Professor Fearing EECS120/Problem Set 3 v 1.01 Fall 2014

Due at 4 pm, Fri. Sep. 26 in HW box under stairs (1st floor Cory) Max 2 names per HW.

1. (18 pts) Multiplication of periodic functions.

Given periodic functions $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_o t}$ and $y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\omega_o t}$ with period T_o:

a. Show that the Fourier coefficents c_k of the signal $z(t) = x(t)y(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_o t}$ can be found from $c_k = \sum_{n=-\infty}^{\infty} a_n b_{k-n}$.

b. Given $x(t) = \cos 16\pi t$ and $y(t) = \cos 10\pi t$ with $T_o = 1.0$, find and sketch the line spectra c_k.

c. Given $x(t) = \cos 16\pi t$ and $y(t) = \Pi(t) * comb(t/2)$ with $T_o = 2.0$, find and sketch the line spectra c_k of $z(t)$.

2. (18 pts) Discrete Fourier Series.

Determine Fourier series coefficients for each of the following discrete time periodic signals. Plot the magnitude and phase of each set of coefficients a_k.

a. Period $N = 12$, $x[n] = 1 - \sin\frac{\pi n}{4}$ for $0 \leq n \leq 11$.

b. Period $N = 4$, $x[n] = 1 - \sin\frac{\pi n}{4}$ for $0 \leq n \leq 3$.

c. $x[n] = 1, 0 \leq n \leq 3$ and $x[n] = 0, 4 \leq n \leq 5$, $N = 6$.

3. (22 pts) Total Harmonic Distortion

Using Parseval's theorem, the ratio of power in the harmonics to the power in the fundamental (total harmonic distortion) is given by: THD $= \frac{\sum_{k=2}^{\infty} |a_k|^2}{|a_1|^2}$.

A signal $x(t) = \cos 2\pi t$ is passed though a saturating amplifier with $y(t) = x(t)$ for $|x(t)| < \frac{\sqrt{3}}{2}$ and with $y(t) = \frac{\sqrt{3}}{2}$ for $x(t) > \frac{\sqrt{3}}{2}$ and with $y(t) = -\frac{\sqrt{3}}{2}$ for $x(t) < -\frac{\sqrt{3}}{2}$.

a) Sketch $y(t)$ and determine the Fourier Series coefficients for $y(t)$ and sketch the line spectra.

b) Calculate the total power in $y(t)$.

c) Calculate the total power in the fundamental frequency of $y(t)$.

d) Calculate the total power in the higher harmonics and the total harmonic distortion.

4. (18 pts) Filtering periodic signals.

A simple microprocessor may not have a digital-to-analog convertor output, but an analog output can be approximated using a variable width periodic pulse followed by a low pass filter. Consider the microprocessor generating an output rectangular wave, with peak 3V, minimum 0V, with frequency 10 kHz ($T_o = 10^{-4}$ sec). The low pass filter has impulse response $h(t) = e^{-t/\tau}u(t)$ with time constant $\tau = 0.01$ sec. For each duty cycle, calculate average voltage output (i.e. a_0), ripple voltage at 10 kHz (i.e. voltage component at $k = \pm 1$), and per cent error from average voltage output due to ripple voltage at 10 kHz.

a. Duty cycle 20%. b. Duty cycle 40%. c. Duty cycle 50%.

5. (24 pts) Fourier Transform

Consider an LTI system with impulse response $h(t) = \frac{\sin 4\pi(t-1)}{\pi(t-1)}$. Find the output $y_i(t)$ for each of the following inputs:

a. $x_1(t) = \cos(2\pi t + 1/2)$ b. $x_2(t) = (\frac{\sin 8\pi t}{\pi t})^2$

c. $x_3(t) = \frac{\sin 4\pi(t+1)}{\pi(t+1)}$ d. $x_4(t) = \Pi(2t) * comb(t)$