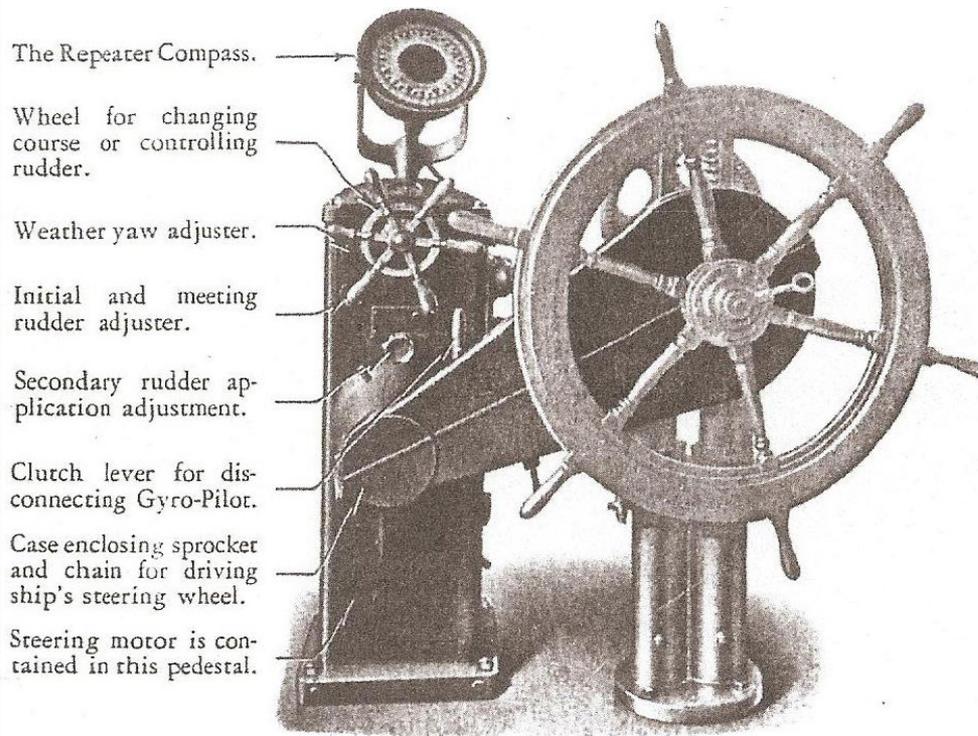


# AUTONOMOUS DRIVING

## a brief EE120+ level review



The Sperry gyropilot (ca. 1922)



Google car (2012)



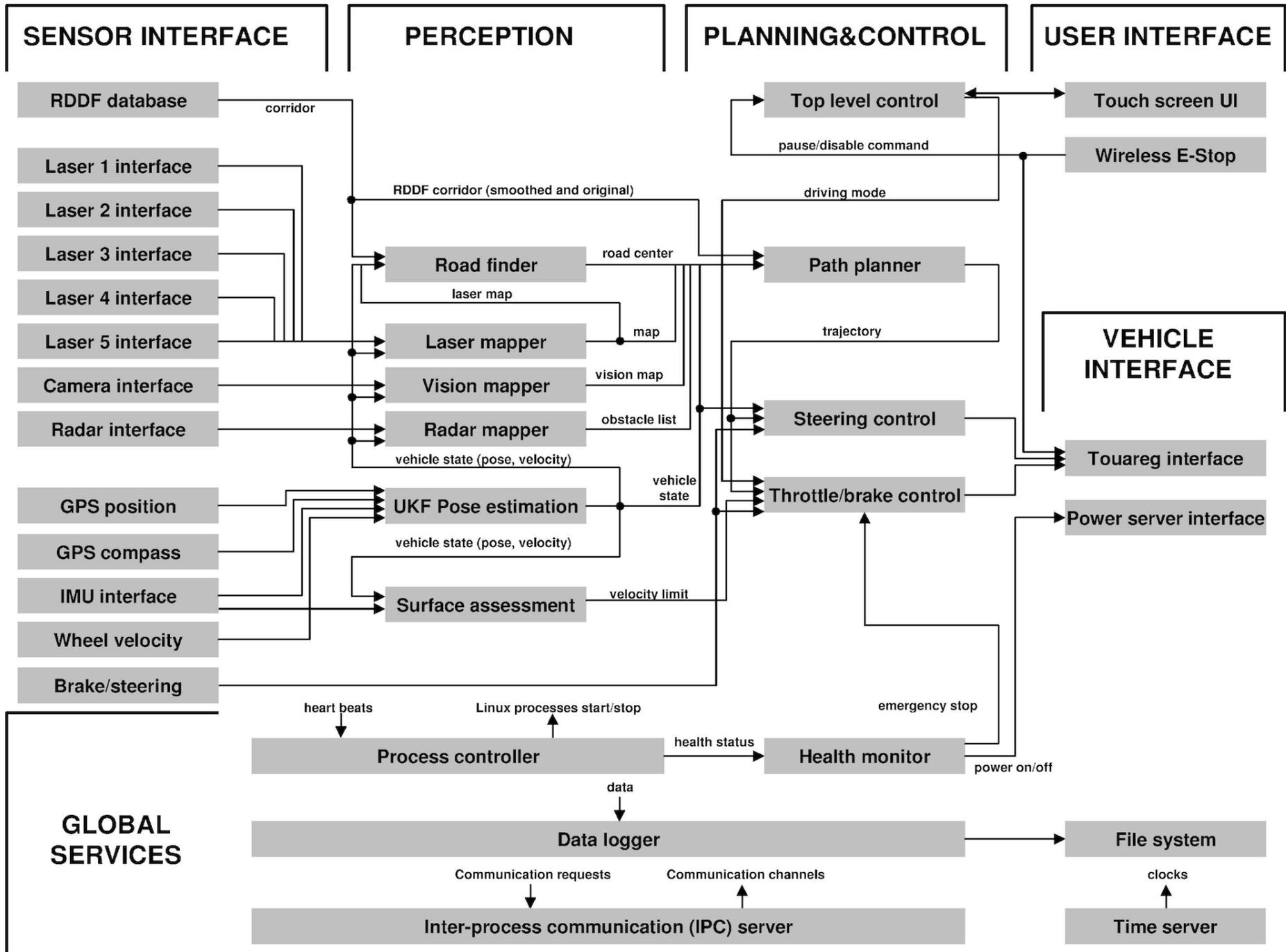
ca. 1957

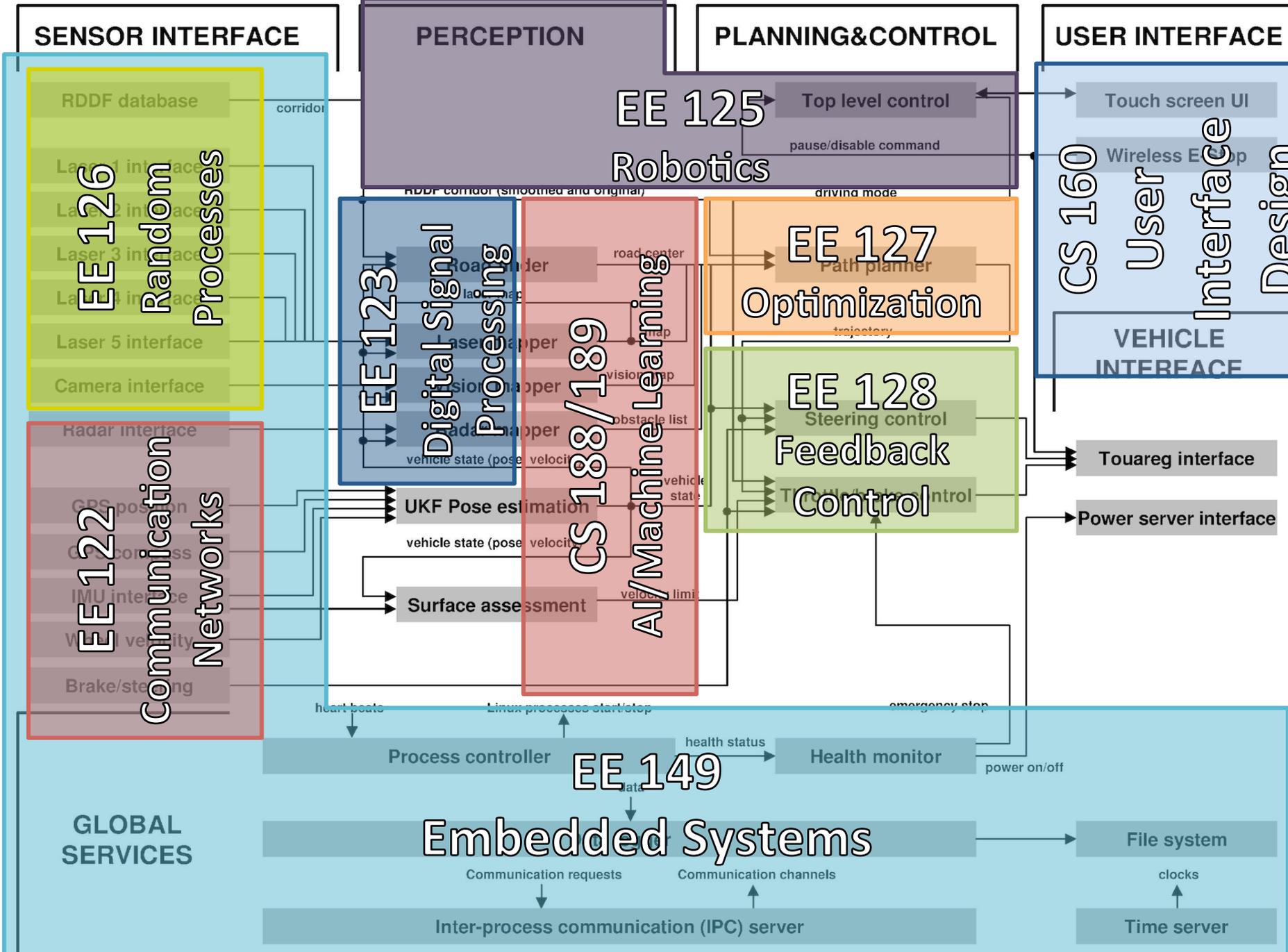
The idea of self-driving cars has existed for a long time. Recent progress was enabled by computation, sensors, radar, mapping, machine learning, and catalyzed by the DARPA Grand Challenges.

# Stanley: The winner of the 2005 DARPA Grand Challenge



Credit: Thrun, Journal of Field Robotics, 23(9), pp. 661-692, 2006. DOI: 10.1002/rob.20147





# Development steps – automated driving

Degree of automation ↑

- Single sensor
- Sensor-data fusion
- Sensor-data fusion + map



## ACC/lane keeping support

Only longitudinal or lateral control



## Integrated cruise assist

Partially automated longitudinal and lateral guidance in driving lane  
Speed range 0-130 kph



## Highway assist

Partially automatic longitudinal and lateral guidance  
Lane change after driver confirmation  
Supervision of surrounding traffic (next lane, ahead, behind)



## Highway pilot

Highly automated longitudinal and lateral guidance with lane changing  
Reliable environment recognition, including in complex driving situations  
No permanent supervision by driver

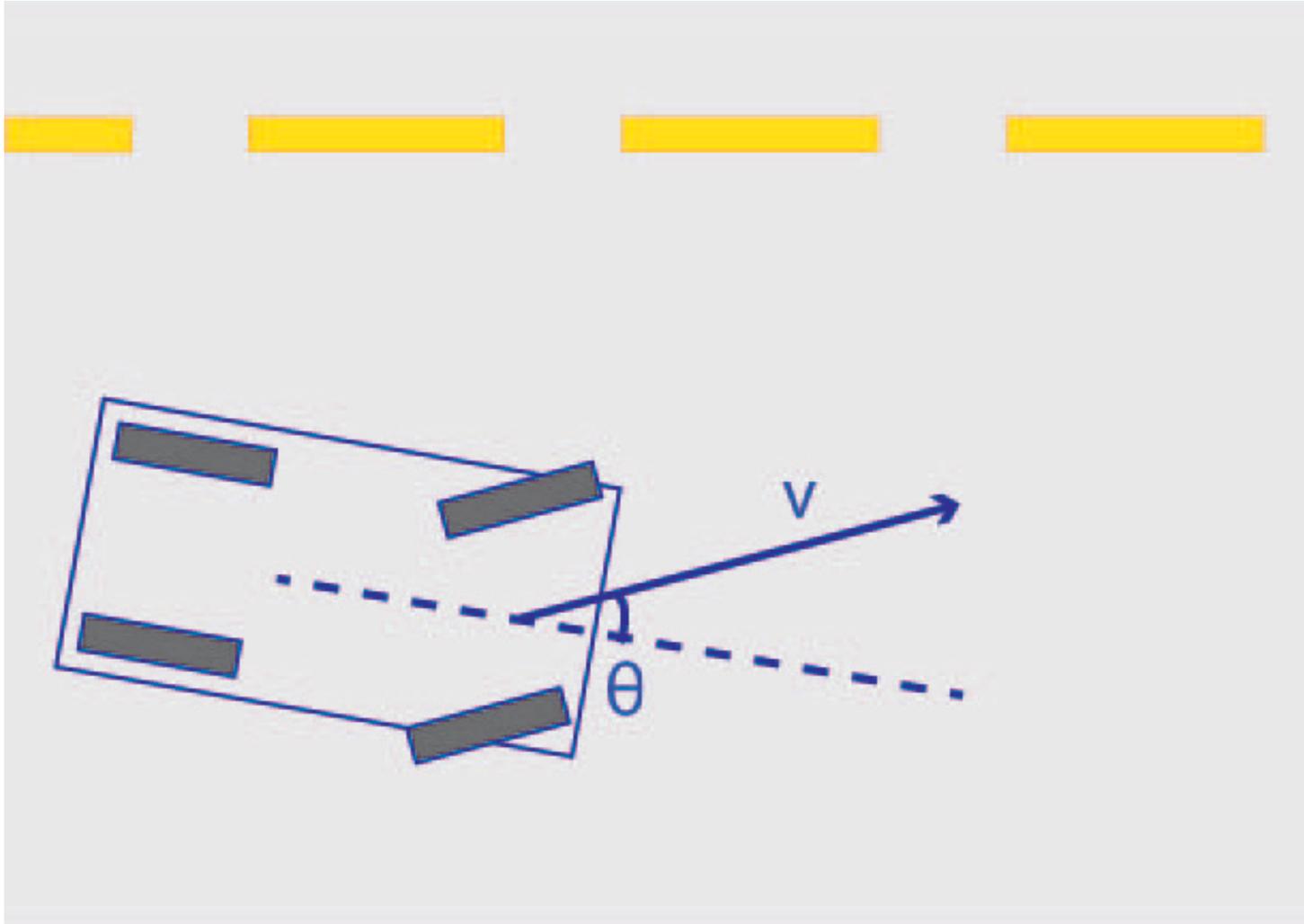


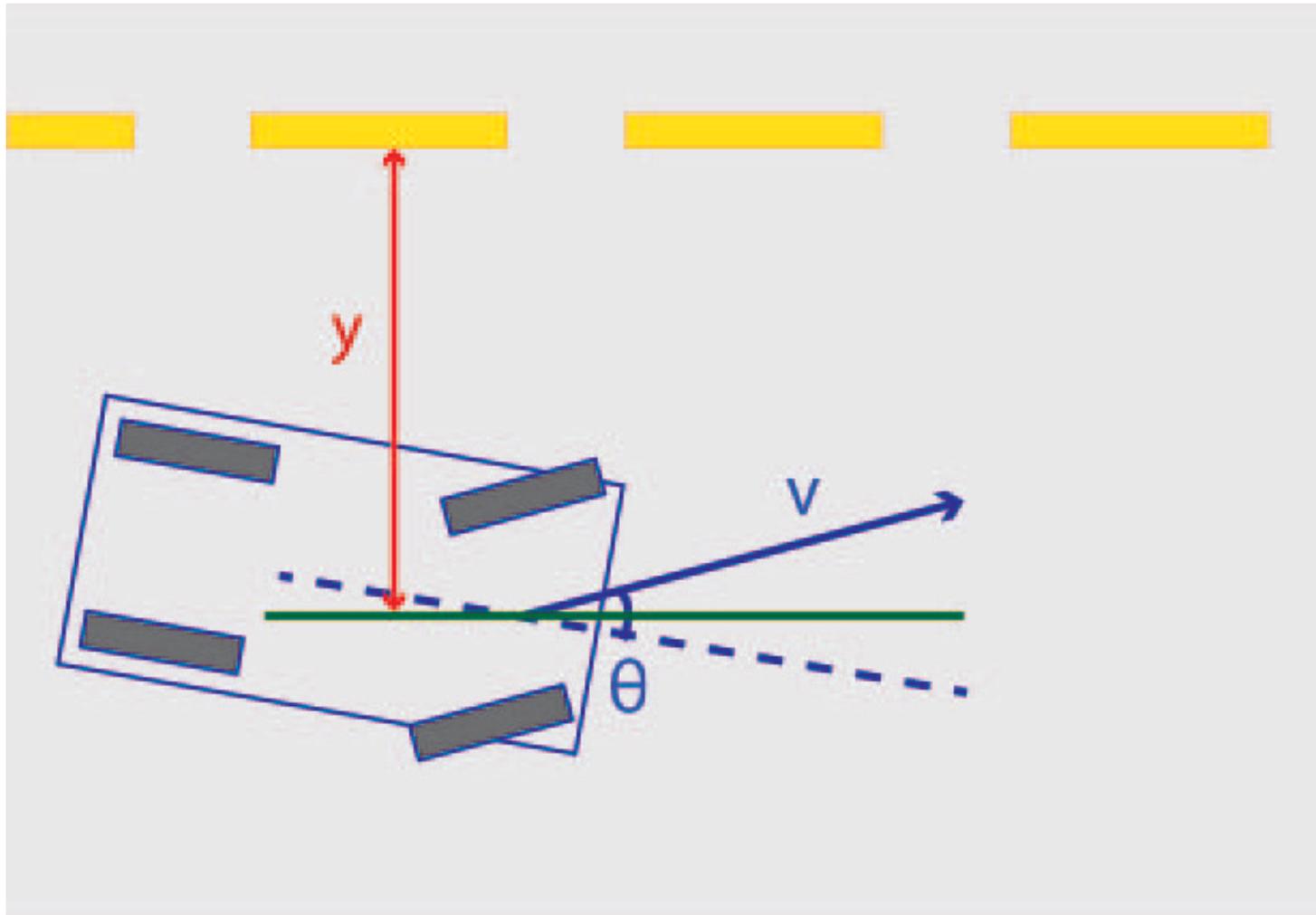
## Auto pilot

Door-to-door commuting (e.g. to work) in urban traffic  
Strictest safety requirements  
No supervision by driver

