EECS 120 Signals & Systems University of California, Berkeley: Fall 2007
Ramchandran September 6, 2007

Homework 2
Due: Thursday, September 13, 2007, at 5pm

Reading OWN Chapter 1 and Chapter 2, Sections 2.1-2.3.

Practice Problems *(Suggestions.)* OWN 1.3, 1.11, 1.17, 2.4, 2.11(a)

Problem 1 *(Review: Complex Numbers)*

Find all complex fourth roots of -16.

Problem 2 *(Graphical Convolution.)*

Graphically convolve (flip-and-drag method) the following pairs of signals. You do not need to write the equations for your results, but clearly label and scale your axes.

(a) $x_1(t) = -\delta(t+2) + 3\delta(t-3)$

$$x_2(t) = \begin{cases} 1+t, & -1 \le t \le 0 \\ 1, & 0 \le t \le 1, \\ 0, & |t| > 1 \end{cases}$$

(b) $x_1(t) = \begin{cases} 1, & |t| \le 1 \\ 0, & |t| > 1 \end{cases}$

$x_2(t) = \sum_{k=-\infty}^{\infty} x_1(t+3k)$

(c) Determine the impulse response $h(t)$ of the LTI when the input $x(t)$ and the output $y(t)$ are shown in Figure 1.

(d) If $y(t) = h(t) * x(t)$, where $h(t)$ and $x(t)$ are shown in Figure 2. Find the value of t at the which $y(t)$ is maximum, and the maximum value of $y(t)$.

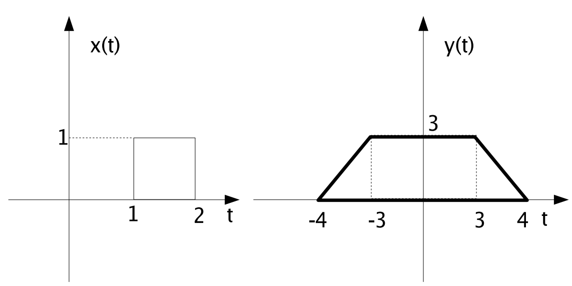

Figure 1: Problem 2(c)

Problem 3 *(Convolution.)*

OWN Problem 2.21 (a), (d).

OWN Problem 2.22 (a), (e).

Problem 4 *(Impulse response and system properties.)*

OWN Problem 2.29 (d), (g).

Problem 5 *(Noise suppression system for airplanes.)*

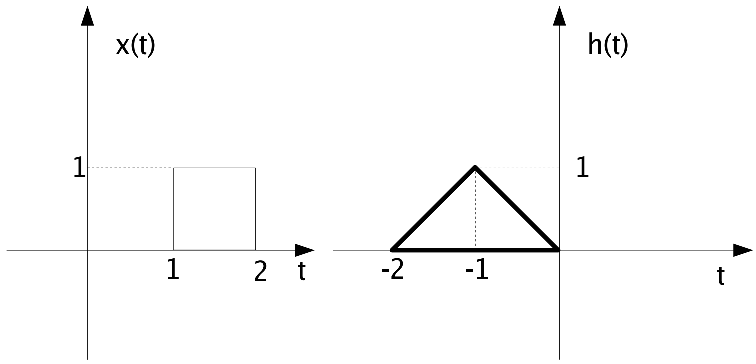

Figure 2: Problem 2(d)

On an airplane, there is a very special type of noise generated by the jet engines. In this simple example, we will assume that the noise is at relatively high frequencies, and that it is nearly periodic, well modeled by

$$x_{noise}[n] \quad = \quad \cos(\frac{3\pi}{4}n) + \frac{1}{2}\cos(\frac{2\pi}{3}n). \tag{1}$$

The goal is to build a filter that suppresses the jet engine noise but still lets you talk to your neighbors and to the flight attendant. We assume that the speech signal can be modeled as

$$x_{speech}[n] \quad = \quad \cos\left(\frac{\pi}{40}n\right). \tag{2}$$

It is suggested to use the system illustrated in Figure 3.

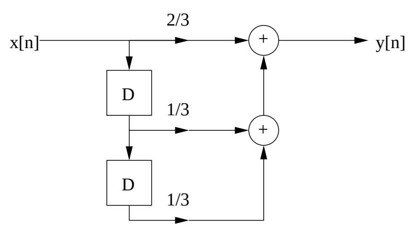

Figure 3: The suggested squeaking-suppression system.

a) Assuming that at the beginning of time, the contents of the two delay elements is zero, show that the system is linear and time-invariant, and find its impulse response $h[n]$.

b) Is the system causal? Memoryless? Stable? Why is stability an interesting issue?

c) Use matlab to give plots of *both* the input and output signals for the following three cases: *(i),* the input to the system is $x_{speech}[n]$, *(ii),* the input to the system is $x_{noise}[n]$, and *(iii),* the input to the system is $x[n] = x_{speech}[n] + x_{noise}[n]$. Does the system attenuate the jet engine noise? What's your opinion regarding the suggested system?

d) One interesting measure of success is to consider the *signal-to-noise ratio* (SNR) before and after the filter. The SNR before the filter is simply the ratio of the power of the signal to the power of the noise. The SNR after the filter, for this example, can be defined as the ratio of the power of the signal, passed through the filter by itself, to the power of the noise, passed through the filter. Use matlab (or paper-pencil if you prefer) to calculate the SNR before and after the filter. What do you observe?

