signals.*)

- (a) Determine the fundamental period of the signal $z(t) = \cos(\frac{\pi}{3}t) + \cos(\frac{2\pi}{5}t)$
(b) Is $w(t) = \cos(\frac{\pi}{3}t) + \cos(t)$ periodic? If so, determine its fundamental period.

Problem 5 (*Transformations of functions.*)

Consider the following function:

$$x(t) = \begin{cases} 1+t, & -1 \leq t \leq 0, \\ 1-2t, & 0 \leq t \leq 1/2, \\ 0, & \text{otherwise.} \end{cases} \quad (4)$$

Sketch, in one figure, $x(t)$, $x(t/2 - 3)$, and $x(2 - t)$. Carefully label both axes in the plot.

Problem 6 (*Periodic discrete-time signals.*)

- (a) Let $y_1[n] = \cos(\frac{\pi}{3}n)$ and $y_2[n] = \cos(\frac{3\pi}{5}n)$. What are their fundamental periods? Plot two periods of each signal in Matlab and clearly mark the end of each period.
(b) Determine the fundamental period of the signal $z[n] = y_1[n] + y_2[n]$. Plot two periods of $z[n]$ in Matlab and clearly mark the end of each period.
(c) Is $z[n] = \cos(2n)$ periodic? If so, what is its period?

Problem 7 (*Properties of systems.*)

Are the following systems linear? Are they time-invariant? In each case, give a short justification using the definitions of these properties.

- (a) $\mathcal{H}\{x(t)\} = x(at + b)$
(b) $\mathcal{H}\{x(t)\} = x(at^2 + b)$

Problem 8 (*Properties of systems.*)

Are the following systems linear? Are they time-invariant? In each case, give a short justification using the definitions of these properties.

- (a) $\mathcal{H}\{x(t)\} = x(at) + b$
(b) $\mathcal{H}\{x(t)\} = \frac{d}{dt}x(t)$

Problem 9 (*Properties of continuous-time systems.*)

For each of the following systems with input $x(t)$ and output $y(t)$, determine whether the system is (i) memoryless, (ii) stable, (iii) causal, (iv) linear, and (v) time invariant. In each case, give a short justification using the definitions of these properties.

- (a) $y(t) = x(t)\frac{e^t + e^{-t}}{2}$
(b) $y(t) = \text{Im}\{x(t)\}$

Problem 10 (*Properties of continuous-time systems.*)

For each of the following systems with input $x(t)$ and output $y(t)$, determine whether the system is (i) memoryless, (ii) stable, (iii) causal, (iv) linear, and (v) time invariant. In each case, give a short justification using the definitions of these properties.

(a) $y(t) = \int_{-\infty}^{t/2} x(3\tau) d\tau$

(b) $y(t) = x(-t/2)$

Problem 11 (*Properties of discrete-time systems.*)

For each of the following systems with input $x[n]$ and output $y[n]$, determine whether the system is (i) memoryless, (ii) stable, (iii) causal, (iv) linear, and (v) time invariant. In each case, give a short justification using the definitions of these properties.

(a) $y[n] = \sum_{k=-\infty}^n x[k+1]$

(b) $y[n] = x[n] \cdot \sum_{k=-\infty}^{\infty} \delta[n-3k]$

Problem 12 (*Convolution of step functions.*)

Recall that if an LTI system has impulse response $h(t)$ and input $x(t)$, the output is given by a convolution $y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$. Recall also that the unit step function is defined as

$$u(t) = \begin{cases} 1, & \text{if } t \geq 0, \\ 0, & \text{otherwise.} \end{cases} \quad (5)$$

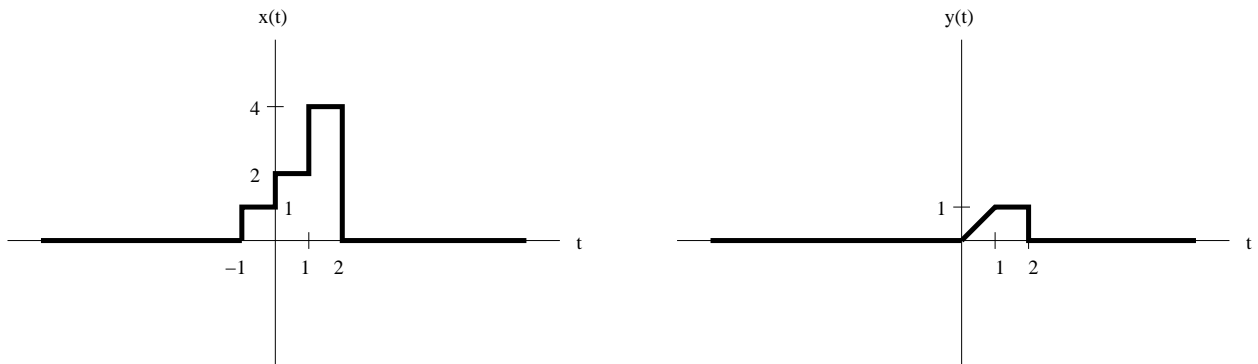
For the following, find the output given the input $x(t)$ and the system impulse response $h(t)$ specified. Sketch (by hand) the input, the impulse response, and the output.

(a) $h(t) = u(t) - u(t-1)$, $x(t) = u(t)$

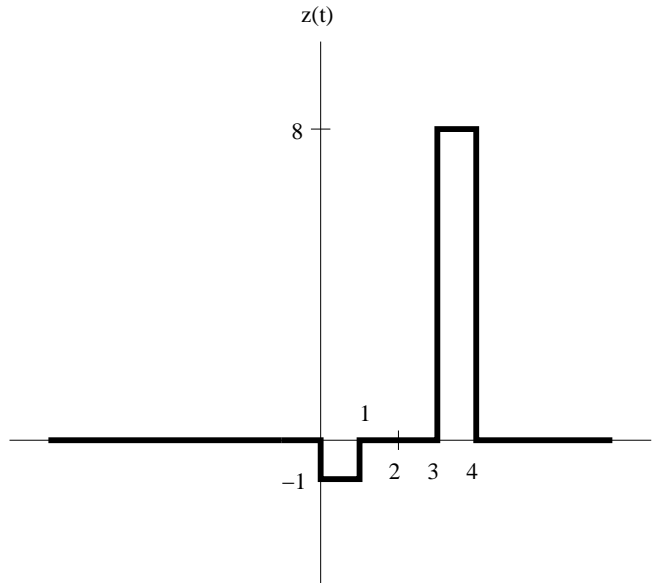
(b) $h(t) = u(t) - u(t-1)$, $x(t) = u(t) - u(t-1)$

Problem 13 (*LTI systems.*)

You are given an LTI system, and told that when the input to the system is $x(t)$, the output is $y(t)$



Sketch the output of the system when the input is $z(t)$, labeling both axes of the plot.



Problem 14 (*Convolution via Matlab.*)

Many homework sets will have a component in Matlab. For this problem, you will use Matlab to convolve exponentials. You can type 'help conv' in Matlab for help on using the function. For the following pairs of functions, first plot $h(t)$ and $x(t)$ in the domain $[-2, 2]$, with .01 between points. Then, plot $y(t) = h(t) * x(t)$ in the same domain.

If the Matlab exercise in this homework requires a lot of effort, you should go through a tutorial on Matlab. Please submit your Matlab code along with the plots.

(a) $h(t) = e^{-t}u(t)$ and $x(t) = u(t)$

(b) $h(t) = e^{-t/2}u(t)$ and $x(t) = e^{-t}u(t)$