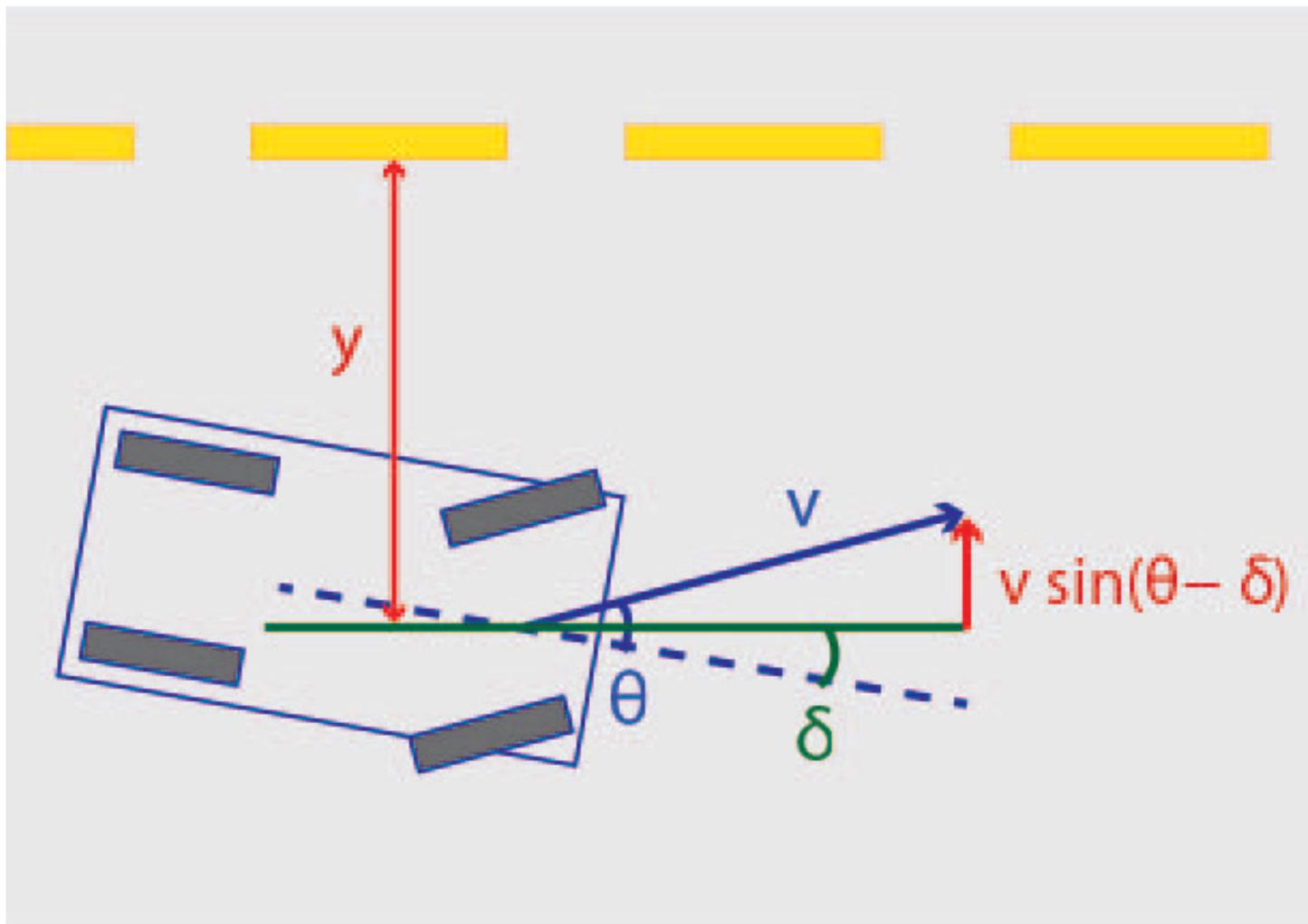


# Topic 1: Steering Control





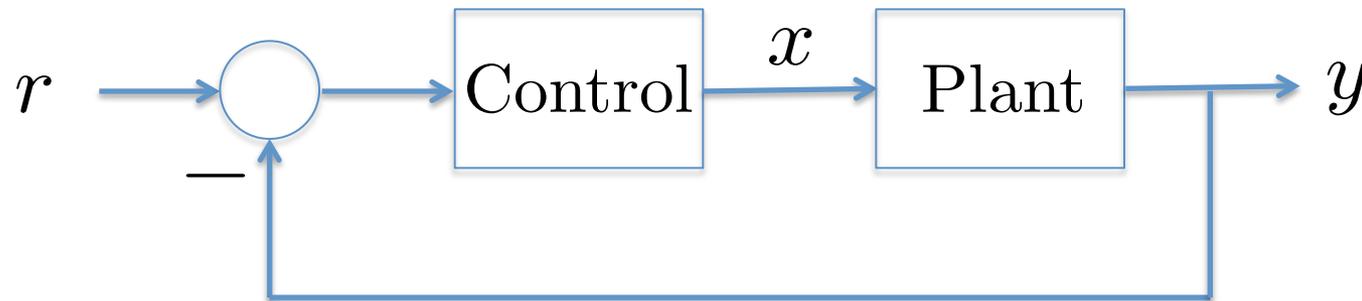
Goal: Maintain  $y(t)$  at desired value



$$\frac{dy(t)}{dt} = -v \sin(\theta - \delta)$$

Define  $x \triangleq \delta - \theta$ . Then, the 'plant' is:

$$\frac{dy(t)}{dt} = v \sin(x(t)) \quad (\text{nonlinear!})$$



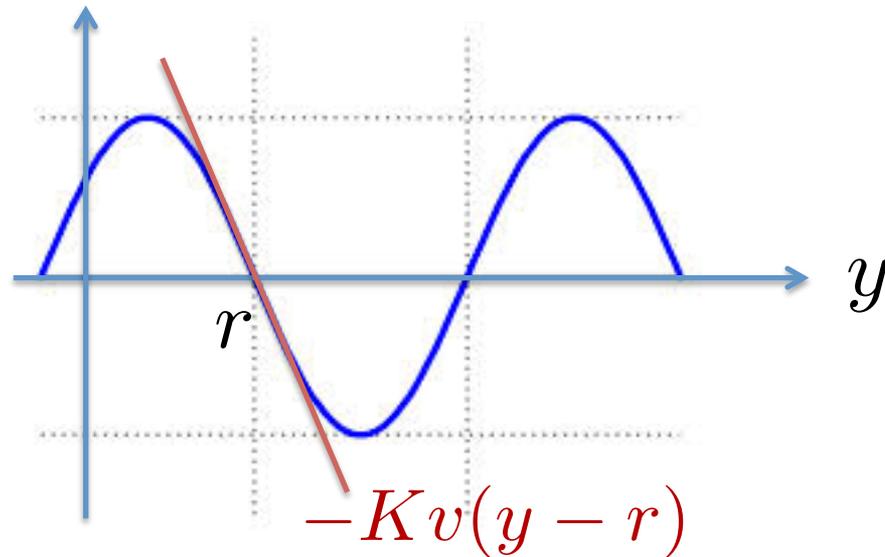
First, try the constant gain control

$$x = K(r - y)$$

Closed-loop:  $\frac{dy}{dt} = v \sin(K(r - y))$

Near  $y = r$ ,

$$\frac{dy}{dt} = v \sin(K(r - y)) \approx -Kv(y - r)$$



Therefore, for constant  $r$ :

$$y(t) - r \approx (y(0) - r)e^{-Kvt}$$

if  $y(0) - r$  small

# Root Locus Interpretation

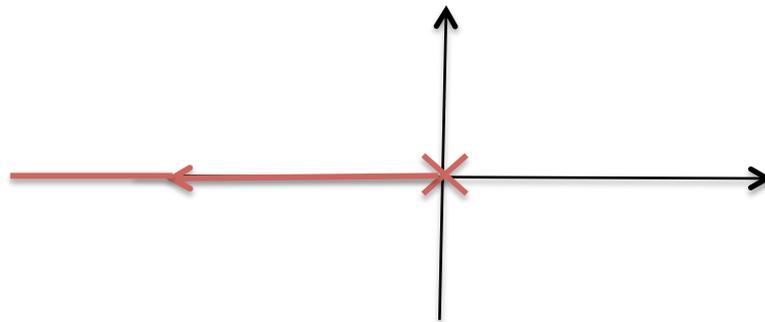
Linearized plant model:

$$\frac{dy}{dt} = v \sin(x) \approx vx$$

$$H_p(s) = \frac{v}{s}$$

Closed-loop poles are the roots of:

$$1 + KH_p(s) = 0 \quad \Rightarrow \quad s = -Kv$$



Steady-state error = 0 ( $H_p(s)$  has pole at 0)

The controller above does not obey the physical steering limit:  $|x| < 90^\circ$

Instead, try the nonlinear controller:

$$x = \tan^{-1}(K(r - y))$$

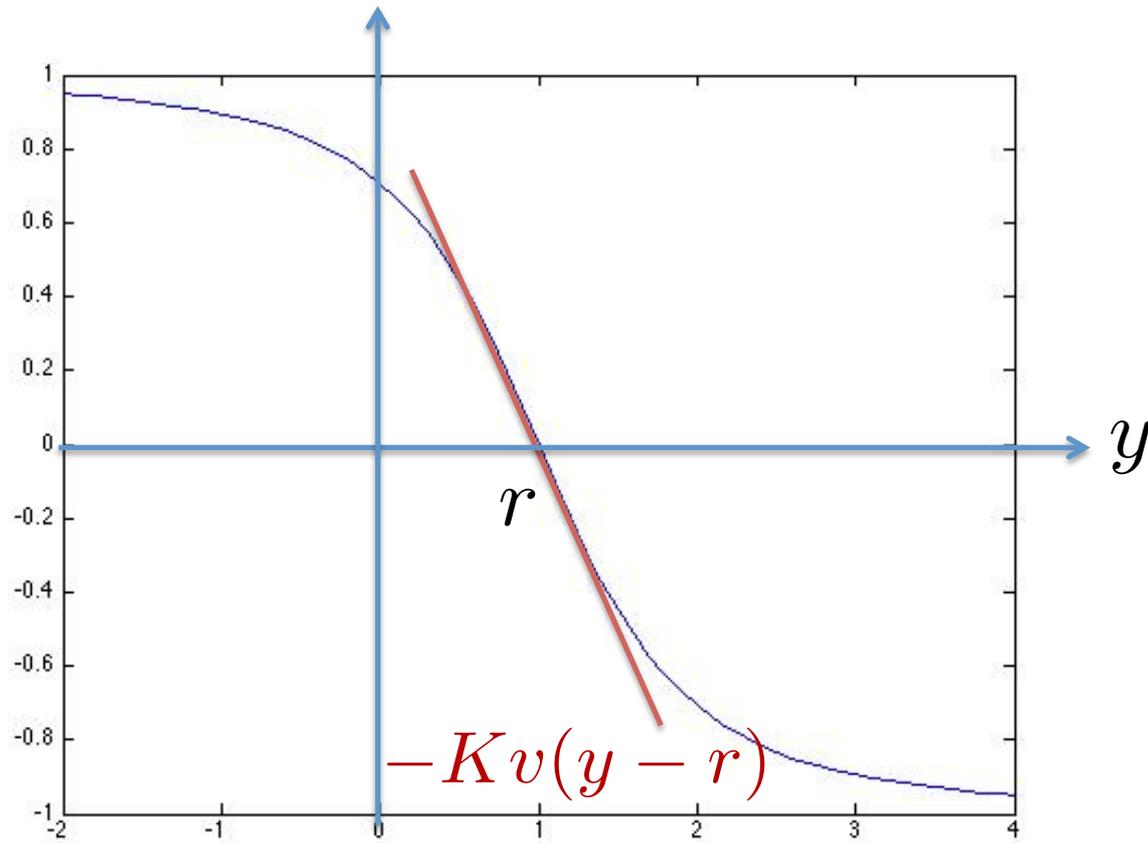
Closed-loop:

$$\frac{dy}{dt} = v \sin(\tan^{-1}(K(r - y)))$$

Substitute:  $\sin(\tan^{-1}(u)) = \frac{u}{\sqrt{1+u^2}}$

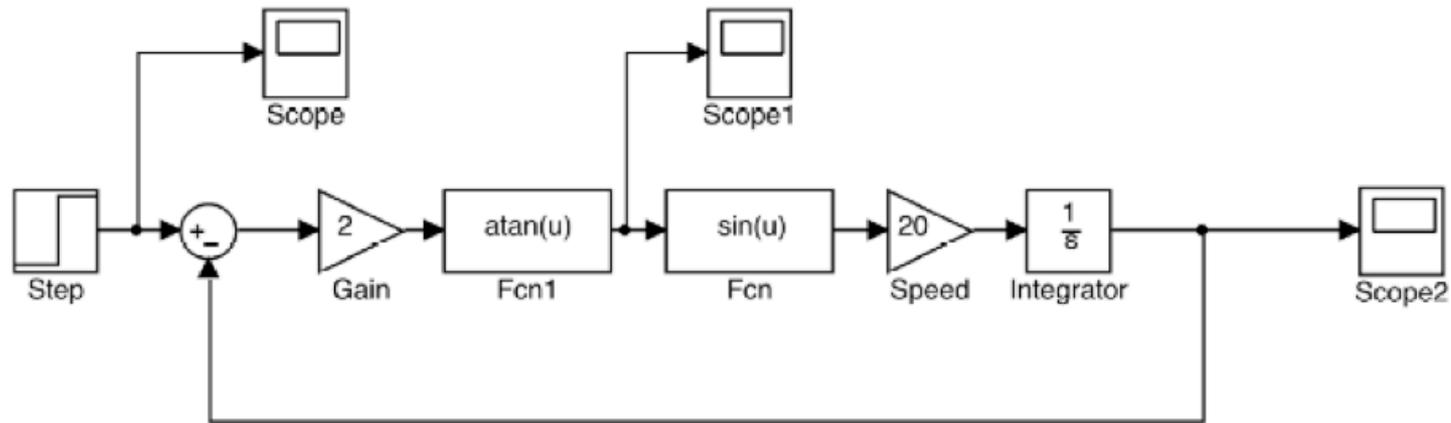
$$\frac{dy}{dt} = v \frac{K(r - y)}{\sqrt{1 + K^2(r - y)^2}}$$

$$\frac{dy}{dt} = v \frac{K(r - y)}{\sqrt{1 + K^2(r - y)^2}}$$

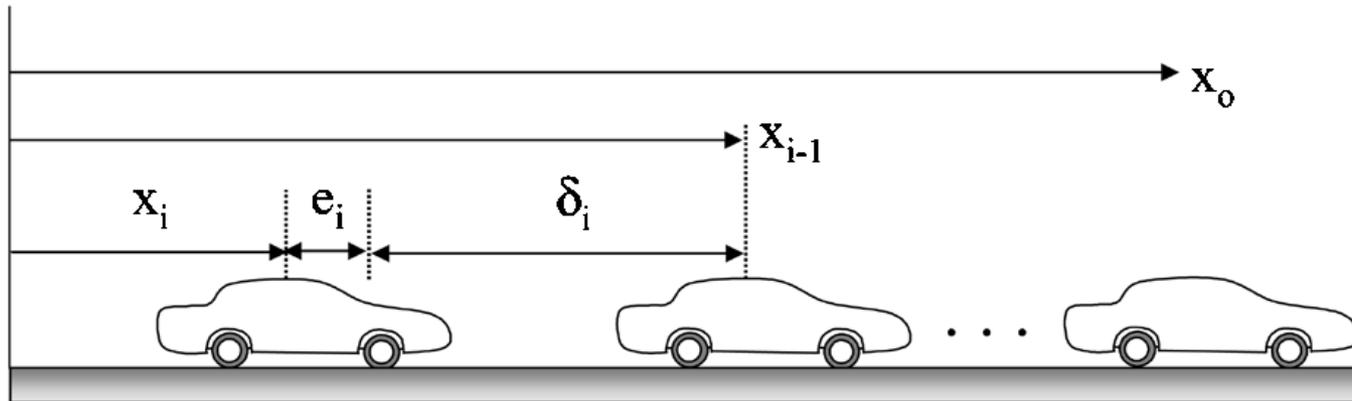


$y(t) \rightarrow r$  for all  $y(0)$

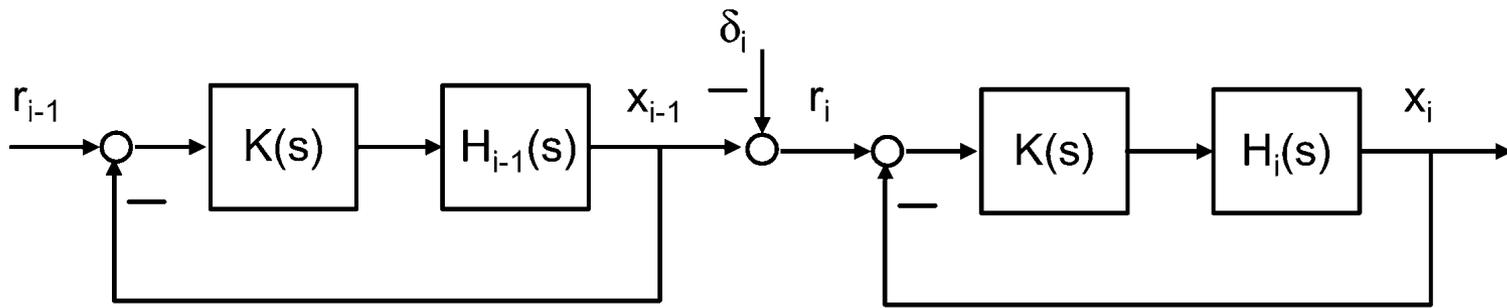
# Simulink Diagram:



# Topic 2: Vehicle Following



Goal: Maintain the gap  $x_i - x_{i-1}$  at  $\delta_i$ ; thus the reference for vehicle  $i$  is  $r_i = x_{i-1} - \delta_i$