Problem 6 *(Matlab, vector bases)*

In addition to answering each part of this question, also turn in a print out of all of the Matlab scripts and m-files that you write.

Go to the ee120 homepage (`http://inst.eecs.berkeley.edu/~ee120/`). Download the file basis.mat and import all the variables in basis.mat into Matlab. Use the command `load('basis.mat')`.

Recall that any four linearly independent vectors form a basis for $\mathbb{R}^4$. The file basis.mat contains 4 such vectors, $\texttt{v1}, \texttt{v2}, \texttt{v3}, \texttt{v4}$.

a) Use Matlab to compute the norm of each of these vectors ($\|\vec{v}_1\|, \|\vec{v}_2\|, \|\vec{v}_3\|,$ and $\|\vec{v}_4\|$). Throughout this problem, you should assume that any values less than 10^{-10} are rounding errors.

Compute the angle between every pair of the given vectors. The angle θ between two vectors $\vec{a}$ and $\vec{b}$ can be found by using the following formula

$$\vec{a}^T\vec{b} = \sum_{i=1}^{4} a(i) * b(i) = \vec{a} \cdot \vec{b} = \|\vec{a}\|\|\vec{b}\|\cos(\theta)$$

It should be clear now that the vectors $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ constitute a special type of basis for $\mathbb{R}^4$. What is the name of this kind of basis?

b) Any vector in $\mathbb{R}^4$ can be expressed as a linear combination of the vectors in a basis set. Moreover, if the basis is the special type that we found $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ to be in *(a)*, then any vector in $\mathbb{R}^4$ can be expressed as a *unique* linear combination of the vectors in the basis.

Assume that an arbitrary vector $\vec{y}$ in $\mathbb{R}^4$ can be written as $\vec{y} = \alpha_1\vec{v}_1 + \alpha_2\vec{v}_2 + \alpha_3\vec{v}_3 + \alpha_4\vec{v}_4$. Use the properties of linear algebra and your results from *(a)* to determine $\vec{y}^T\vec{v}_1$, $\vec{y}^T\vec{v}_2$, $\vec{y}^T\vec{v}_3$, and $\vec{y}^T\vec{v}_4$. Be sure to show your complete derivation, not just the answer.

c) The workspace basis.mat also contains three more vectors, $\vec{x}_1, \vec{x}_2, \vec{x}_3$, in $\mathbb{R}^4$. We know that each of these can be expressed in the form $\alpha_1\vec{v}_1 + \alpha_2\vec{v}_2 + \alpha_3\vec{v}_3 + \alpha_4\vec{v}_4$. Write a program to find the alphas for each of the x_i vectors. Turn in both the script of your program and the alphas for the three vectors.

Problem 7

When the dot-com bubble went bust in the year 2000, the average annual salary of graduates took a big dive to the bottom. One of the students realized that he could increase his income by putting his net annual income in a savings account. At the beginning of each subsequent year he deposits his net income of the previous year ($1500). At the end of the year, he earns 6% of the of the total amount he had in the account at the beginning of the year. Write a difference equation that describes the evolution of the bank account as a function of time and find the impulse response of the system and determine its properties (i.e. causality, stability ...).

(Optional Problem) *(Review: Difference Equations)*

Consider the system shown in Fig. 4. Balls with numbers on them go in the box. At any time, there can be at most two balls in the box. At each time, the box releases one of the two balls, chosen arbitrarily. Let input $X[n]$ denote the number on the ball that goes in at time n, the state $S[n]$ be the sum of the numbers on the two balls inside the box (before one is released), and the output $Y[n]$ be the ball that is released. At time $n+1$, $X[n+1]$ enters the box, and $Y[n+1]$ is released, and so on.

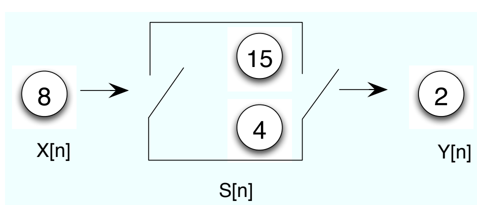

Figure 4: Optional problem

a) Derive a difference equation connecting $X[\cdot]$, $Y[\cdot]$, and the output $S[\cdot]$. Represent it as a block-diagram.

b) (should be fun!) Suppose the inputs $X[n]$'s are chosen such that $X[n] = Y[n] + S[n-1]$. Show that the ratio $\frac{S[n]}{S[n-1]}$ converges to $\frac{1+\sqrt{5}}{2}$ as $n \to \infty$, regardless of the initial state of the system.