Quiz 1

Name:

SID:

Instructions:

- **Make sure you write your name and SID**.
- The exam is closed book.
- Use of calculators is allowed.
- No partial credit will be given.

Formulas that may be of use:

- Geometric series $\sum_{k=l}^{m} \alpha^k = \frac{\alpha^l - \alpha^{m+1}}{1-\alpha}$
- Continuous time Fourier series:
  $x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{+\infty} a_k e^{jk(2\pi/T)t}$,
  $a_k = \frac{1}{T} \int_T x(t) e^{-jk\omega_0 t} dt = \frac{1}{T} \int_T x(t) e^{-jk(2\pi/T)t} dt$,
  $\frac{1}{T} \int_T |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |a_k|^2$
- Discrete time Fourier series:
  $x[n] = \sum_{k=<N>} a_k e^{jk\omega_0 n} = \sum_{k=<N>} a_k e^{jk(2\pi/N)n}$,
  $a_k = \frac{1}{N} \sum_{n=<N>} x[n] e^{-jk\omega_0 n} = \frac{1}{N} \sum_{n=<N>} x[n] e^{-jk(2\pi/N)n}$,
  $\frac{1}{N} \sum_{n=\langle N \rangle} |x[n]|^2 = \sum_{k=\langle N \rangle} |a_k|^2$

Problem 1 *(Linear Systems)*

a) A popular way to transmit radio signals is called Amplitude Modulation (AM). For transmitting a signal $x(t)$, the transmitter multiplies it with $\cos(\Omega t)$, where Ω is some fixed large frequency. The transmitted signal, $y(t)$, is, therefore

$$y(t) = x(t)\cos(\Omega t)$$

Consider the system with input $x(t)$ and output $y(t)$. For given (fixed) Ω ,is the system

linear?	Yes	time invariant?	No	stable?	Yes

b) Is the system with input $x(t)$ and output $y(t) = x(at+b)$

linear?	Yes	time invariant?	No	stable?	Yes

Problem 2 *(Graphical Convolution)*

Graphically convolve the two signals $x[n]$ and $h[n]$ shown in Fig. 1 to evaluate the output $y[n]$ at the following points

Answer: (a) y[3]= 3 (b) y[4] = 7

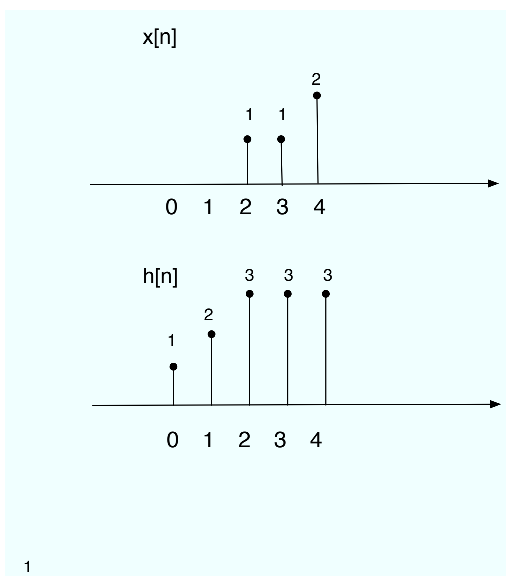

Figure 1: Problem 2

Problem 3 *(Frequency Response)*

$y(t)$ is the output of an LTI system with frequency response $H(j\omega)$ shown below in Figure 2. When the input $x(t) = e^{\frac{j\pi t}{6}} + \sin(\frac{5\pi t}{8} + \frac{\pi}{4})$ is fed into the system, determine $y(0), y(10)$.

$$H(j\omega) = \begin{cases} 1 & \text{if } \frac{\pi}{12} < |\omega| < \frac{\pi}{2} \\ 0 & \text{otherwise} \end{cases}$$

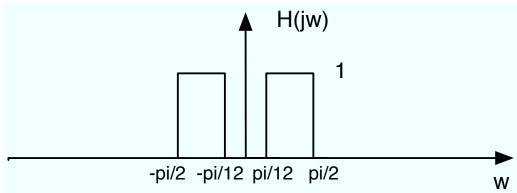

Figure 2: Problem 3

Answer $\boxed{y(0) = 1}$

Answer $\boxed{y(10) = e^{j5\pi/3}}$

Problem 4 *(Continuous Time Fourier Series)*

Plot $x(t)$ for a sequence of Fourier series coefficients $\{a_k\}$, given as follows :

$a_0 = 2,\ a_1 = 1,\ a_{-1} = 1$, and $a_k = 0$ for all k other than $k = -1, 0, 1$. Assume that the period of the signal is $T = 2\pi$.

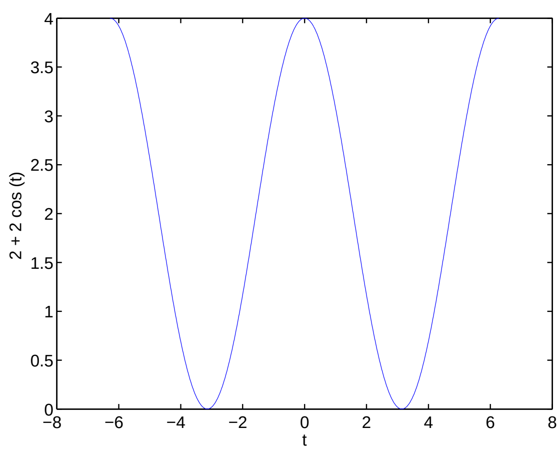

Figure 3: Problem 2