# Vehicle Model

$$H(s) = e^{-T_d s} \frac{k}{s^2 + 2\zeta\omega_n s + \omega_n^2} \frac{1}{s}$$

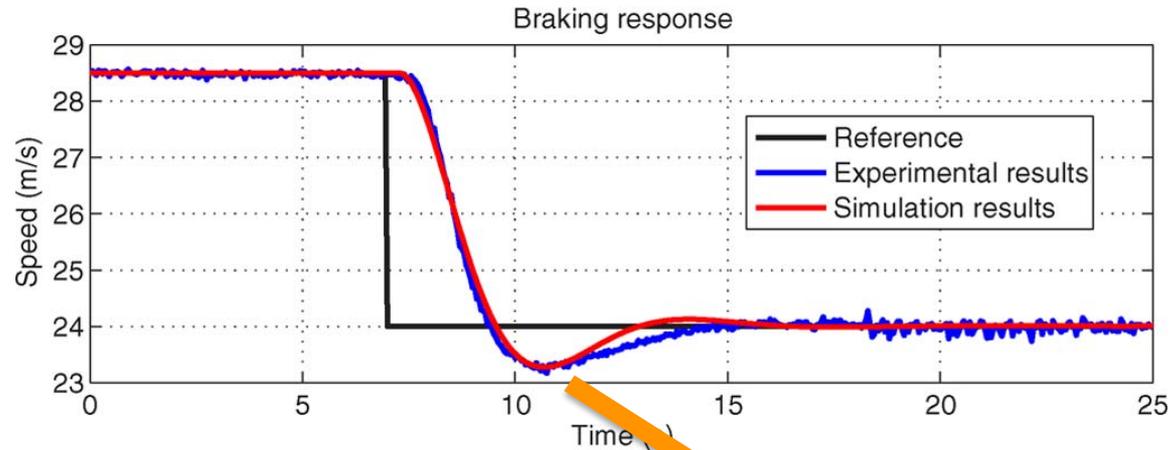
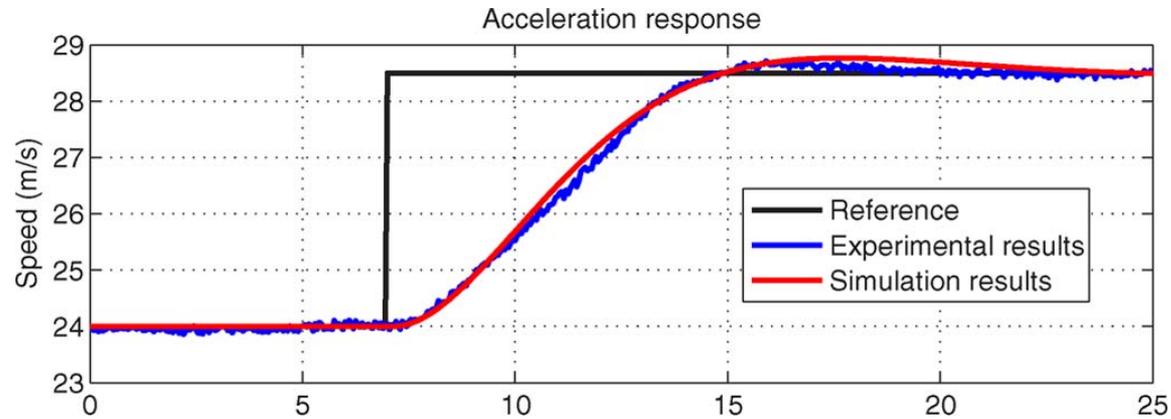
$\triangleq H_v(s)$  (velocity)

position

Model validated and parameters identified for experimental vehicles by PATH (Partners for Advanced Transit and Highways) researchers at UC Berkeley



	$k$	$\zeta$	$\omega_n$	$T_d$
<b>Accelerating</b>	0.156	0.661	0.396	0.146
<b>Braking</b>	1.136	0.5	1.067	0.287



Time (s)

overshoot due to engine braking

Credit: (Milanes et al., 2014)

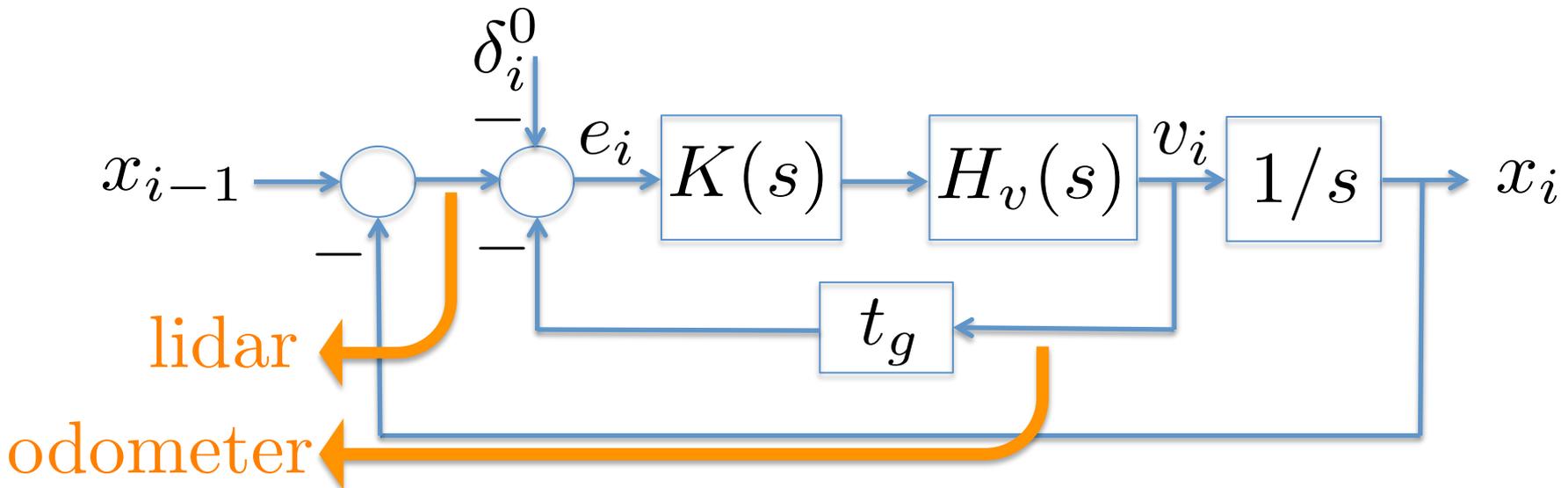
# Time Gap Control

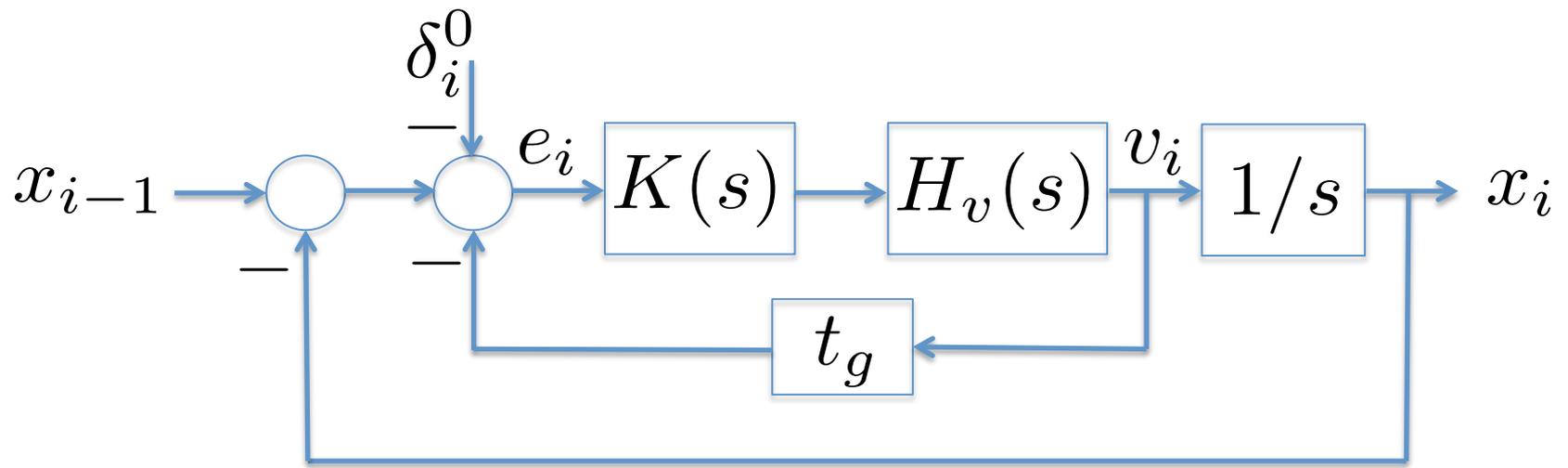
Larger  $\delta_i$  with increasing speed:

$$\delta_i = t_g v_i(t) + \delta_i^0$$

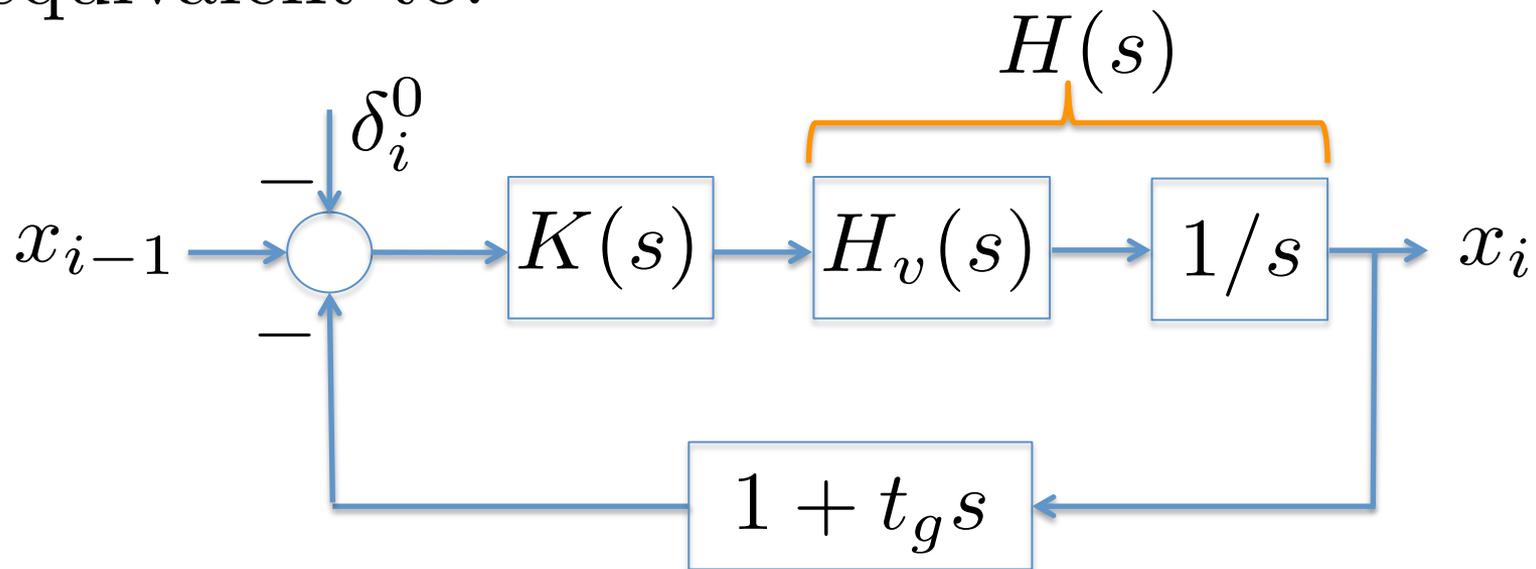
where  $t_g$  is a fixed time gap (e.g.,  $t_g = 1$  sec means about one vehicle length per 10 mph).

Error:  $e_i = x_{i-1} - x_i - v_i t_g - \delta_i^0$





is equivalent to:



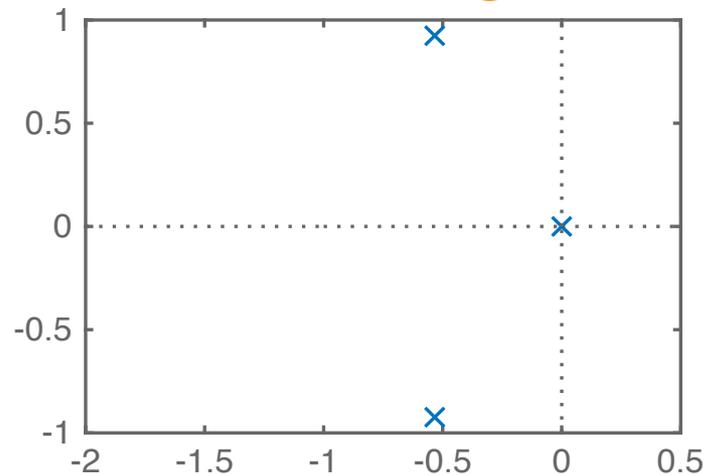
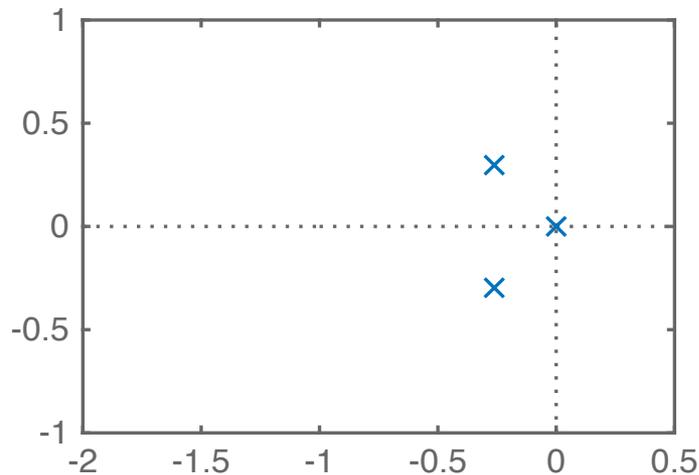
Note  $t_g = 0$  recovers the fixed distance control.

Sketch root locus for cst.gain control and  $T_d = 0$

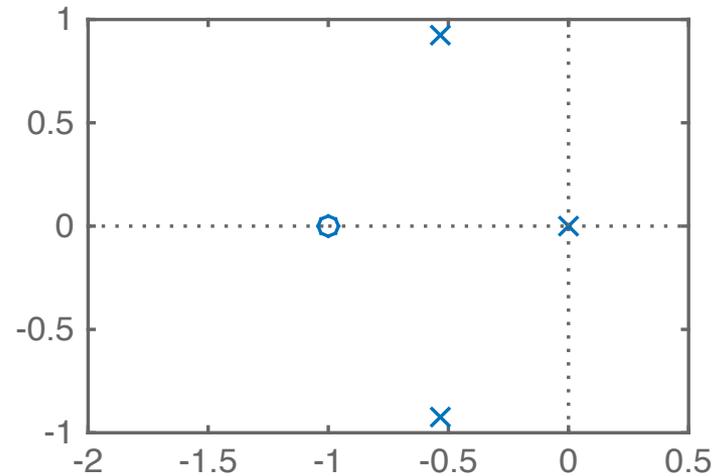
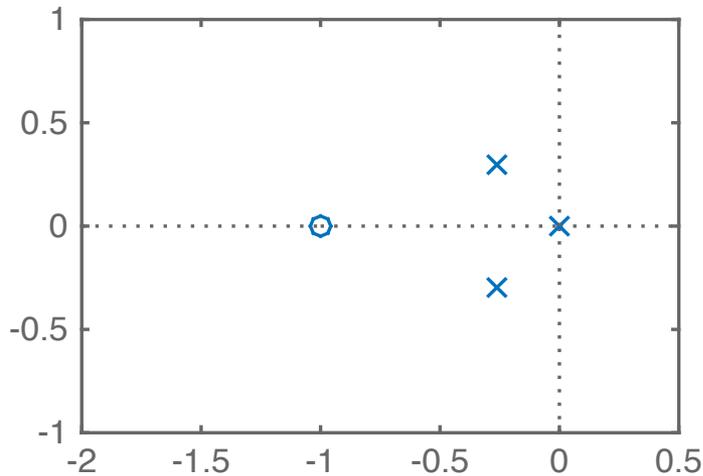
acceleration

braking

$t_g = 0$



$t_g = 1$



All-pass approximation for delay:

$$e^{-T_d s} \approx \frac{1 - \frac{T_d}{2} s}{1 + \frac{T_d}{2} s}$$

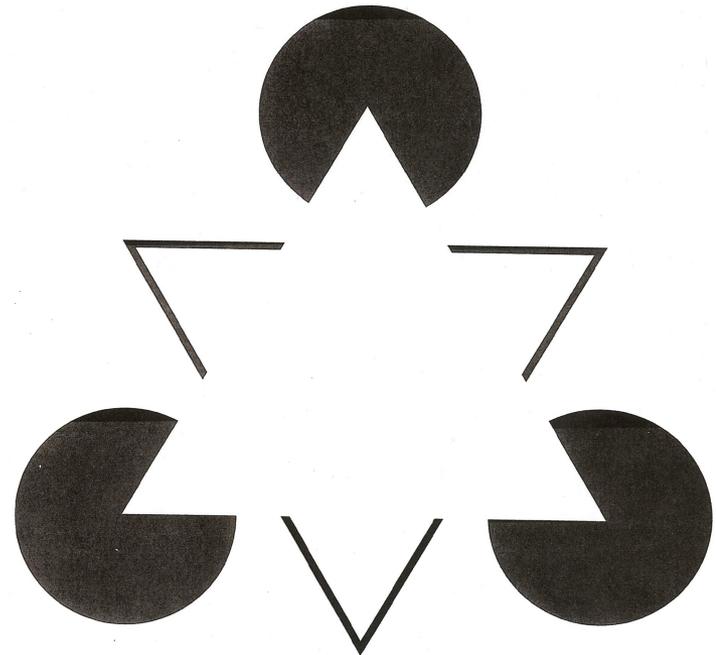
Augment transfer function with this all pass  
and design controller using Matlab rltool

# Topic 3: Edge Detection

- Detection of object boundaries is critical in computer vision (e.g., detecting lane markings).
- Visual cortex detects light-dark discontinuities. The brain interpolates between contours.



