Homework 6

Problem 1: For $\omega_s = 2\pi$:

\[X(e^{j\omega})\]

Nyquist frequency: $5\pi$
But we can sample at $\omega_s = \frac{5}{2}\pi$
(Which is the smallest rate that avoids aliasing)

We want the aliased copies to be non-overlapping

$\Rightarrow$ $\omega_s$ must be a multiple of the bandwidth:

$\omega_s = k (\omega_2 - \omega_1)$ where $k$ integer.

Therefore the lowest sampling frequency is

$\omega_s = \frac{2\omega_2}{C}$ where $C = \left\lfloor \frac{\omega_2}{\omega_2 - \omega_1} \right\rfloor$ = number of complete copies of
the spectrum that can fit in $\omega_s$.

Preprocessing: We can always shift the bandpass signal

Preprocessing and make a lowpass representation that can be sampled conventionally.

\[x(t) \xrightarrow{\text{cos}(\omega_1 t)} x(t) \xrightarrow{\text{Low Pass}} x(t) \xrightarrow{\text{normal sampling}} x(t) \xrightarrow{\text{Bandpass Filtering}} x(t)\]
\[ Y(t) = \left[ x(t) + A \right] \cos(wct + \theta_c) \]

\[ Y_1(t) = (x(t) + A) \cos(wct + \theta_c) \cdot \cos(wct) = \left[ x(t) + A \right] \left[ \frac{1}{2} \cos(2\theta_c) + \frac{1}{2} \cos(2wct + \theta_c) \right] \]

\[ Y_2(t) = \left[ x(t) + A \right] \left[ \frac{1}{2} \cos(2\theta_c) \right] \]

\[ Y_3(t) = (x(t) + A)^2 \cdot \frac{1}{4} \cos^2(\theta_c) \]

And

\[ Y_1'(t) = (x(t) + A) \cos(wct + \theta_c) \cdot \sin(wct) = (x(t) + A) \left[ \frac{1}{2} \sin(2wct + \theta_c) - \frac{1}{2} \sin(\theta_c) \right] \]

\[ Y_2'(t) = (x(t) + A) \left( -\frac{1}{2} \sin \theta_c \right) \]

\[ Y_3'(t) = (x(t) + A)^2 \cdot \frac{1}{4} \sin^2 \theta_c \]

Therefore

\[ Y_5(t) = \sqrt{(x(t) + A)^2 \cdot \frac{1}{4} \cos^2(\theta_c) + (x(t) + A)^2 \cdot \frac{1}{4} \sin^2(\theta_c)} = \left| x(t) + A \right| \sqrt{\frac{1}{4} (\cos^2 \theta_c + \sin^2 \theta_c)} \]

\[ = \frac{1}{2} \left| x(t) + A \right| = \frac{1}{2} (x(t) + A) \quad \text{since} \quad x(t) + A \quad \text{is positive} \]

Therefore we recover \( x(t) \) by rescaling and subtracting \( \theta \).
Problem 4.1 8.29

(a) Spectrum after modulation with sinusoid ($Y_1(j\omega)$)

(b) The spectrum of $Y_2$ will be:

Notice that $P(j\omega) = 2S(j\omega)$ so they are proportional.
(Since the upper side band is canceled out by its opposite.)
We want to show that $S_2(t) = X(t)$.

The spectrum of $S_1$ will be:

And therefore the filtered signal $S_2(j\omega) = X(j\omega)$.