EECS 120 Signals & Systems — University of California, Berkeley: Spring 2004
Gastpar — February 5, 2004

Homework 3
Due: Tuesday, February 17, 2004, at 4pm

Reading OWN Chapter 3.

Practice Problems *(Suggestions.)* OWN 3.3, 3.4, 3.16.

Problem 1 *(Your new job at BT&T, continued.)* — 20 Points

In class, we saw that high frequencies can be suppressed to some extent by the system

$$y[n] \quad = \quad x[n] + \alpha y[n-1], \tag{1}$$

and by picking $0 < \alpha < 1$. As seen in class, the frequency response of this system is given by

$$H\left(e^{j\omega}\right) \quad = \quad \frac{1}{1 - \alpha e^{-j\omega}}. \tag{2}$$

(a) In Homework 2, a colleague of yours at BT&T designed a BART-squeaking suppression filter:

$$y[n] \quad = \quad \frac{1}{2}x[n] + \frac{1}{4}x[n-1] + \frac{1}{4}x[n-2]. \tag{3}$$

Determine the frequency response $G(e^{j\omega})$ of this filter.

(b) Print, in *one* matlab plot, both $|G(e^{j\omega})|$ and $|H(e^{j\omega})|/2$, setting $\alpha = 0.5$, for $-\pi < \omega < \pi$. The reason why we divide $|H(e^{j\omega})|$ by two is because we want a frequency response magnitude of 1 at frequency zero. Such a division does not influence the system behavior otherwise. Which system is "better"? Give a detailed answer (2-3 sentences).

Remark: If you'd like to get a better feeling for what the system with frequency response $|H(e^{j\omega})|$ does for different α, plot also (in a different plot) $\alpha|H(e^{j\omega})|$ for $\alpha = 0.1$ and for $\alpha = 0.99$. What do you observe? (Again, to get a nice plot, we multiply $|H(e^{j\omega})|$ by α.)

Problem 2 *(Fourier series representation.)* — 20 Points

(a) OWN 3.22 (a), *only* for the signal in Figure (c). Then, also give the Fourier series of $y(t) = x(2t)$, where $x(t)$ is the signal of Figure (c). *Hint:* Think twice before evaluating any integrals.

(b) OWN 3.23 (d).

(c) OWN 3.28 (b).

(d) OWN 3.39.

Problem 3 *(Analysis of the fly visual system.)* — 25 Points

One of the goals of contemporary neuroscience is to understand the functioning of the visual systems of organisms. The blowfly is often used as an example. Researchers encounter the following problem: They can show a *stimulus* $x(t)$ in front of the fly's eye, and they can measure an electrical signal $y(t)$ in the fly's brain, but the actual system itself remains a black box. Researchers often consider the instantaneous *spiking rate* of the measured neuron to be the output signal $y(t)$. Clearly, in that case, the output $y(t)$ should not be negative.

(a) (10 Points) An often observed property in sensory processing in organisms is that, when a (constant) stimulus is switched on, it gives a nice spiking response initially that dies off rapidly. The following is a simplified measurement trace:

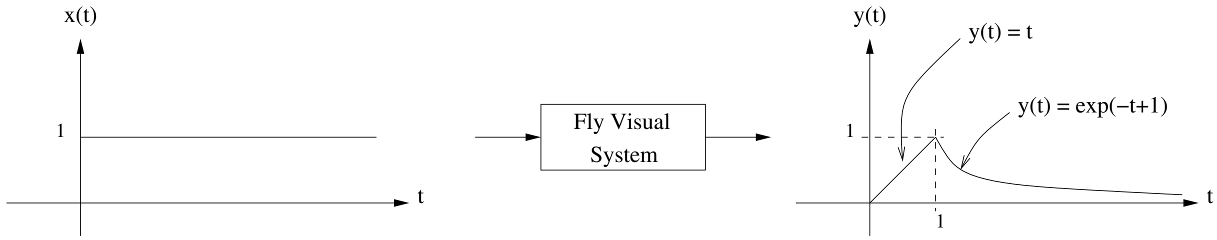

Figure 1: An experimentally observed stimulus-response behavior.

Assuming that the fly visual system is an LTI system, sketch its impulse response $h(t)$. *Remark:* A sketch is sufficient, but you may also determine the impulse response explicitly.

(b) (5 Points) Is your answer from Part *(a)* a good model for the fly visual system? Explain. *Hint:* Think about how the system respond to inputs other than the unit step.

(c) (10 Points) A popular alternative model is the so-called *energy model*, where the observed spiking rate $y(t)$ depends on the stimulus $x(t)$ as

$$y(t) \quad = \quad [(x * g)(t)]^2 . \tag{4}$$

For the measurement data in Figure 1, sketch the function $g(t)$. Then, sketch the predicted spiking rate $y(t)$ when the stimulus $x(t) = u(t) - u(t-2)$, using the model described by Equation (4). Does your prediction make sense? Briefly comment on why the energy model makes more sense than our earlier model. *Hint:* Split $g(t)$ into two parts as $g(t) = g_1(t) + g_2(t)$, and evaluate the convolution separately for each part.