Credit: (H. Cheng, 2011)

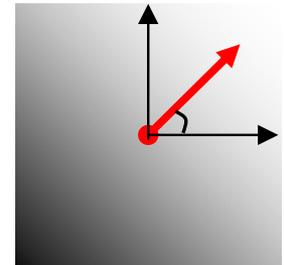
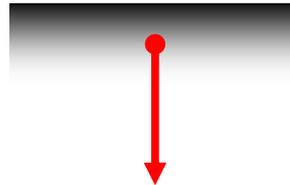
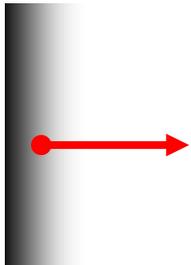


Credit: (Kanizsa, 1979)

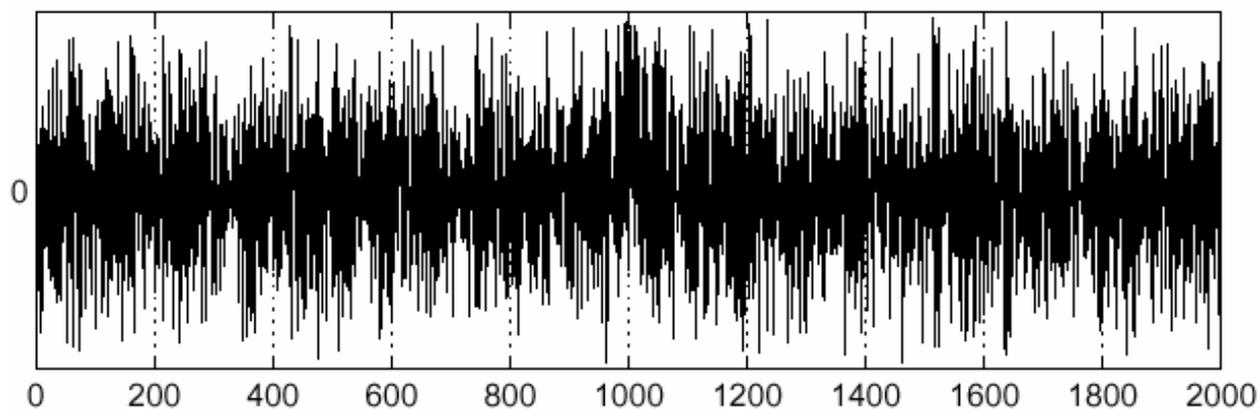
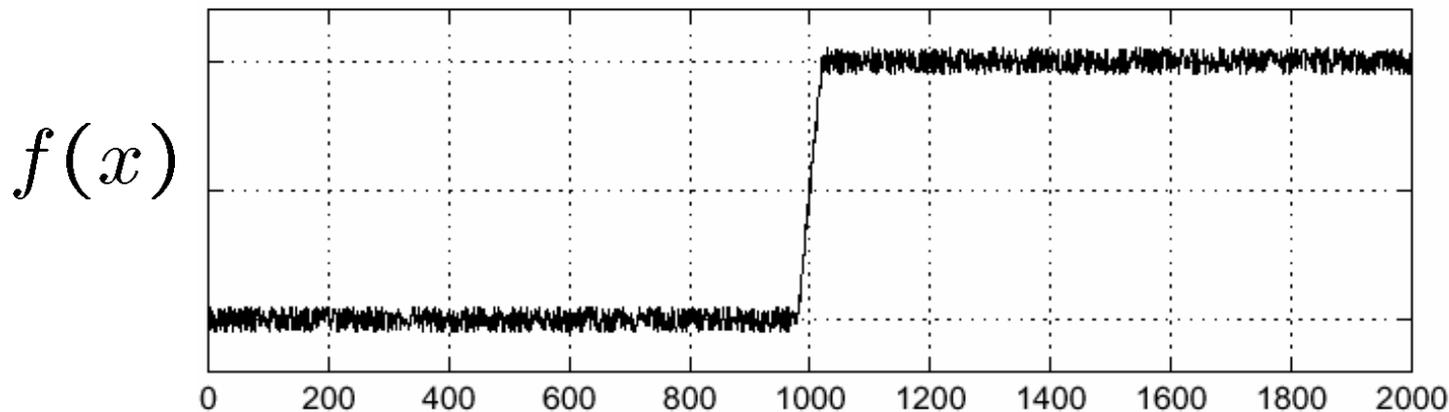
Edge detection algorithms seek sharp gradients in the image.

$$\nabla f(x, y) = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \end{bmatrix}$$

The gradient points in the direction of most rapid change in intensity:



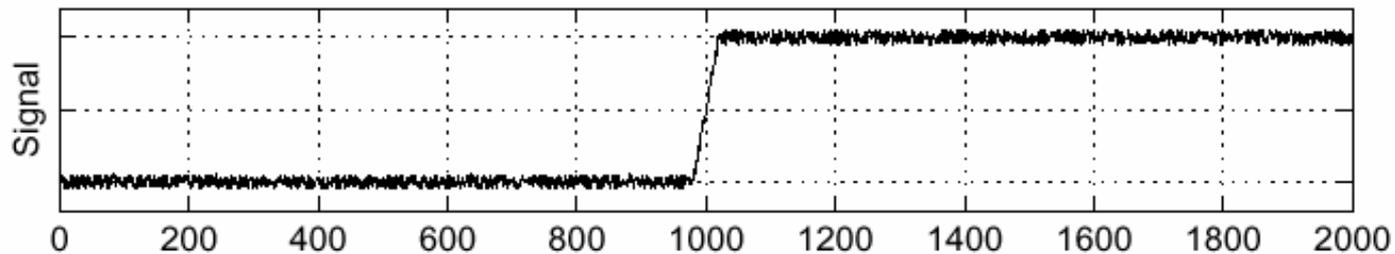
Problem: Algorithms based on derivatives are sensitive to noise



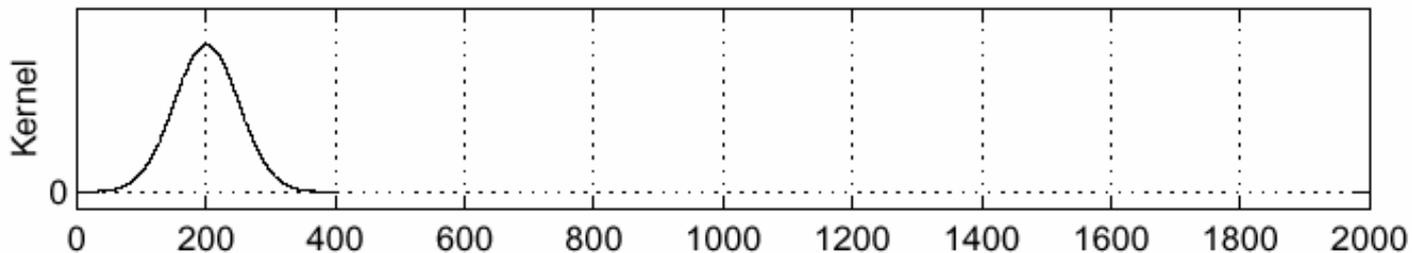
Credit: Efros, Computational Photography

# Solution: Low-pass filter before differentiation

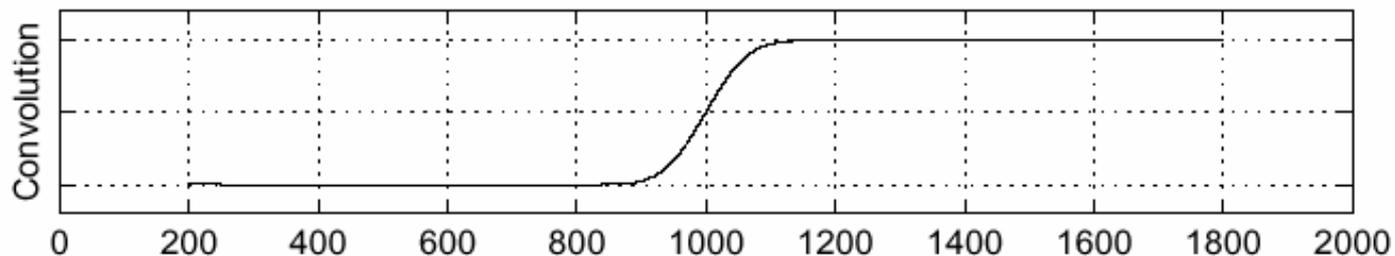
$f(x)$



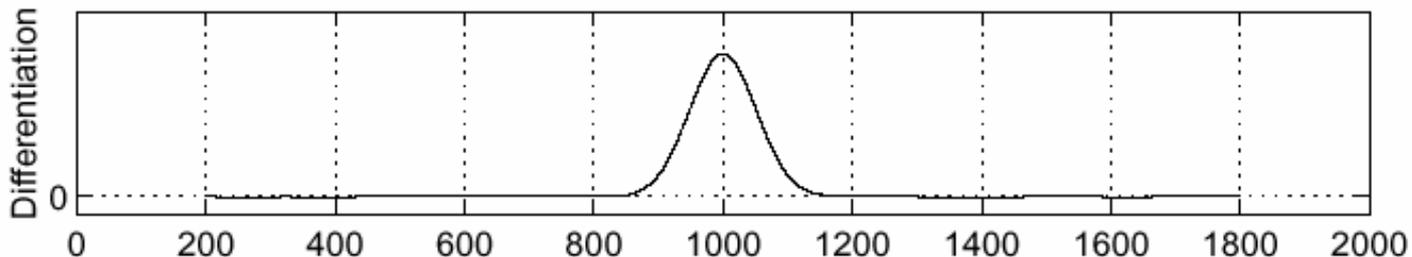
$h(x)$



$h(x) * f(x)$



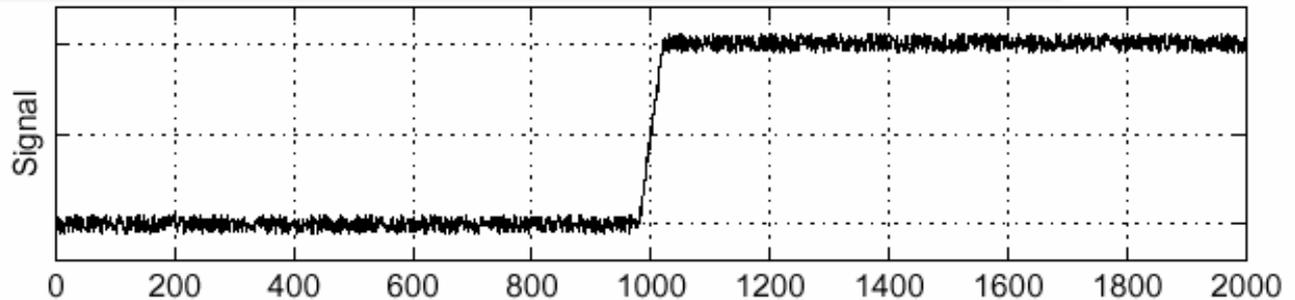
$\frac{d}{dx} \{h * f\}$



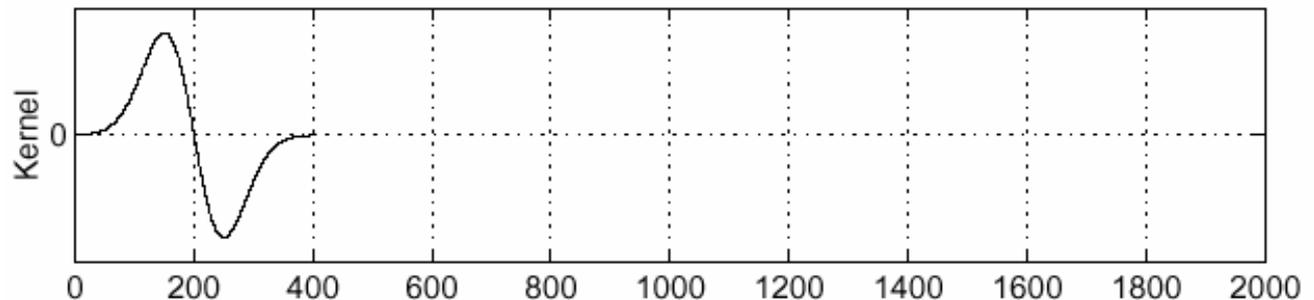
Save one operation using the property:

$$\frac{d}{dx} \{h(x) * f(x)\} = \frac{dh(x)}{dx} * f(x)$$

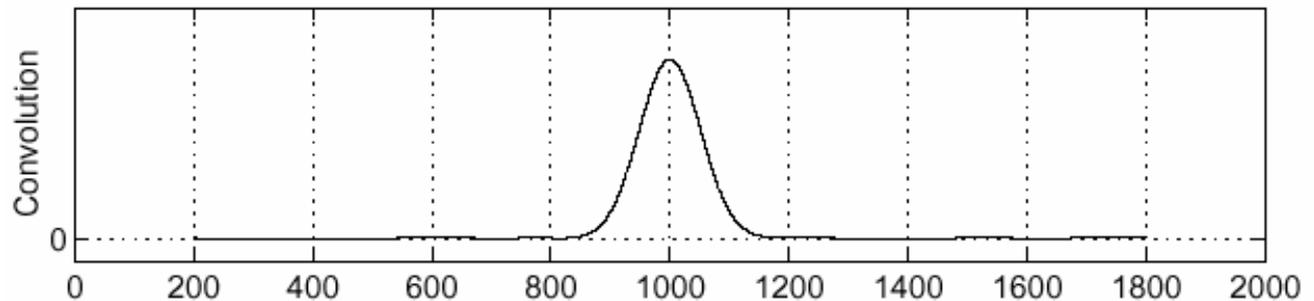
$f(x)$



$\frac{dh(x)}{dx}$



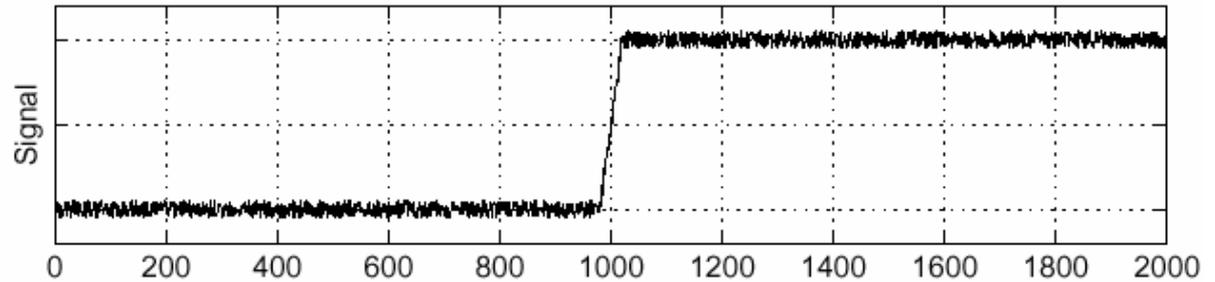
$\frac{dh(x)}{dx} * f(x)$



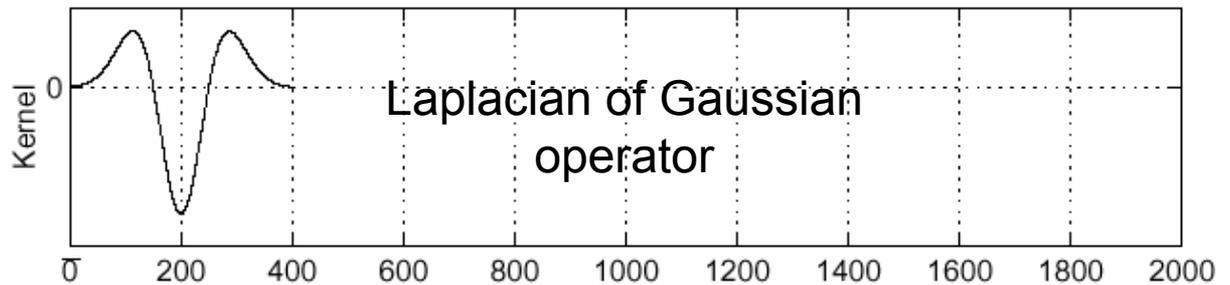
Credit: Efros, Computational Photography

The peaks of  $\frac{d}{dx} \{h * f\} = \frac{dh}{dx} * f$  are the zero  
 crossings of  $\frac{d^2}{dx^2} \{h * f\} = \frac{d^2h}{dx^2} * f$

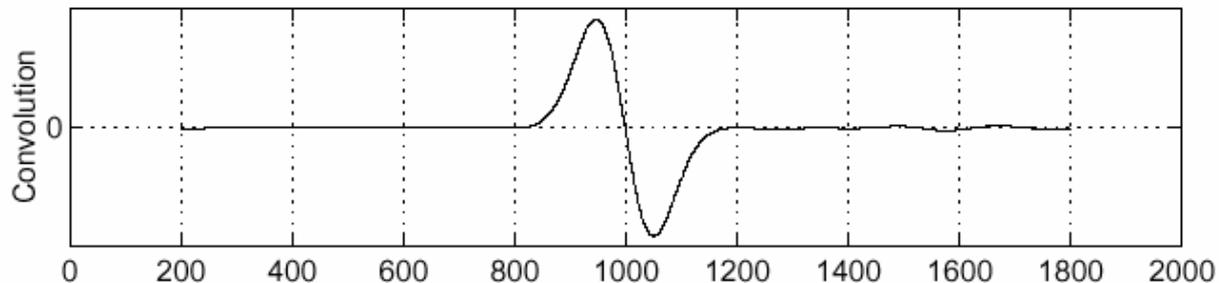
$f(x)$



$\frac{d^2h}{dx^2}$



$\frac{d^2h}{dx^2} * f$

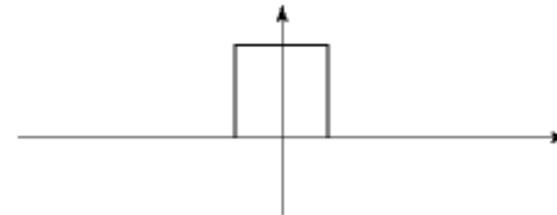
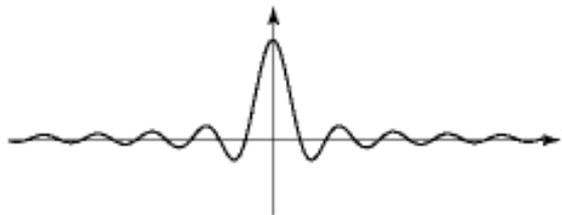
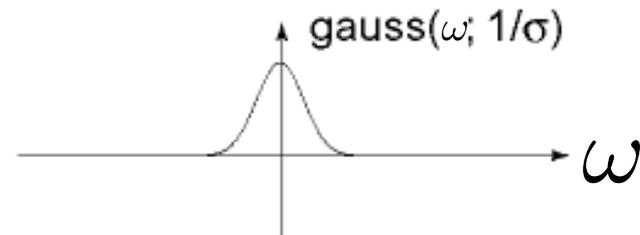
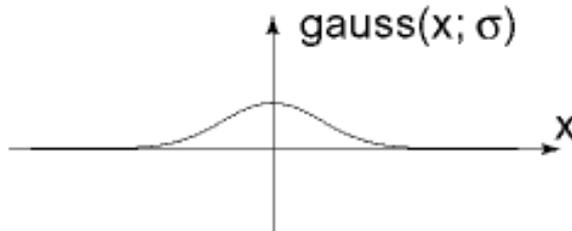
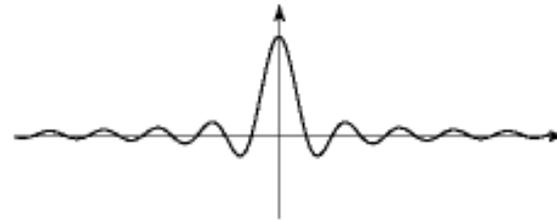
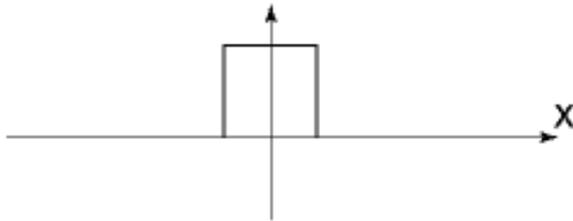


