
Homework 3

Due: Thursday, September 20, 2007, at 5pm
Homework GSI: Mary Knox

(Send your grades to ee120.gsi@gmail.com, check the course webpage for instructions)

Reading OWN Chapters 2 and 3.

Practice Problems (*Suggestions.*) OWN 3.3, 3.4

Review (will not be graded) (*Orthogonality.*)

\mathbf{S} is the subspace of \mathbb{R}^5 spanned by $\vec{v}_1 = [1 \ 4 \ 2 \ 0 \ 5]^\top$, $\vec{v}_2 = [1 \ -1 \ 0 \ 0 \ 9]^\top$, $\vec{v}_3 = [2 \ 4 \ -8 \ 0 \ 16]^\top$, and $\vec{v}_4 = [3 \ 2 \ 2 \ 0 \ 23]^\top$.

(i) Find an orthonormal basis for \mathbf{S}

(ii) Express $\vec{v}_1, \vec{v}_2, \vec{v}_3$, and \vec{v}_4 in terms of the basis you found in part (a).

Review (will not be graded) (*Frequency responses.*)

Find the outputs when the given inputs are fed into the following systems.

(a) $H(j\omega) = \frac{1}{2j\omega}$, $x(t) = 2e^{j2t} - \cos(-\pi t)$

(b) $h(t)$ and $x(t)$ are shown in Figure 1.

(c) $h[n] = (\frac{1}{4})^n u[n]$, $x[n] = 3e^{j\frac{\pi}{4}(n-2)} - \sin(\frac{5\pi}{4}n)$

Problem 1 (*Noise suppression system for airplanes, continued.*)

High frequencies can be suppressed to some extent by the system

$$y[n] = x[n] + \alpha y[n-1], \quad (1)$$

and by picking $0 < \alpha < 1$. The frequency response of this system is given by

$$H(e^{j\omega}) = \frac{1}{1 - \alpha e^{-j\omega}}. \quad (2)$$

(a) In Homework 2, you considered a noise suppression filter:

$$y[n] = \frac{2}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]. \quad (3)$$

Determine the frequency response $G(e^{j\omega})$ of this filter.

(b) Plot, in *one* Matlab figure, both $\frac{3}{4}|G(e^{j\omega})|$ and $|H(e^{j\omega})|/2$, for $\alpha = 0.5$ and $-\pi < \omega < \pi$. Which system is “better”? Give a detailed answer (2-3 sentences).

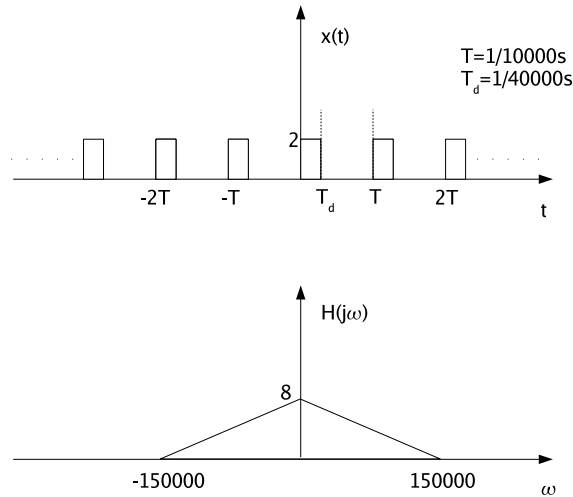


Figure 1: Review Problem

Note: The reason why we divide $|H(e^{j\omega})|$ by 2 is because we want a frequency response magnitude of 1 at frequency zero. Such a division does not influence the system behavior otherwise.

Remark: If you'd like to get a better intuition for how the system with frequency response $H(e^{j\omega})$ behaves for different α , plot (in a separate figure) $\alpha|H(e^{j\omega})|$ for $\alpha = 0.1$ and $\alpha = 0.99$. What do you observe? (Again, to get a nice plot, we multiply $|H(e^{j\omega})|$ by α .)

Problem 2 (*Continuous-time Fourier series.*)

(a) OWN 3.22 (a), only for Figure (f). Then, give the CTFS for the signal $y(t) = 0.5x(t - 1)$, where $x(t)$ is the signal in Figure (f).

(b) OWN 3.40 (a)

Problem 3 (*CTFS Properties.*)

OWN 3.44

Problem 4 (*CTFS Properties.*)

$x(t)$ is periodic with fundamental period T and Fourier series coefficients a_k . Derive the Fourier series coefficients of the following signal in terms of a_k .

