Professor Fearing EECS120/Problem Set 5 v 1.01 Fall 2014

Due at 4 pm, Fri. Oct. 10 in HW box under stairs (1st floor Cory)

Midterm 1 in class Wed. Oct. 15

1. (18 pts) DTFT properties (Lec 7,8 OW Ch 5)

Given $\mathcal{F}\{x[n]\} = X(e^{j\omega})$, find the DTFT of the following signals in terms of $X(e^{j\omega})$ for $x[n]$ complex.

a. $Re\{x[n]\}$

b. $x^*[-n]$

c. $Even\{x[n]\}$

2. (21 pts) DTFT exercises (Lec 8,9, 5.5)

Given $x[n] = \sin(\pi n/8) - 2\cos(\pi n/4)$ is input to an LTI system with the following impulse response. Determine the output $y[n]$ for each input:

a. $h_1[n] = \frac{\sin(\pi n/6)}{\pi n}$

b. $h_2[n] = \frac{\sin(\pi n/6)}{\pi n} + \frac{\sin(\pi n/2)}{\pi n}$

c. $h_3[n] = \frac{\sin(\pi n/6)}{\pi n}\frac{\sin(\pi n/3)}{\pi n}$

3. (21 pts) LDE and DTFT (Lec 9, OW 5.8)

a. Given an LTI system with input $x[n]$ and output $y[n]$ described by the LDE: $y[n] + \frac{1}{2}y[n-1] = x[n] - \frac{1}{4}x[n-1]$, find the impulse response of the inverse, $g[n]$ such that $g[n] * h[n] = \delta[n]$ and the difference equation for the inverse.

For the following DTFTs, find the corresponding causal difference equation, and determine minimum number of additions, multiplies, and delays required for implementation.

b.

$$H(e^{j\omega}) = \frac{e^{2j\omega} - \sqrt{2}e^{j\omega} + 1}{e^{2j\omega}},$$

c.

$$H(e^{j\omega}) = \frac{e^{2j\omega}}{e^{2j\omega} - \frac{3}{4}\sqrt{2}e^{j\omega} + 9/16}$$

4. (20 pts) Sampling, Reconstruction, and Interpolation (Lec 8, 4.5)

A signal $x(t) = \cos(2\pi t)$ is sampled at 8 Hz:

$$\tilde{x}(t) = x(t) \cdot \sum_{n=-\infty}^{n=\infty} \delta(t - \frac{n}{8}) \tag{1}$$

a) Sketch $X(j\omega)$ and $\tilde{X}(j\omega)$.

b) Find and sketch $X(e^{j\omega})$ for $x[n] = \cos(\frac{\pi n}{4})$ and compare to $X(j\omega)$ and $\tilde{X}(j\omega)$ from part a.

c) Find an ideal reconstruction filter $R(j\omega)$ such that $\hat{X}(j\omega) = R(j\omega)\tilde{X}(j\omega)$ and show that the reconstructed signal $\hat{x}(t)$ is identical to $x(t)$.

d) In the time domain, find an expression for $\hat{x}(t)$ (in terms of $\tilde{x}(t)$), and evaluate $\hat{x}(t = \frac{1}{6})$, either in closed form or numerically. (Note that the reconstruction filter interpolates between samples to find this value. If the signal is bandlimited, the ideal LPF does exact interpolation.)

5. (20 pts) Digital Receiver in iPython (Lec 9, 8.1, 8.2)

Two analog signals are sent over a radio channel using a signal $x(t) = m(t)\cos\omega_c t + n(t)\sin\omega_c t$, where $m(t), n(t)$ represent the useful information, for example voice signals. Assume $\omega_c = 2\pi 500\text{kHz}$. You are given a data file `signal.wav` (for compactness, not an audio file) which is $x(t)$ sampled at 4 MHz. Download `PS5-receiver.ipynb` from the class web page. Design and implement an iPython function to recover $m(t)$ and $n(t)$ from the receiver signal. Turn in a block diagram of the receiver functions, sketches of approximate DTFT spectra at key points, iPython code for the receiver functions, and plots of $m(t)$ and $n(t)$. Useful functions may include `numpy.sinc` and `numpy.convolve`.