# Gaussian Filter

Removes noise while minimizing spatial smoothing

Spatial domain

Frequency domain



Credit: Efros, Computational Photography

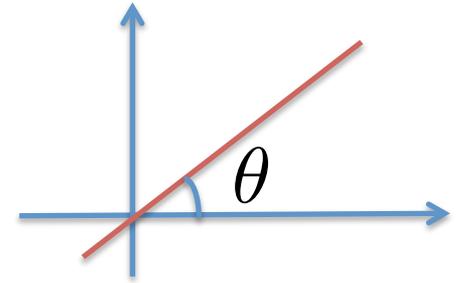
# 2D Gaussian Filter

$$h(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2 + y^2}{2\sigma^2}}$$

Directional derivatives along  $[\cos \theta \ \sin \theta]$ :

First derivative:

$$\begin{aligned} D_\theta f(x, y) &= \nabla f \cdot [\cos \theta \ \sin \theta] \\ &= \frac{\partial f(x, y)}{\partial x} \cos \theta + \frac{\partial f(x, y)}{\partial y} \sin \theta \end{aligned}$$



Second derivative:

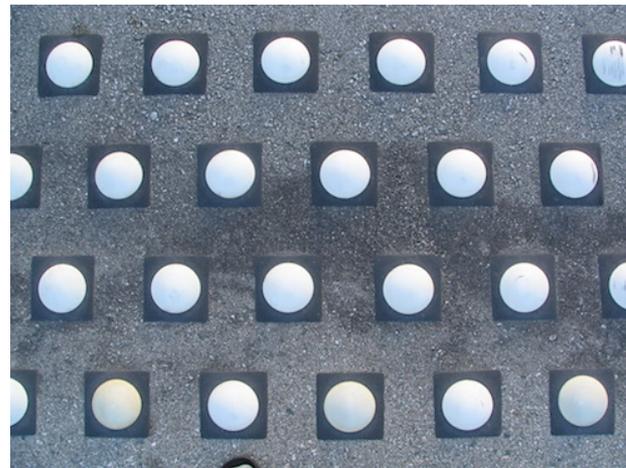
$$D_\theta^2 f(x, y) = \frac{\partial^2 f(x, y)}{\partial x^2} \cos^2 \theta + \frac{\partial^2 f(x, y)}{\partial x \partial y} \cos \theta \sin \theta + \frac{\partial^2 f(x, y)}{\partial y^2} \sin^2 \theta$$

Therefore, we can detect edges in the  $\theta$  direction from the zero crossings of:

$$D_{\theta}^2 \{h(x, y) * f(x, y)\} = D_{\theta}^2 h(x, y) * f(x, y)$$

Directional derivatives are suitable to detect lanes or stop markings:

Choose direction  $\theta$  to be orthogonal to them.



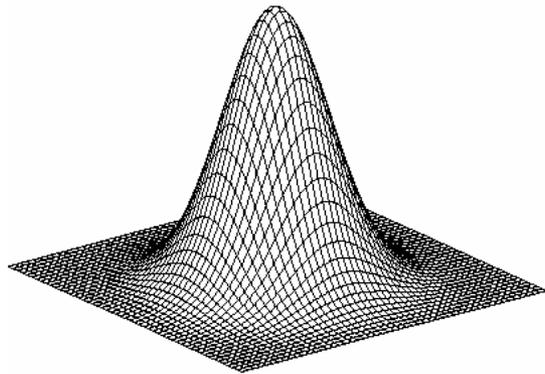
What about isotropic edges?

When there is no directionality (e.g., circular objects, such as Botts dots), use the Laplacian:

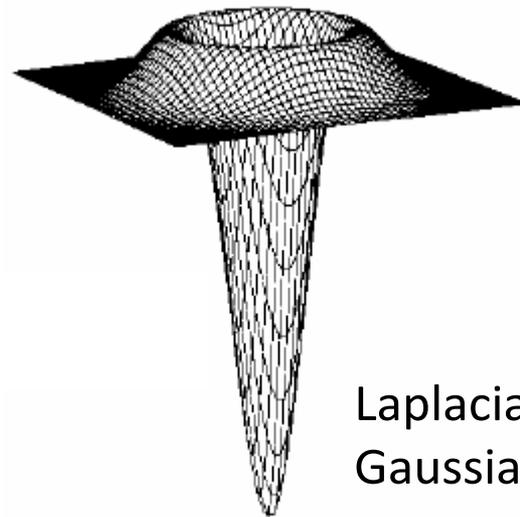
$$\nabla^2 f(x, y) = \frac{\partial^2 f(x, y)}{\partial x^2} + \frac{\partial^2 f(x, y)}{\partial y^2}$$

which is invariant under rotations of the image.  
Look for zero crossings of:

$$\nabla^2 \{h(x, y) * f(x, y)\} = \nabla^2 h(x, y) * f(x, y)$$



Gaussian



Laplacian of  
Gaussian

# Finite Difference Approximations of Derivatives

Estimate derivatives from samples of  $f(x)$ :

$$\left. \frac{df(x)}{dx} \right|_{x=n\Delta} \approx \frac{1}{\Delta} \{f(n\Delta + \Delta) - f(n\Delta)\}$$

(forward difference)

$$\frac{1}{\Delta} \{f(n\Delta) - f(n\Delta - \Delta)\}$$

(backward difference)

$$\frac{1}{2\Delta} \{f(n\Delta + \Delta) - f(n\Delta - \Delta)\}$$

(central difference)

Taylor series to assess the approximation accuracy:

$$f(n\Delta + \Delta) = f(n\Delta) + \Delta f'(n\Delta) + \frac{\Delta^2}{2} f''(n\Delta) + \mathcal{O}(\Delta^3)$$

$$f(n\Delta - \Delta) = f(n\Delta) - \Delta f'(n\Delta) + \frac{\Delta^2}{2} f''(n\Delta) + \mathcal{O}(\Delta^3)$$

Forward difference:

$$\frac{1}{\Delta} \{f(n\Delta + \Delta) - f(n\Delta)\} = f'(n\Delta) + \mathcal{O}(\Delta)$$

Backward difference:

$$\frac{1}{\Delta} \{f(n\Delta) - f(n\Delta - \Delta)\} = f'(n\Delta) + \mathcal{O}(\Delta)$$

Central difference:

$$\frac{1}{2\Delta} \{f(n\Delta + \Delta) - f(n\Delta - \Delta)\} = f'(n\Delta) + \mathcal{O}(\Delta^2)$$

smaller error

# Second Derivative Approximation

$$\frac{1}{\Delta^2} \{f(n\Delta + \Delta) - 2f(n\Delta) + f(n\Delta - \Delta)\} = f''(n\Delta) + \mathcal{O}(\Delta^2)$$

## Finite Difference Approximations in 2D

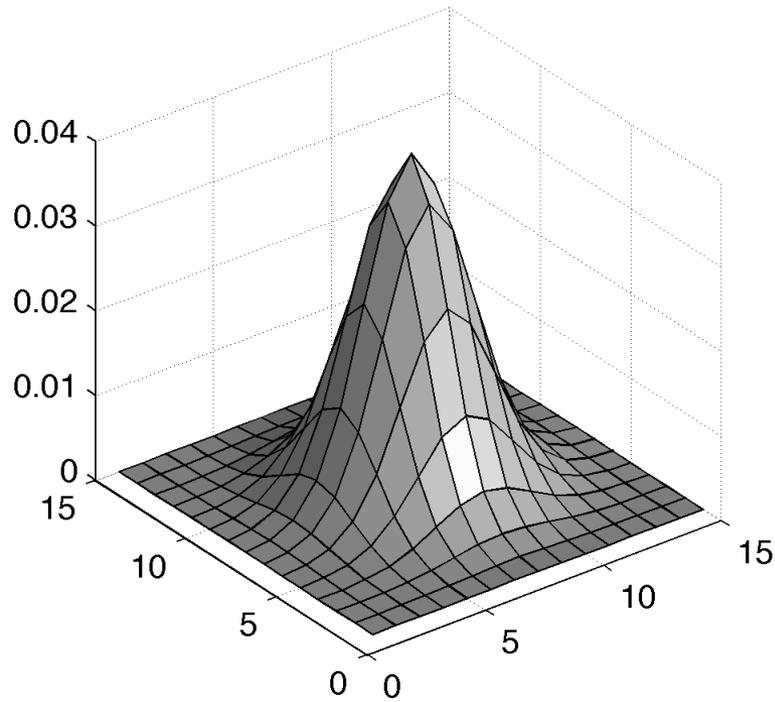
$$\frac{\partial}{\partial x} f(x, y) \Big|_{(n_1\Delta, n_2\Delta)} \approx \frac{1}{\Delta} \{f(n_1\Delta + \Delta, n_2\Delta) - f(n_1\Delta, n_2\Delta)\}$$

$$\frac{\partial}{\partial y} f(x, y) \Big|_{(n_1\Delta, n_2\Delta)} \approx \frac{1}{\Delta} \{f(n_1\Delta, n_2\Delta + \Delta) - f(n_1\Delta, n_2\Delta)\}$$

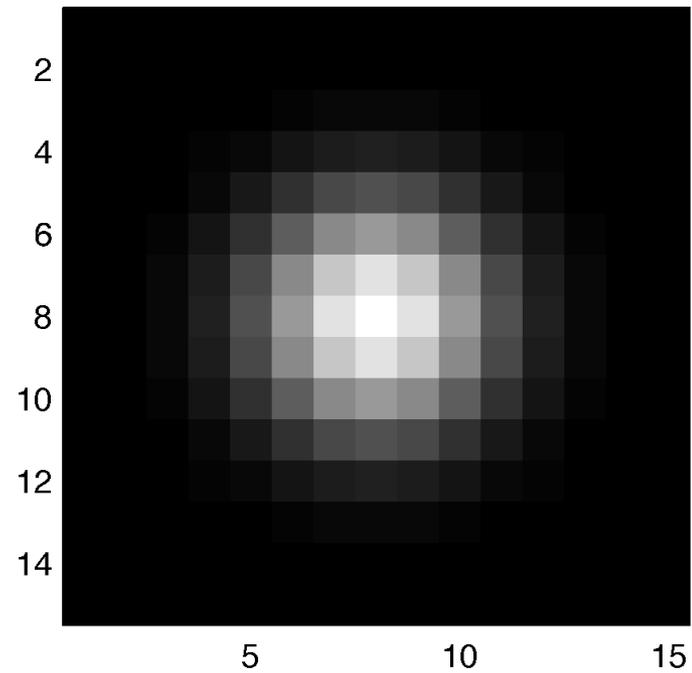
$$\begin{aligned} \nabla^2 f(x, y) \approx & \frac{1}{\Delta^2} \{f(n_1\Delta + \Delta, n_2\Delta) + f(n_1\Delta - \Delta, n_2\Delta) \\ & f(n_1\Delta, n_2\Delta + \Delta) + f(n_1\Delta, n_2\Delta - \Delta) \\ & - 4f(n_1\Delta, n_2\Delta)\} \end{aligned}$$

# Gaussian Filter and its Derivatives in MATLAB

Gaussian Filter



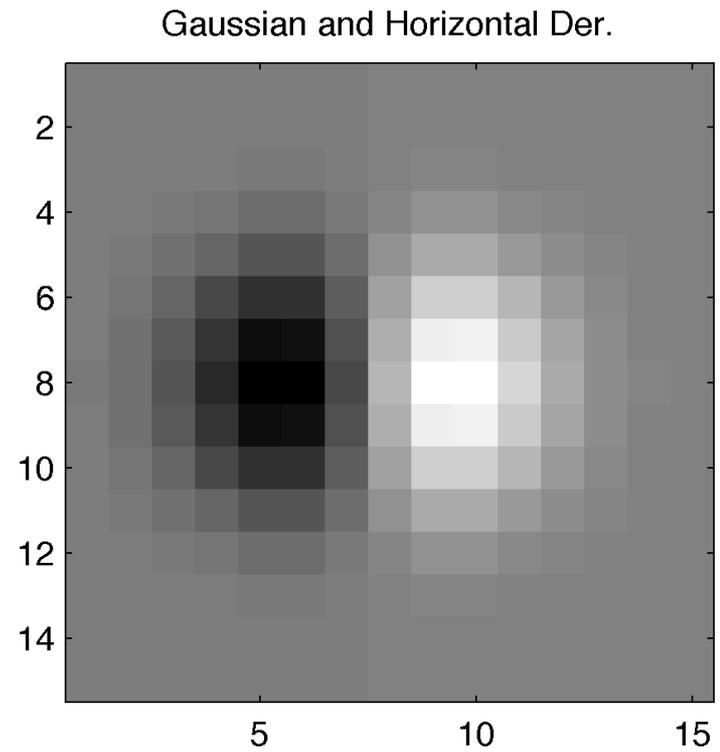
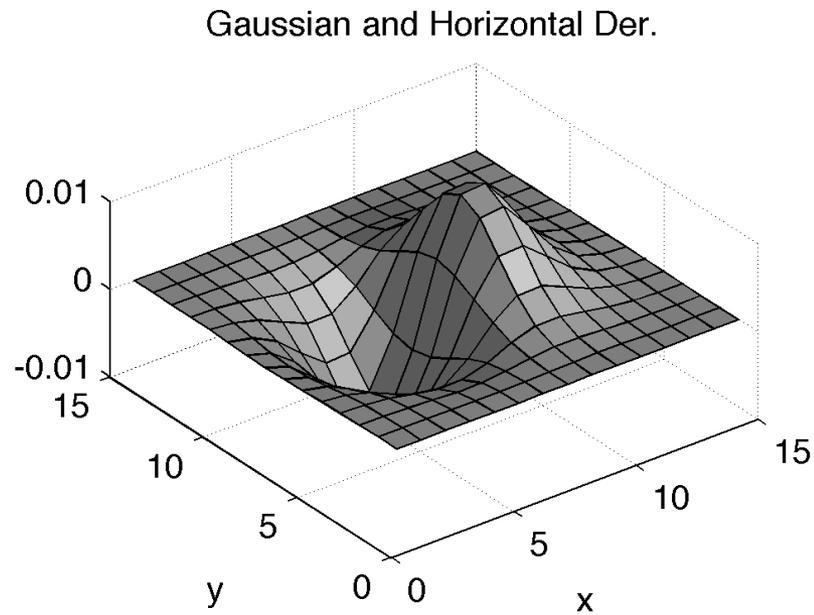
Gaussian Filter



### Gaussian Filter, 15x15 pixels, $\sigma=2$

```
>> g=fspecial('gaussian',15,2);
```

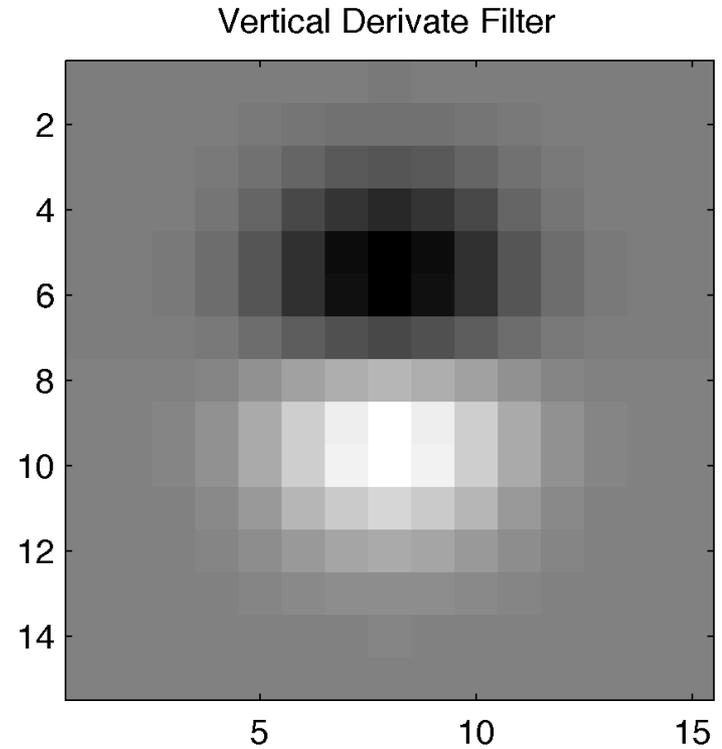
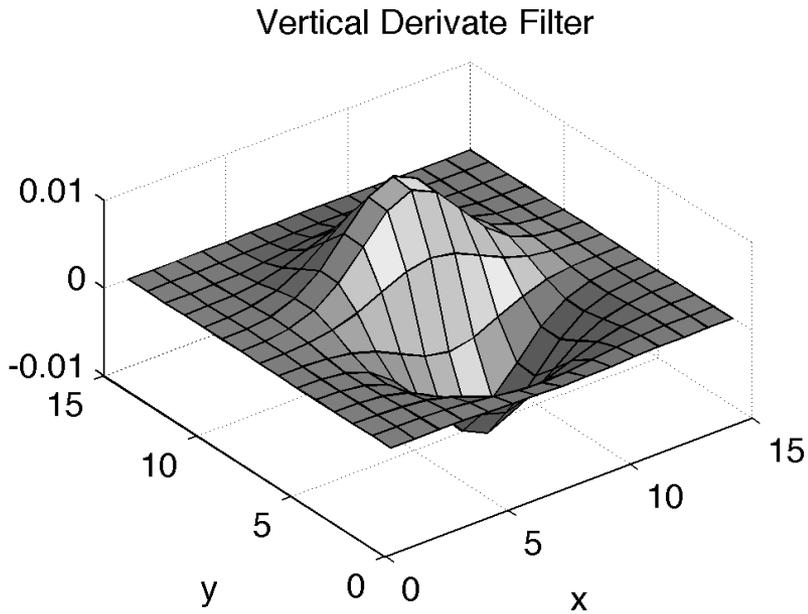
$$h(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$