Problem 4 *(Review: Differential Equations.)* 15 Points

In the course Math 54, you learned how to solve differential equations. This is an important tool in the analysis of systems, too. In this homework problem, we review a simple example. To refresh your memory, you can study Example 2.14, p. 118, in OWN.

Consider the following differential equation:

$$5\frac{d}{dt}y(t) + 10y(t) \quad = \quad \cos(t/5). \tag{5}$$

(a) Find the homogeneous solution to the differential equation.

Hint: The homogeneous solution is the solution to the differential equation $5\frac{d}{dt}y^{(h)}(t) + 10y^{(h)}(t) = 0$. For a first-order (i.e., only first derivative appears) linear differential equation with constant coefficients, it is found by applying $y^{(h)}(t) = ce^{rt}$, where the constant r is found by plugging $y^{(h)}(t)$ into the homogeneous part of the differential equation. Observe that any constant c satisfies the equation.

(b) Find the particular solution to the differential equation.

Hint: The particular solution is induced by the right hand side of the equation. Since this is $\cos(\omega t)$, the right approach is $y^{(p)}(t) = c_1 \cos(\omega t) + c_2 \sin(\omega t)$. To find the values of the constants c_1 and c_2, plug $y^{(p)}(t)$ directly into Equation (5).

(c) What is the complete solution to the differential equation given in Equation (5)? Suppose that the initial condition is $y(t = 0) = 0$. This specifies the constant c of the homogeneous solution.

Problem 5 *(Discrete-time periodic signals and LTI systems.)* 20 Points

In this problem, we study the response of linear time-invariant systems to discrete-time periodic signals.

(a) (5 Points) Show that the convolution sum of a discrete-time periodic signal $x[n]$ of fundamental period N with an arbitrary (not necessarily periodic) discrete-time signal $h[n]$ is a discrete-time periodic signal $y[n]$ of period N.

(b) (5 Points) The periodic signal $x[n]$ with period $N = 9$ consists of the infinite repetition of the vector $\mathbf{x} = (2, 2, 1, 1, 1, 0, -1, 0, 1)^T$. Suppose this signal is passed through the system H specified by the impulse response

$$h[n] \quad = \quad \delta[n] + 0.6\delta[n-1] + 0.3\delta[n-2] + 0.1\delta[n-3]. \tag{6}$$

Determine one period of the output signal. That is, determine the vector $\mathbf{y}$ whose infinite repetition corresponds to the signal $y[n]$. *Hint:* You may use matlab to do this if you prefer.

(c) (5 Points) Verify (using matlab) that the 9×9 matrix H_9,

$$H_9 \quad = \quad \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0.1 & 0.3 & 0.6 \\ 0.6 & 1 & 0 & 0 & 0 & 0 & 0 & 0.1 & 0.3 \\ 0.3 & 0.6 & 1 & 0 & 0 & 0 & 0 & 0 & 0.1 \\ 0.1 & 0.3 & 0.6 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0.3 & 0.6 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0.3 & 0.6 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.1 & 0.3 & 0.6 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1 & 0.3 & 0.6 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1 & 0.3 & 0.6 & 1 \end{pmatrix} \tag{7}$$

satisfies $\mathbf{y} = H_9\mathbf{x}$, with $\mathbf{x}$ and $\mathbf{y}$ as in Part *(b)*. Convince yourself that H_9 characterizes the system response to any periodic signal whose period is $N = 9$ (by selecting other input signals $x[n]$ of period $N = 9$). You do not need to turn in any documentation of this work. Then, find the matrix H_3 that characterizes the system response to any periodic signal whose period is $N = 3$. Explain (1-2 sentences) why the particular structure of H_9 and especially H_3 makes sense.

(d) (5 Points) Create a new file with filename `ImpRespMatrix.m`. This is called an m-file. Write the following contents into the m-file:

```
 function H = ImpRespMatrix(h, N);
% This function returns a matrix H that permits to easily find
% the output of the LTI system with impulse response h, when the
% input is a periodic signal with period N.
[ W, L ] = size(h);
if W>1, h = h'; L = W; end; % just to make sure h is a row vector...
% append zeros to h to obtain a length that is a multiple of N:
hpadded = [ h, zeros(1, ceil(L/N)*N-L) ];
hN = zeros(1, N); for k=0:ceil(L/N)-1, hN = hN + hpadded(k*N+1:(k+1)*N); end;
H = zeros(N, N); for k=1:N, H(k, :)  = [ hN(k:-1:1), hN(N:-1:k+1) ]; end;
```

This program constructs the matrix H as in Equation (7) for arbitrary impulse response $h[n]$ and arbitrary period N of the input signal. To call this m-file from the matlab command line, simply type

```
> N = 9;
> h = [ 1 0.6 0.3 0.1 ];
> H = ImpRespMatrix(h, N);
```

Use this program to determine the system output $y[n]$ when the input is $x[n] = \cos(\frac{\pi}{64}n)$ and the impulse response is

$$h[n] \quad = \quad \begin{cases} \frac{1}{n+1}, & 0 \le n \le 150, \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

Turn in a matlab printout of 3 periods of the input signal $x[n]$ (as a dashed line) and overlaid the output signal $y[n]$ (as a solid line). *Hint:* First determine the period length of the input signal. *Remark:* We will continue this problem in the next homework.