$\text{Odd}\{x(t - \frac{T}{2})\}$ ($\text{Odd}\{f(t)\}$ denotes the odd part of the signal $f(t)$)

Problem 5 (*Frequency responses.*)

OWN 3.16

Problem 6 (*Discrete-time Fourier series.*)

OWN 3.28 (b)

Problem 7 (*Parseval's Relation.*)

$$f(t) = u(t - \frac{1}{2}) - u(t + \frac{1}{2})$$

$$g(t) = f(t) * f(t)$$

$$y(t) = \sum_{n=-\infty}^{\infty} g(t - 4n)$$

Find a signal $\hat{y}(t)$ that has the following properties:

- (i) $\hat{y}(t)$ is linear sum of N complex exponentials.
- (ii) $\hat{y}(t)$ retains at least 90% of the power of $y(t)$.
- (iii) N is the minimum number such that property (ii) holds.

Problem 8 (CTFS for LTI systems.) Suppose that the signal $x(t) = e^{j200\pi t}$ is the input to a series RC circuit and has Fourier series coefficients a_k . Let $y(t)$ be the voltage across the capacitor. The impulse response of this system is given by $h(t) = \frac{1}{RC}e^{-t/RC}u(t)$. For simplicity assume that $R = 1\Omega$ and $C = 1F$.

- (a) Show that $y(t)$ is also periodic with the same period as $x(t)$.
- (b) Solve for the Fourier series coefficients of $y(t)$. Note that the Fourier series coefficients have the form $b_k = c_k a_k$ where c_k are complex constants.
- (c) Find c_k .

Problem 9 (Fourier Series and Gibbs phenomenon - Matlab.)

In this problem we will use Matlab to examine the convergence of Fourier Series and Gibbs Phenomenon. One of the main reasons that Fourier series were so controversial when they were discovered by Fourier was that people didn't believe that an infinite sum of sinusoids could result in discontinuous functions like the square wave. In fact the series converges if the function being approximated satisfies certain conditions (see OWN 3.4), but there is a peculiar behavior that occurs in the convergence at points of discontinuity. This behavior is called Gibbs phenomenon.

- (a) Let $p(t)$ be periodic with period 1. Then, define one period of $p(t)$ as:

$$p(t) = \begin{cases} 1, & 0 \leq t < \frac{1}{2} \\ -1, & -\frac{1}{2} \leq t < 0 \end{cases}$$

Compute the Fourier coefficients of $p(t)$, and call them c_k .

- (b) We want to look at how the partial sum

$$p_N(t) = \sum_{k=-N}^N c_k e^{j2\pi kt}$$

converges to $p(t)$ as $N \rightarrow \infty$. Let's start with $N = 10$.

Open a new m-file. First, create a vector \mathbf{k} of length 21 containing the integers from -10 to 10 .
 $\mathbf{k} = (-10:10);$

Create a vector \mathbf{t} containing 201 time samples¹ in the interval $[-.5, .5]$.

```
t = linspace(-.5, .5, 201);
```

Next, you can create the function $p(t)$ as follows.

```
p = (t >= 0);  
p = 2.*p - 1;
```

Now create a vector \mathbf{c} of Fourier coefficients using the function you computed in part (a). Hint: You may need to manually enter c_k for $k = 0$.

```
c = YOUR_FUNCTION(k);
```

Finally, you can compute $p_{10}(t)$.

```
y = zeros(size(t));  
for i=1:length(c)  
y = y + c(i)*exp(j*2*pi*k(i).*t);  
end  
y = real(y);
```

Note: Analytically, we do not need to take the real part of \mathbf{y} , but computationally we need to because of rounding errors.

Now plot $p(t)$ and $p_{10}(t)$ in the same figure.

```
plot(t, y, 'b-');  
hold on;  
stairs(t, p, 'k--');  
hold off;
```

You should see that $p_{10}(t)$ resembles $p(t)$, but it overshoots and undershoots around $t = 0$.

Now compute $p_{100}(t)$ and $p_{1000}(t)$. Hint: Use smaller spacings between time samples in \mathbf{t} to get a better idea of the behavior of $p_N(t)$ near $t = 0$. Another hint: Type `help COMMAND` in Matlab to see a description of any of the commands we are using.

Plot, in the same figure, $p(t)$, $p_{10}(t)$, $p_{100}(t)$ and $p_{1000}(t)$. You should see Gibbs' phenomenon - the overshoot and undershoot around the discontinuity at $t = 0$.

What is the value of the partial sum approximations $p_N(t)$ at $t = 0$? Does it agree with the value of the function $p(t)$ at $t = 0$?

- (c) What is the approximate value of the maximum overshoot? The maximum overshoot is defined as the maximum value of $|p(t) - p_N(t)|$ near $t = 0$. Does the overshoot get smaller as the number of terms in the sum increases, or does it stay constant?
- (d) When does this maximum overshoot occur for $p_{10}(t)$, $p_{100}(t)$, and $p_{1000}(t)$? What is the relationship between when the maximum overshoot occurs and the number of terms in the sum?

¹How far apart should the time samples be? We will see when we discuss sampling that the answer depends on the highest frequency component of the signal.