## Combine Gaussian Filter with Horizontal Derivative

```
>> dx=conv2(g,[-1,1],'same');
```

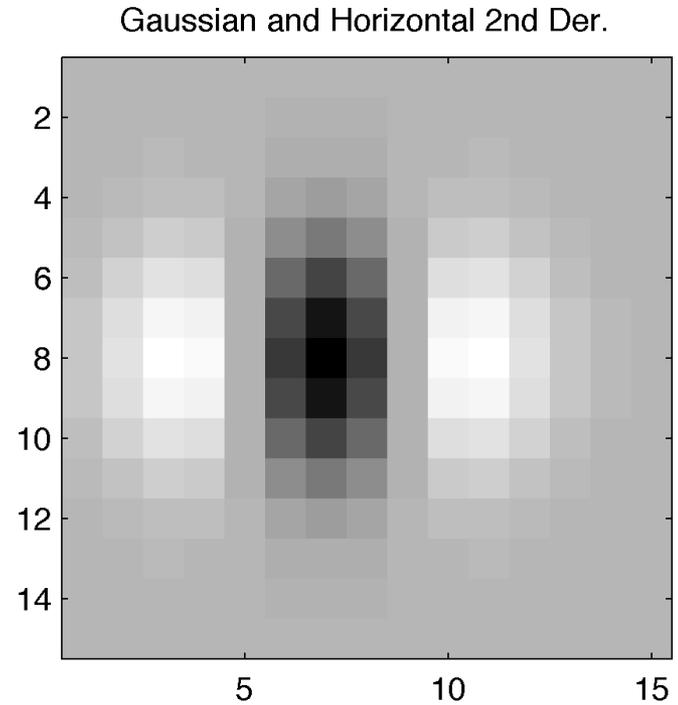
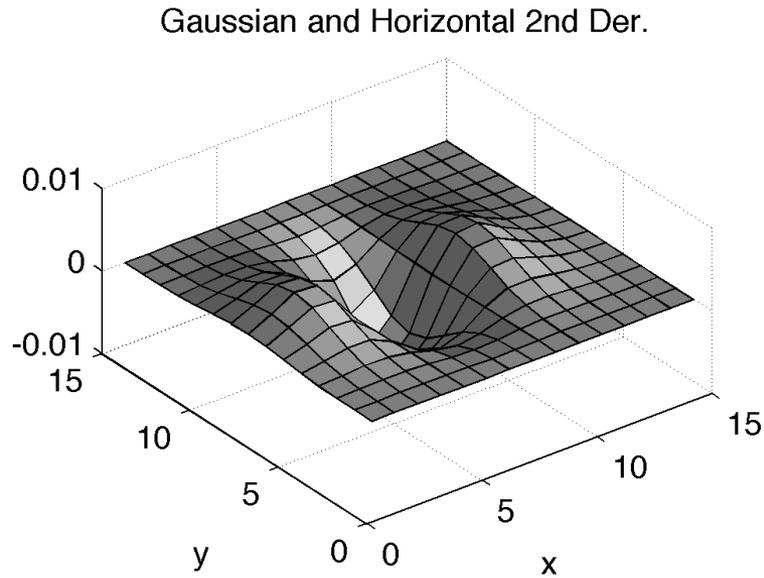
$$\frac{\partial}{\partial x} h(x, y)$$



## Vertical Derivative Filter (with Gaussian)

```
>> dy=conv2(g,[-1;1],'same');
```

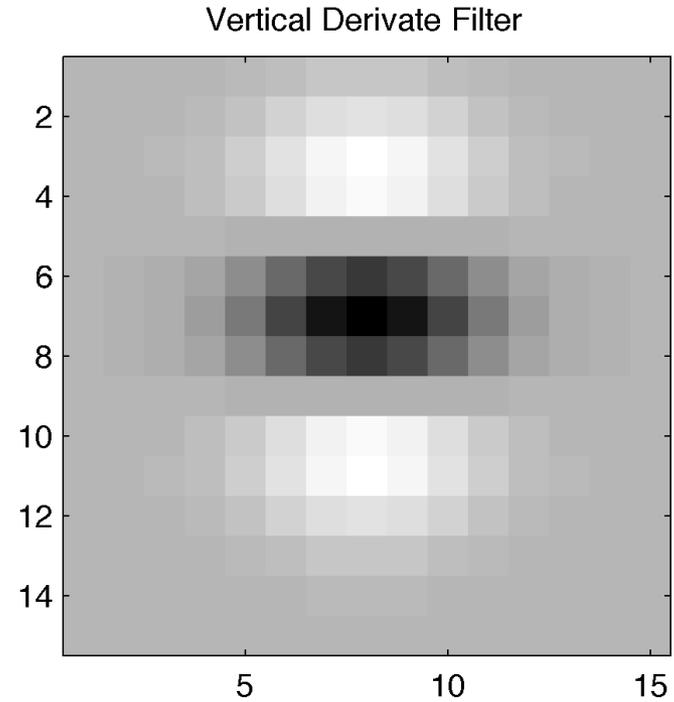
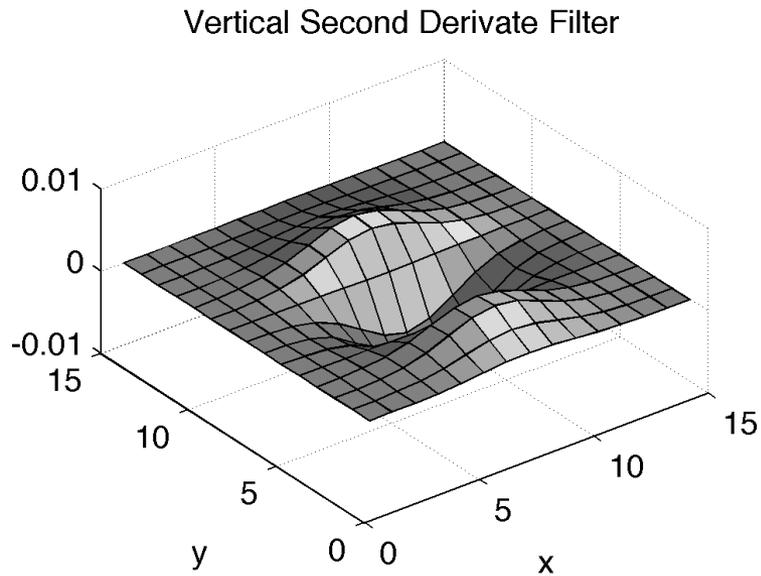
$$\frac{\partial}{\partial y} h(x, y)$$



## Second Derivative of Gaussian in Horizontal Dir.

```
>> ddx=conv2(dx,[-1,1],'same')
```

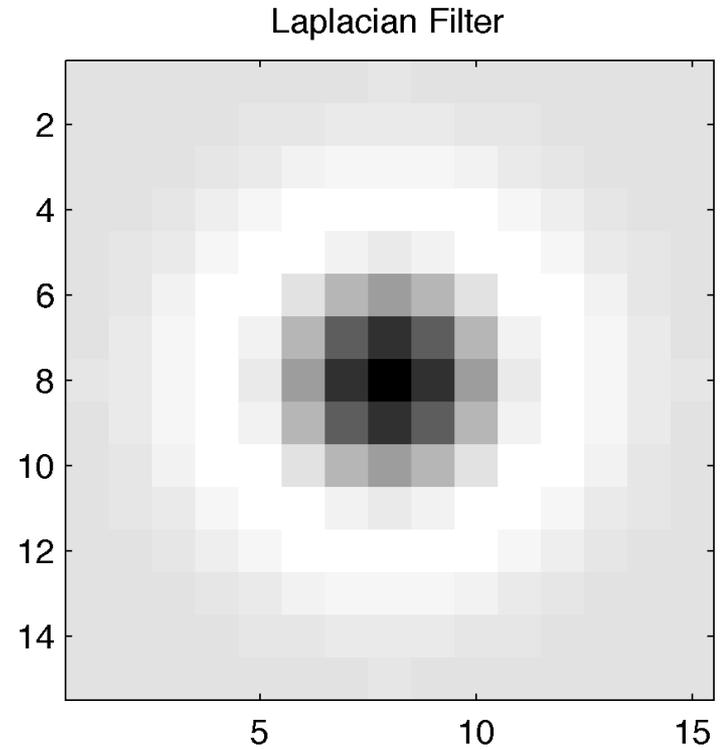
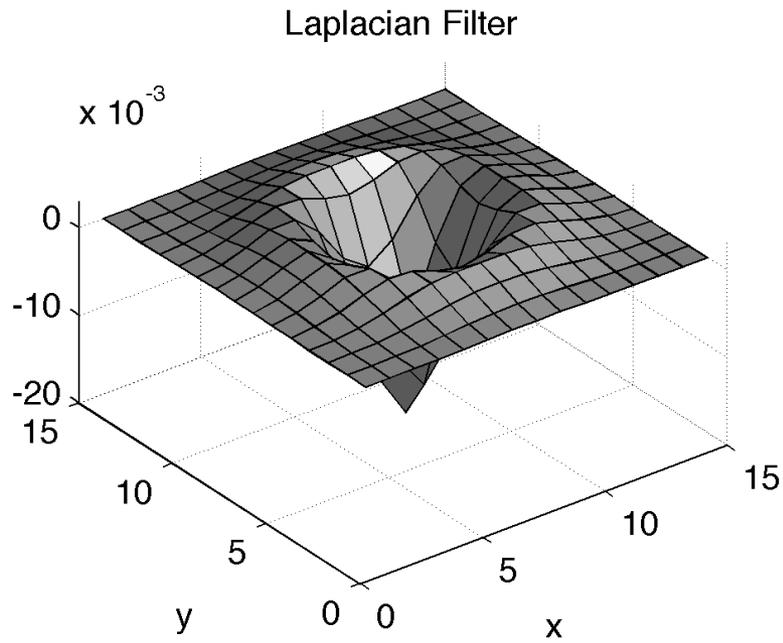
$$\frac{\partial^2}{\partial x^2} h(x, y)$$



## Vertical Second Derivative Filter (with Gaussian)

```
>> dy=conv2(g,[-1;1],'same');
```

$$\frac{\partial^2}{\partial y^2} h(x, y)$$

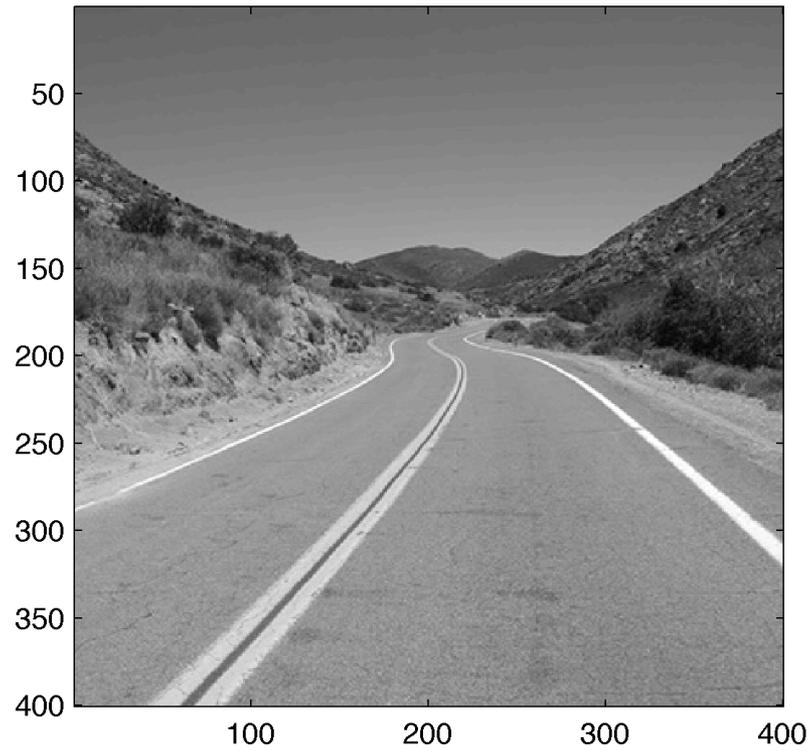


## Laplacian Filtering

```
>> lg=fspecial('log',15,2);
```

$$\nabla^2 h(x, y)$$

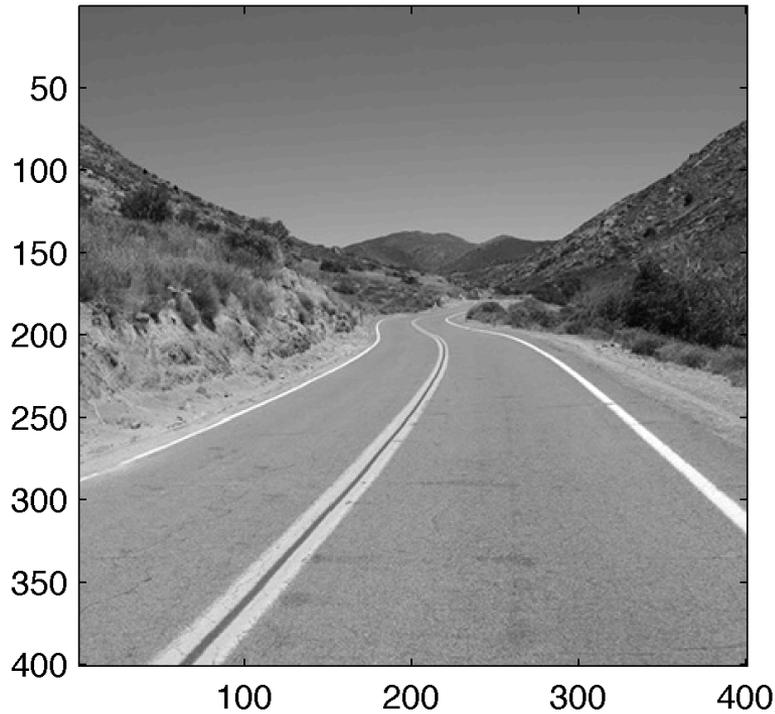
# Edge Detection Examples with MATLAB



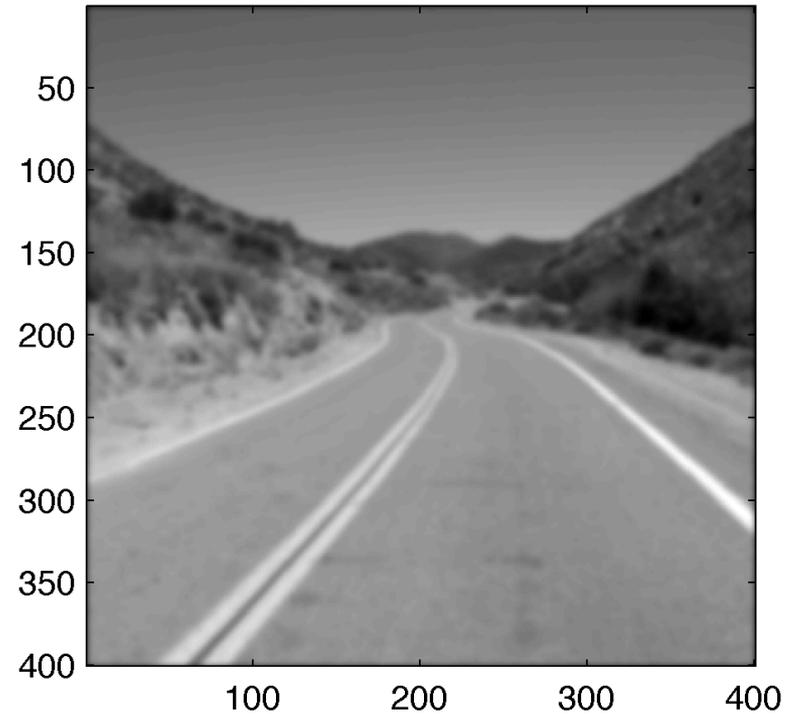
## Load An Image

```
>> road=imread('road.jpg');  
>> imagesc(road); %Scale values and display image
```

Original Image



Gaussian Filtered Image

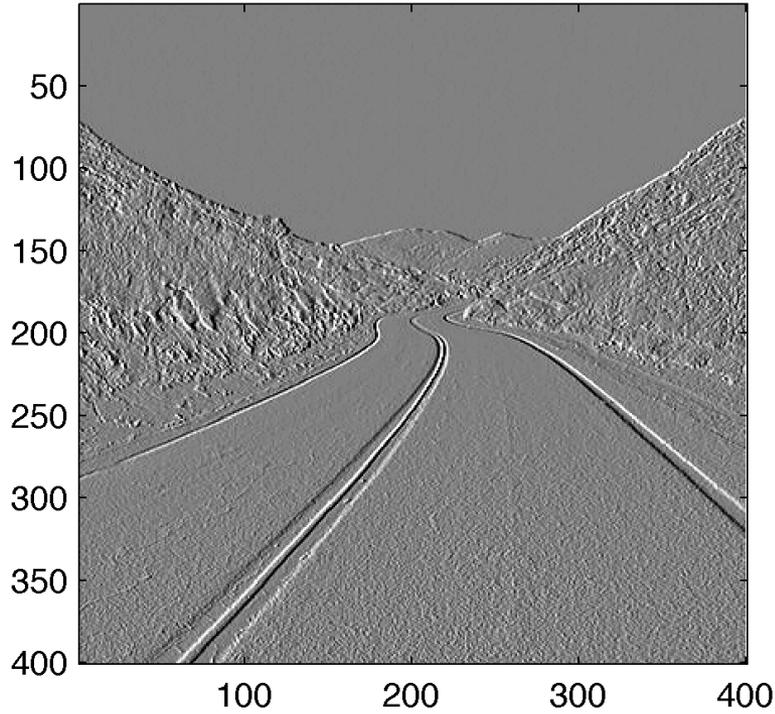


## Gaussian Filtering

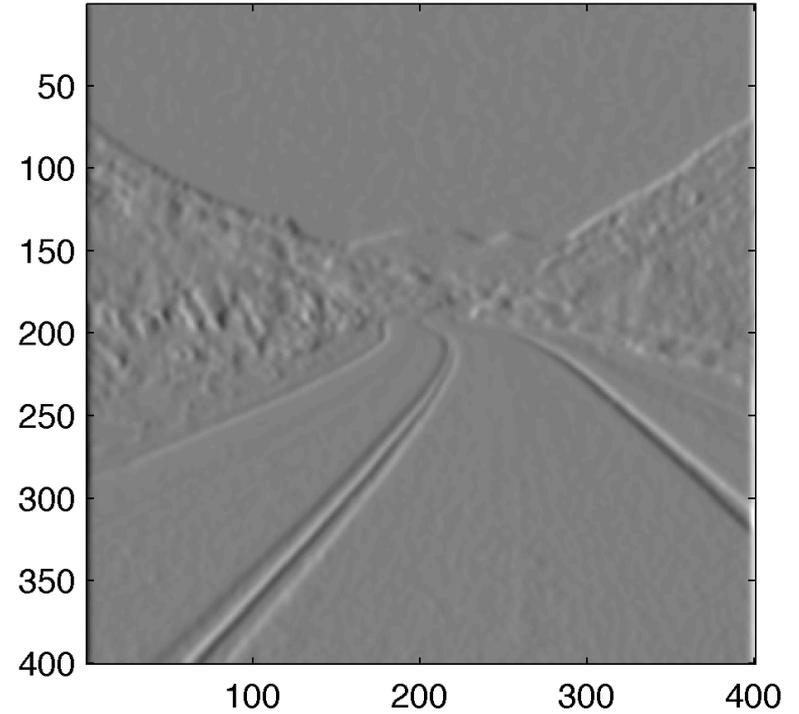
```
>> groad=conv2(road,g,'same');
```

$$f(x, y) \text{ vs. } h(x, y) * f(x, y)$$

Horizontal Derivative (noisy)



Horizontal Derivative after Filtering (Less Noise)



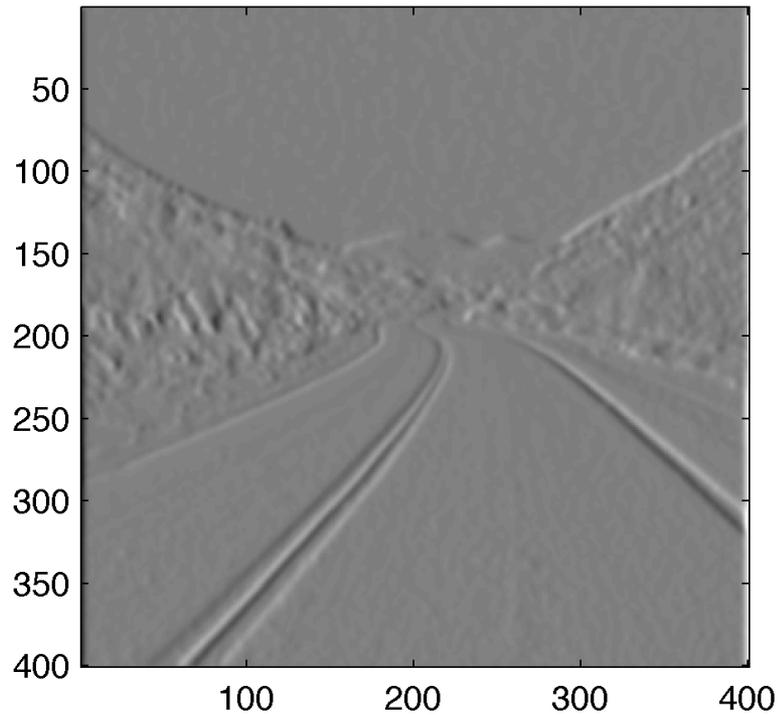
## Edge Detection Along Horizontal Direction

```
>> conv2(road,[-1,1],'same')
```

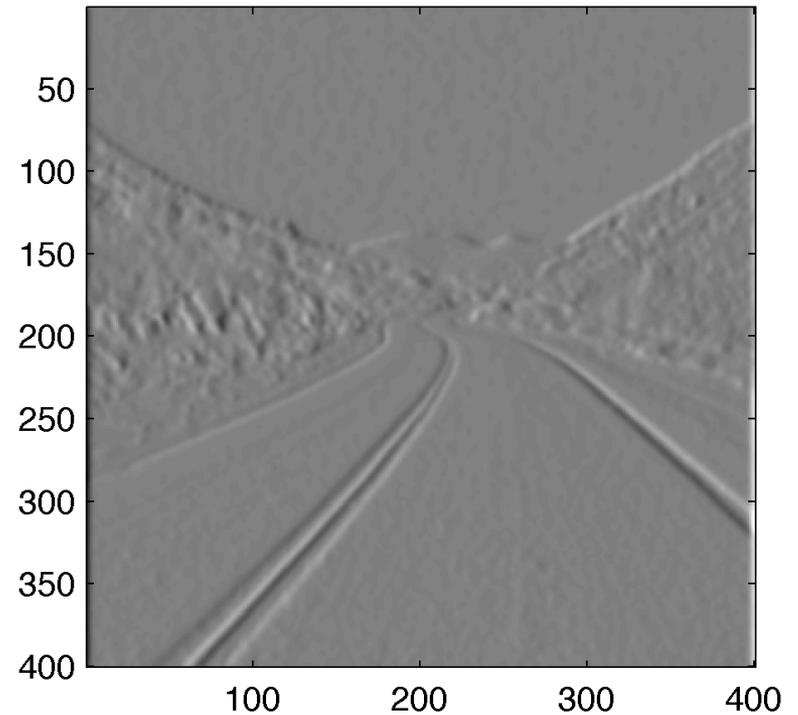
```
>> conv2(groad,[-1,1],'same')
```

$$\frac{\partial}{\partial x} f(x, y) \text{ vs. } \frac{\partial}{\partial x} \{h(x, y) * f(x, y)\}$$

Gaussian Filter, Then Derivative



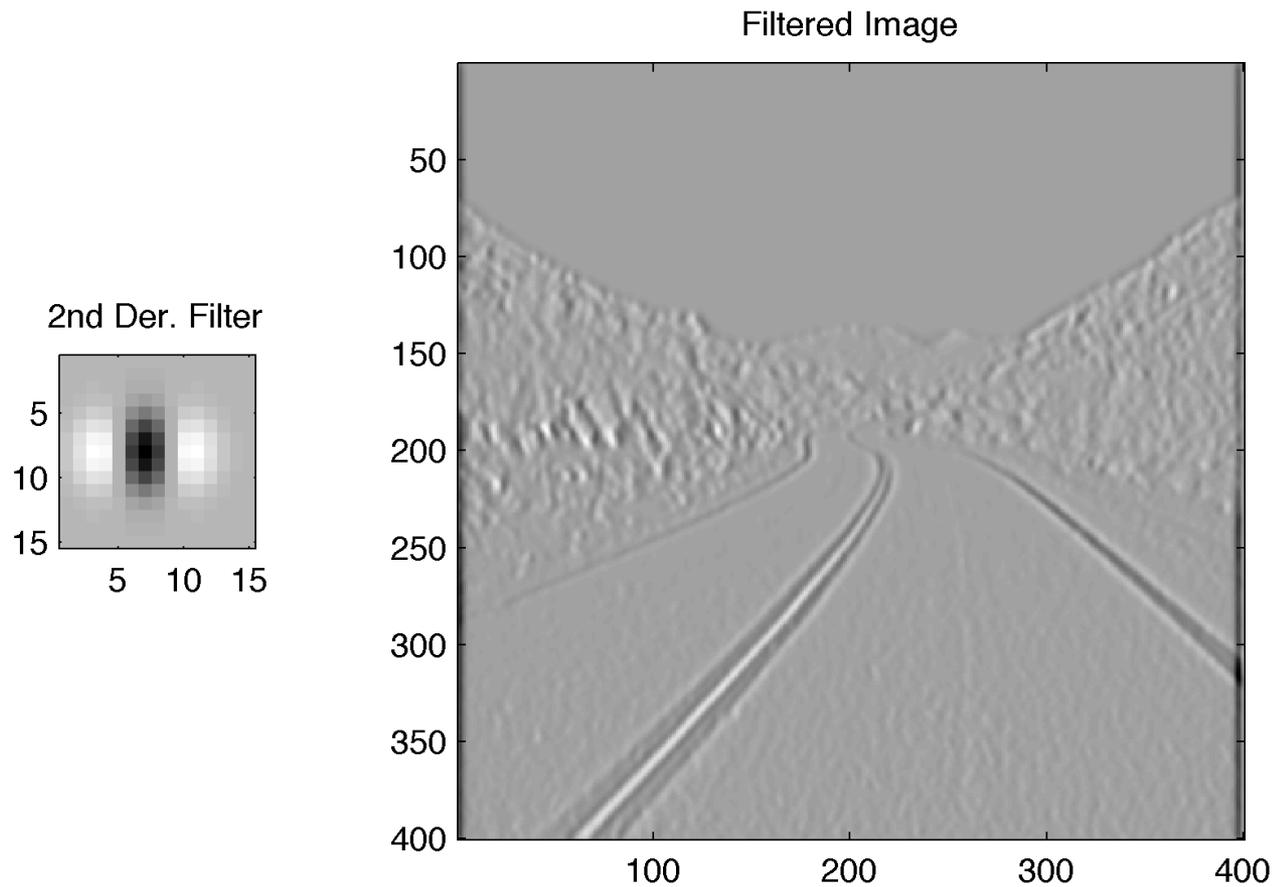
Combined Filter



## Same Result

```
>> conv2(groad,[-1,1],'same')  
>> conv2(road,dx,'same')
```

$$\frac{\partial}{\partial x} \{h * f\} = \frac{\partial h}{\partial x} * f$$



## Second Derivative in Horizontal Direction

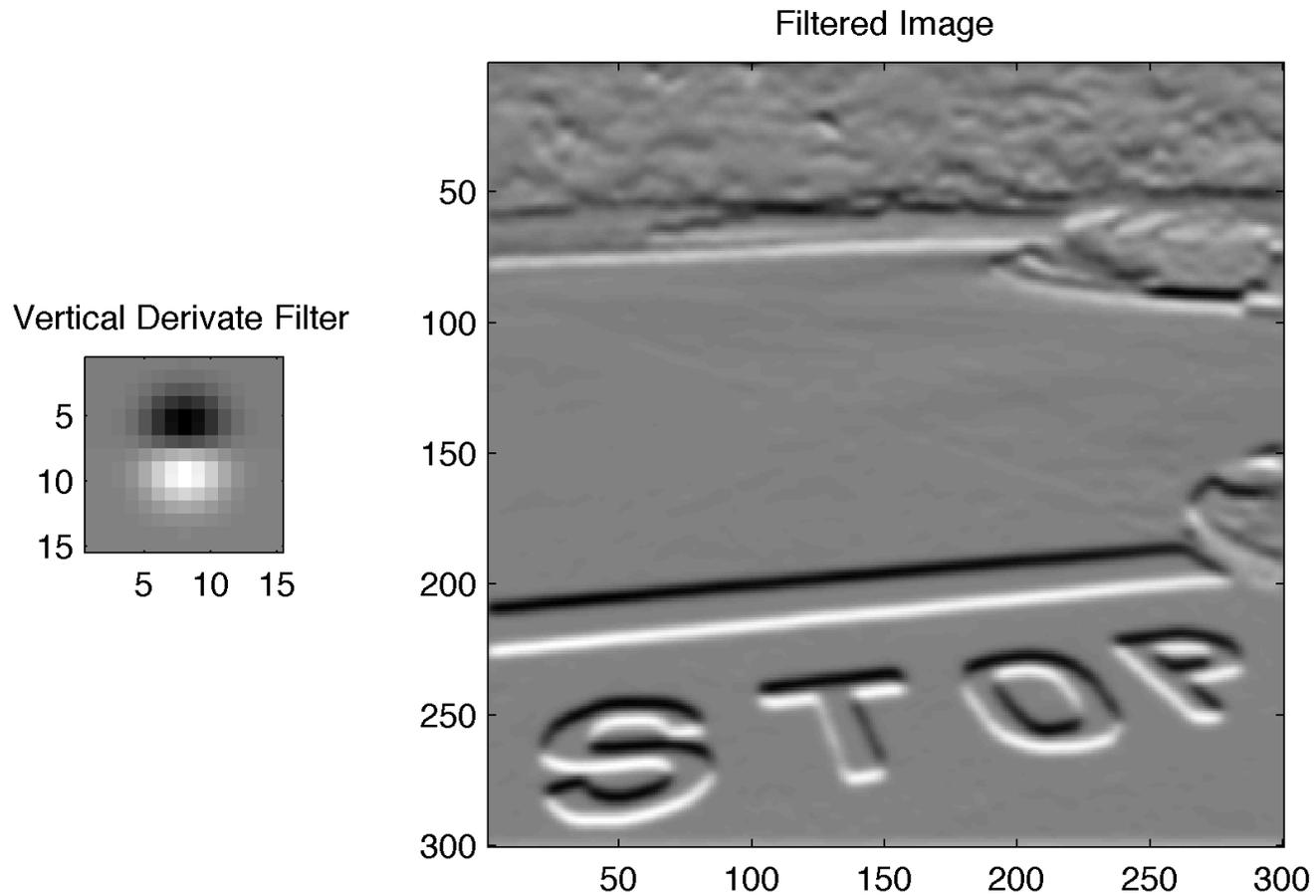
```
>> conv2(road,ddx,'same')
```

$$\frac{\partial^2}{\partial x^2} h(x, y) * f(x, y)$$



## Stop Line Detection

```
>> stop_im = imread('stop.jpg')
```

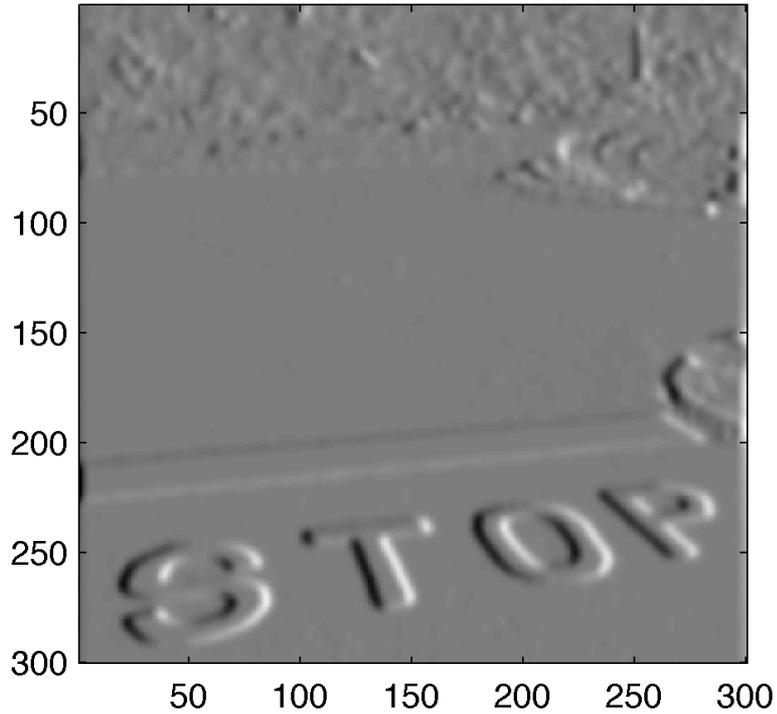


## Detecting Stop Bar using Vertical Derivative

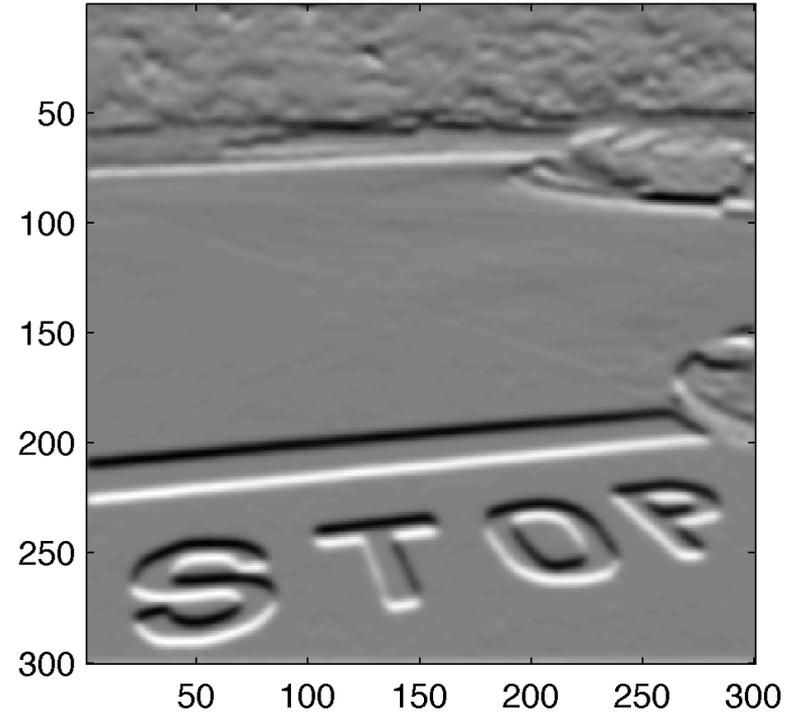
```
>> conv2(stop_im,dy,'same')
```

$$\frac{\partial h}{\partial y} * f$$

dx Filtered Image



dy Filtered Image



## Comparing Horizontal and Vertical Derivative

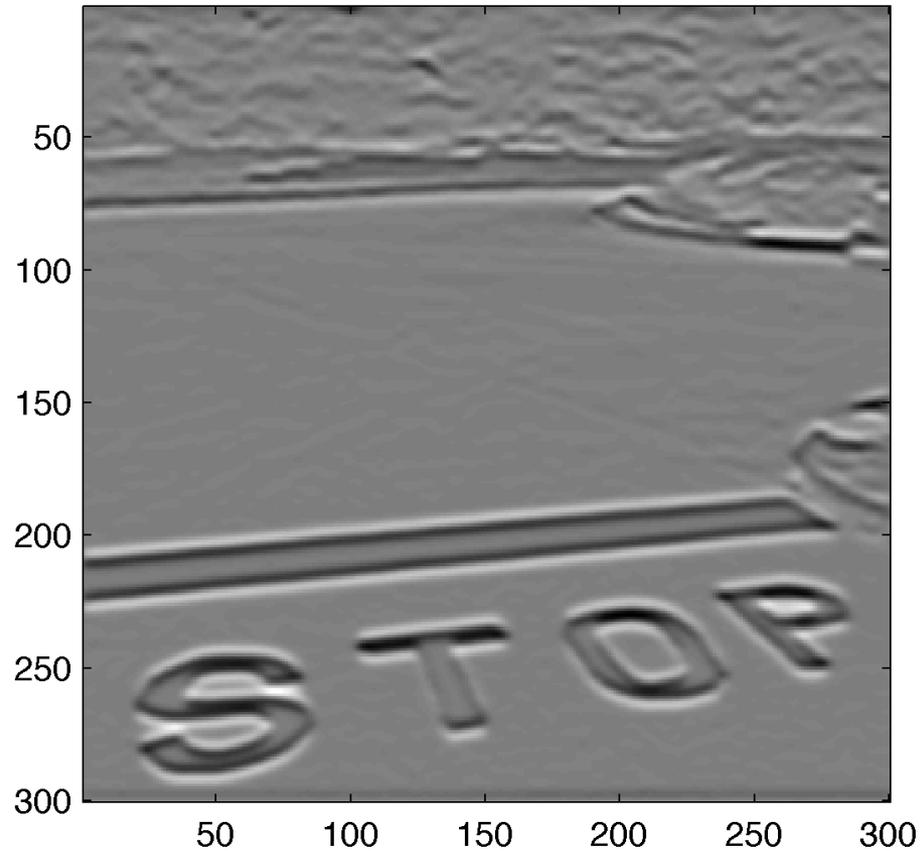
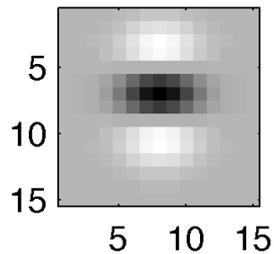
```
>> conv2(stop_im,dx,'same')
```

```
>> conv2(stop_im,dy,'same')
```

$$\frac{\partial h}{\partial x} * f \text{ vs. } \frac{\partial h}{\partial y} * f$$

Filtered Image

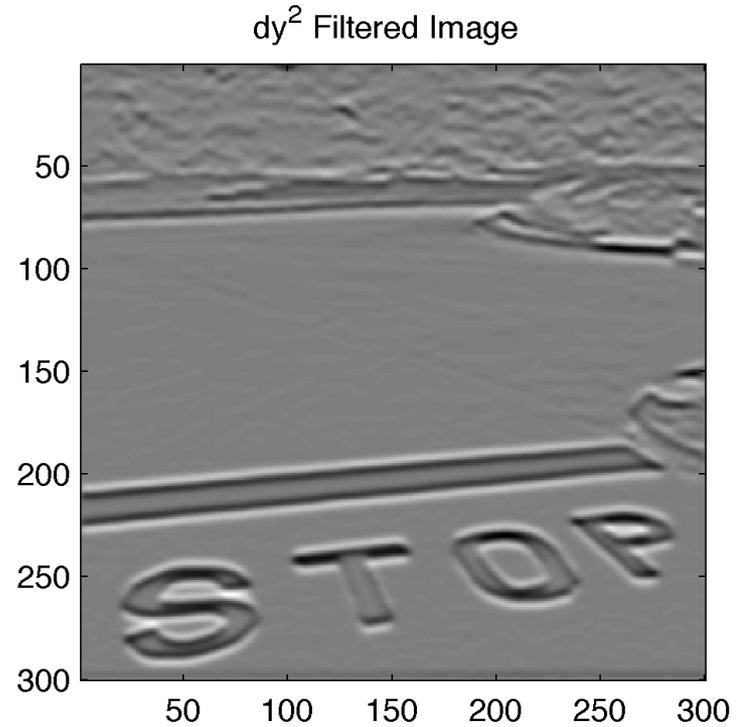
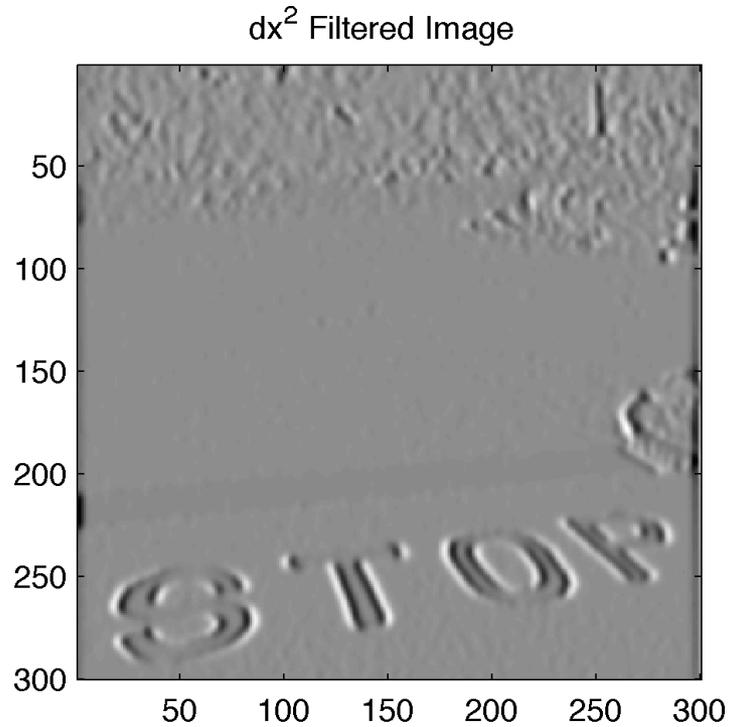
Vertical Derivate Filter



### Detecting Stop Bar using Vertical Second Derivative

```
>> conv2(stop_im,dy,'same')
```

$$\frac{\partial^2}{\partial y^2} h(x, y) * f(x, y)$$

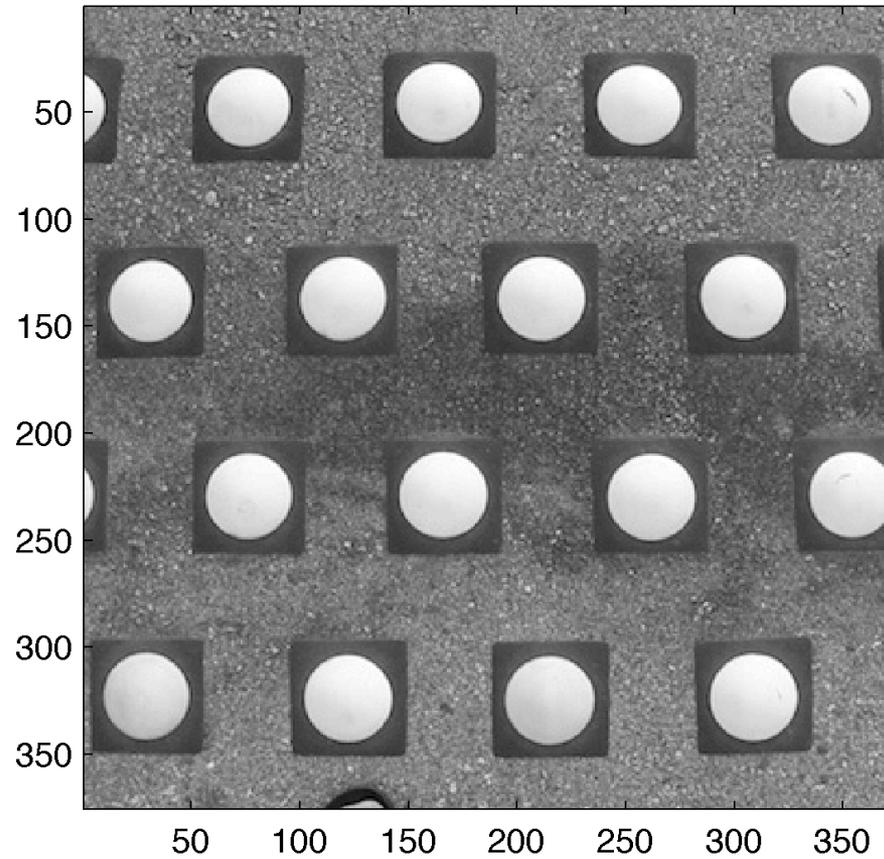


### Comparing Horizontal and Vertical Second Derivatives

```
>> conv2(stop_im, ddx, 'same')
```

```
>> conv2(stop_im, ddy, 'same')
```

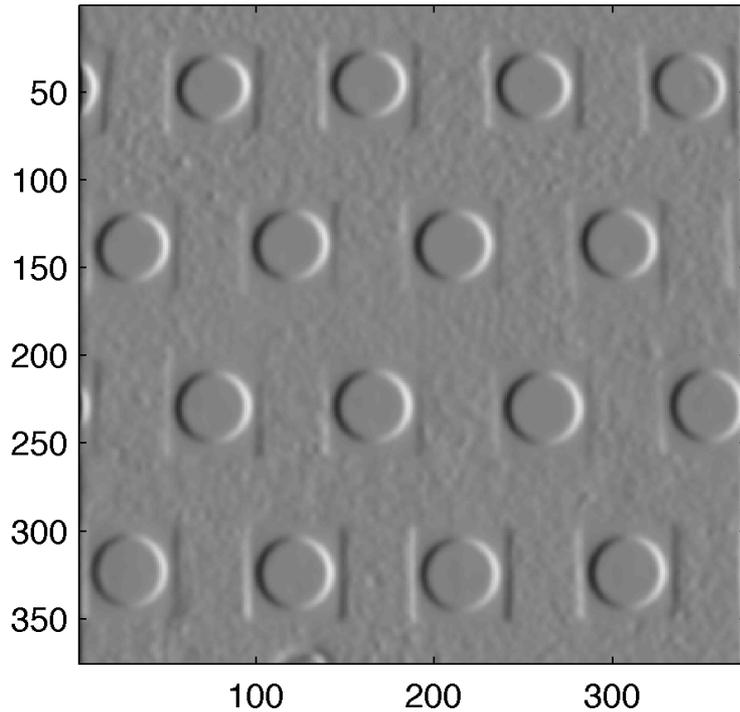
$$\frac{\partial^2}{\partial x^2} h(x, y) * f(x, y) \text{ vs. } \frac{\partial^2}{\partial y^2} h(x, y) * f(x, y)$$



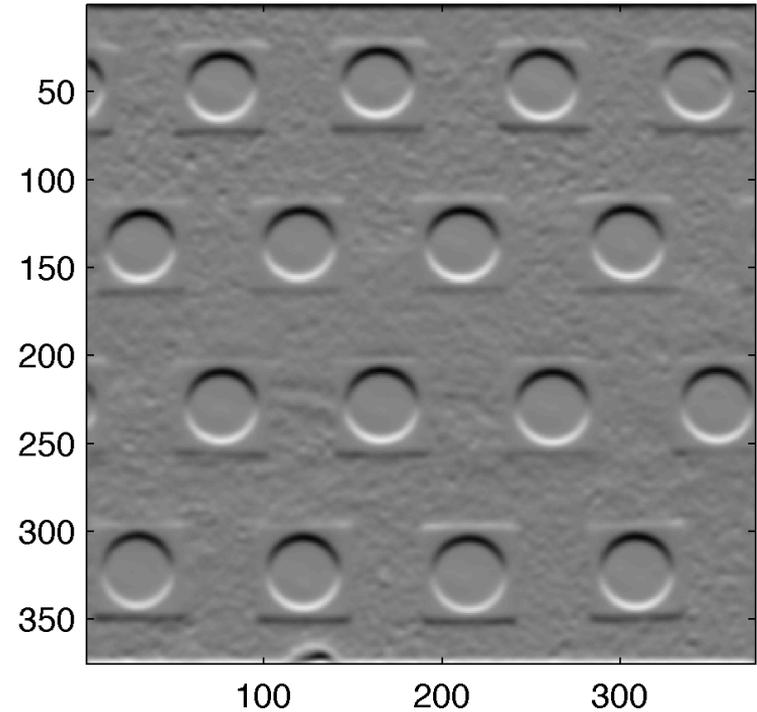
## Detecting Botts Dots

```
>> botts=imread('botts.jpg');
```

dx Filtered Image



dy Filtered Image



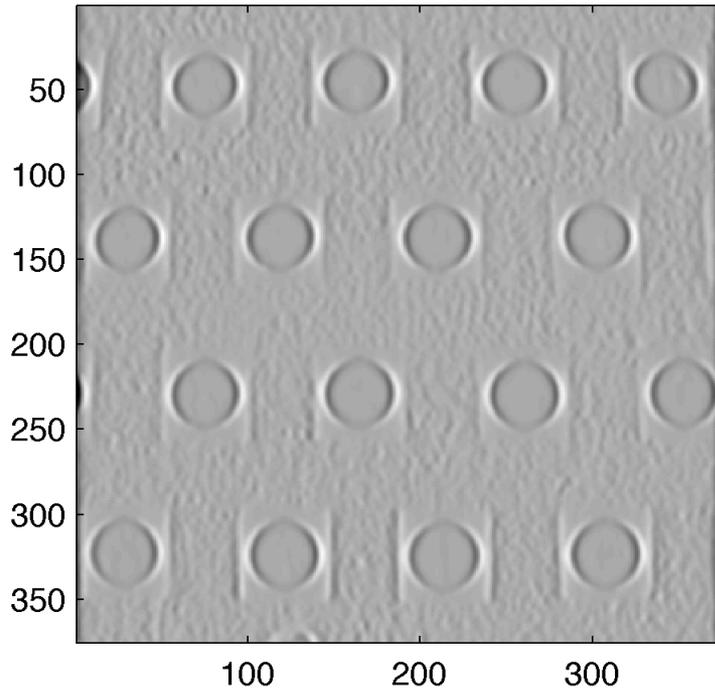
## Horizontal and Vertical Derivative Filtering

```
>> conv2(botts,dx,'same')
```

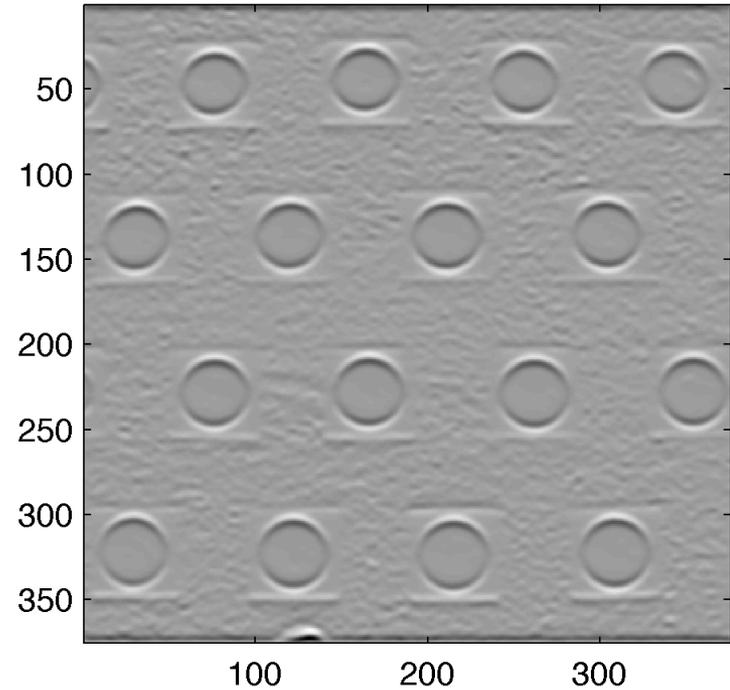
```
>> conv2(botts,dy,'same')
```

$$\frac{\partial}{\partial x} h(x, y) * f(x, y) \text{ vs. } \frac{\partial}{\partial y} h(x, y) * f(x, y)$$

$dx^2$  Filtered Image



$dy^2$  Filtered Image



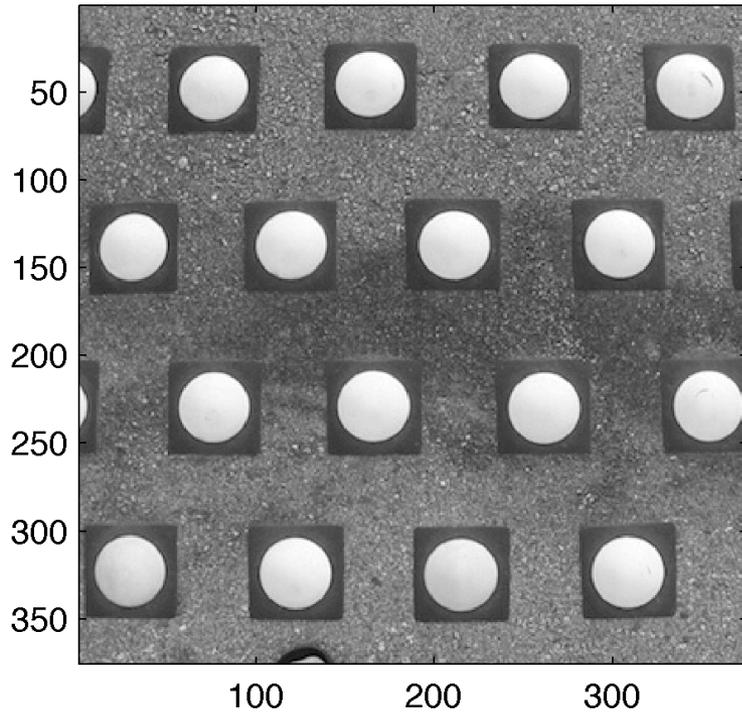
## Horizontal and Vertical Second Derivative Filtering

```
>> conv2(botts, ddx, 'same')
```

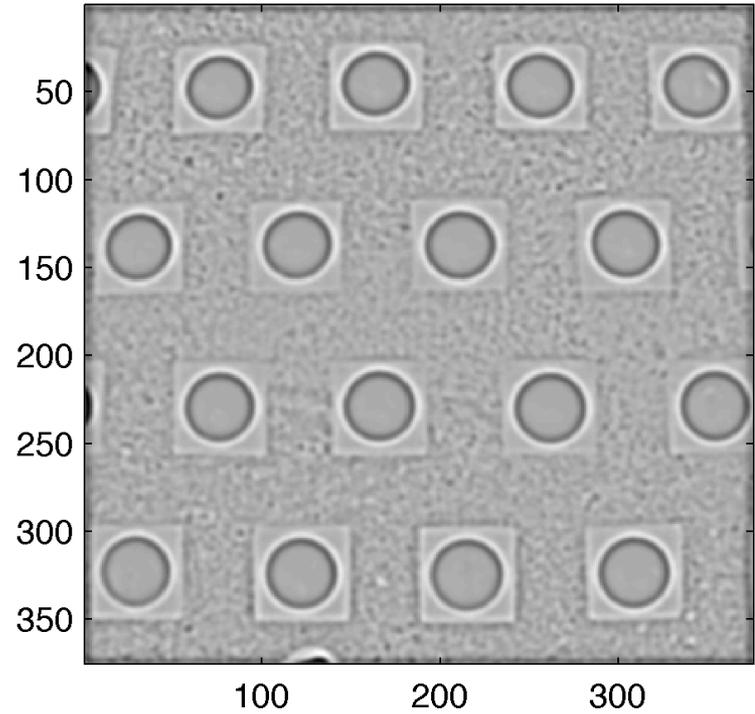
```
>> conv2(botts, ddy, 'same')
```

$$\frac{\partial^2}{\partial x^2} h(x, y) * f(x, y) \text{ vs. } \frac{\partial^2}{\partial y^2} h(x, y) * f(x, y)$$

Original Image



Laplacian Filtered Image



## Original versus Laplacian

```
>> conv2(botts,lg,'same')
```

$$\nabla^2 h * f$$