
Homework 1
Due: Thursday, January 27, 2004, at 9:00am

Reading OWN Chapter 1.

For any Matlab problems, submit computer generated plots and code.

Problem 1 (*Complex numbers.*)

(a) With two real numbers, a and α , a complex number can be specified in its *polar form* as

$$z = ae^{j\alpha},$$

where j is the imaginary unit (i.e., the root of -1). In terms of a and α , express the magnitude $|z|$, the phase $\arg(z)$, the real part $Re\{z\}$, the imaginary part $Im\{z\}$, and the complex conjugate z^* .

(b) Express the following complex numbers in Cartesian and polar coordinates (remember to consider the range of α). Draw them in the complex plane.

$$e^{j3\pi}, \frac{1}{2}e^{j\pi/3}, (\frac{1}{2}e^{j\pi/3})^*, 1 - 2j, \frac{1-2j}{1+j}, j^j$$

(c) Express $\frac{a+bj}{c+dj}$ in polar form.

(d) Consider the complex-valued function

$$H(j\omega) = \frac{1}{1 + 2j\omega},$$

where ω is a real number. Sketch $|H(j\omega)|$ and $\arg(H(j\omega))$ versus ω for $-5 < \omega < 5$. *Remark:* Writing $H(j\omega)$ is standard engineering practice, even though strictly speaking, it would be more logical to write $H(\omega)$.

Problem 2 (*Elementary functions and their graphs.*)

(a) Let $y(t) = e^{j2\pi t}$.

Express, as functions of t : $Re\{y(t)\}$, $Im\{y(t)\}$, $|y(t)|$, $\angle y(t)$.

Sketch by hand, in one (large) figure, for $-1 \leq t \leq 1$, the following functions: $y_1(t) = Im\{y(t)\}$, $y_2(t) = Im\{y(2t)\}$, $y_3(t) = Im\{y(2t - .5)\}$, $y_4(t) = Im\{y(-2t - .5)\}$. Clearly label the sketches and their zero-crossings.

(b) Determine the fundamental period of the signal $z(t) = \cos(\frac{\pi}{2}t) + \cos(\frac{4\pi}{5}t)$.

(c) Is $z(t) = \sin(3t) + \sin(\pi t)$ periodic? If so, what is its period?

Problem 3 (*Discrete-time signals.*)

(a) Let $y_1[n] = \cos(\frac{\pi}{2}n)$, $y_2[n] = \cos(\frac{4\pi}{5}n)$. What are their fundamental periods? Plot two periods of $y_1[n]$ and $y_2[n]$ in Matlab and clearly mark the end of each period.

(b) Determine the fundamental period of the signal $z[n] = y_1[n] + y_2[n]$. Plot two periods of $z[n]$ in Matlab and clearly mark the end of each period.

(c) Is $z[n] = \sin(3n)$ periodic? If so, what is its period?

Problem 4 (*Properties of systems.*)

Are the following systems linear? time invariant? In each case, give a short justification using the definitions of these properties.

(a) $\mathcal{H}[x(t)] = x(at + b)$

(b) $\mathcal{H}[x(t)] = x(at^2 + b)$

(c) $\mathcal{H}[x(t)] = x(at) + b$

(d) $\mathcal{H}[x(t)] = \frac{d}{dt}x(t)$

Problem 5 (*Properties of systems.*)

For each of the following systems with input $x(t)$ and output $y(t)$, determine whether the system is (i) memoryless, (ii) stable, (iii) causal, (iv) linear, and (v) time invariant. In each case, give a short justification using the definitions of these properties.

(e) $y(t) = e^{-t}x(t)$

(f) $y(t) = \int_{-\infty}^{t/2} x(2\tau)d\tau$

(g) $y(t) = x(-t) + 1$

(h) $y[n] = \begin{cases} x[n], & n \geq 1 \\ 0, & n = 0, \\ x[n], & n \leq -1 \end{cases}$

(i) $y[n] = \begin{cases} x[n], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$

(j) $y[n] = \text{Odd}\{x[n]\}$

Problem 6 (*Convolution of step functions*)

Recall that if an LTI system has impulse response $h(t)$ and input $x(t)$, the output is given by a convolution $y(t) = h(t) \star x(t) = \int_{-\infty}^{\infty} h(\tau)x(t - \tau)d\tau$. Recall also that the unit step function can be defined as

$$u(t) = \begin{cases} 1 & \text{if } t \geq 0 \\ 0 & \text{if } t < 0 \end{cases}$$

For the following, find the output given the input and the system specified. Draw, by hand, the input, the impulse response, and the output.

(a) $h(t) = u(t) - u(t - 1)$
 $x(t) = u(t)$

(b) $h(t) = u(t)$
 $x(t) = u(t)$

(c) $h(t) = u(t) - u(t - 1)$
 $x(t) = u(t) - u(t - 1)$

(d) In general, what does convolving steps and pulses with steps and pulses result in?

Problem 7 (*Matlab and convolution with exponentials*)

Most of the homeworks will have a major component in Matlab. For this problem, you will use Matlab's 'conv' function to convolve exponentials. You can type 'help conv' in Matlab for help on using the function. For the following, plot $h(t)$ and $x(t)$ on the domain $[-2, 2]$ with .01 between points. Then plot the resulting convolution. To create the time axis, you can use, for example, 't = [-2:0.01:2]'. If the Matlab exercises in this homework require a lot of effort, you should go through a tutorial on Matlab. Don't forget to submit code with the plots.

(a) $h(t) = e^{-t}u(t)$
 $x(t) = u(t)$

(b) $h(t) = e^{-t}u(t)$
 $x(t) = e^{-2t}u(t)$

Problem 8 (*Matlab and discrete time Fourier Transform*)

Recall that for a discrete signal $x[n]$, the discrete time Fourier Transform (DTFT) is given by $X(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$. We will cover the DTFT in detail this semester, but you should be able to find the transforms of the following signals through your knowledge from EE 20. In the following $u[n]$ and $\delta[n]$ are the discrete unit step and unit impulse functions respectively. Find the DTFT of each. Then, using MATLAB, plot each of the functions on the domain $[-7, 7]$. Also plot the magnitude of the DTFT of the signal that you derive with domain $[-\pi, \pi]$. Later on this semester, we will use functions in Matlab's signal processing toolbox to study Fourier Transforms.

Hint: $e^{-j\omega n} + e^{j\omega n} = 2 \cos(\omega n)$.

(a) $x[n] = u[n + 2] - u[n - 3]$

(b) $x[n] = (\delta[n] + \delta[n - 2] + \delta[n + 2]) - (\delta[n - 1] + \delta[n + 1])$

(c) In what frequencies (low or high) is the energy in each signal concentrated? Explain why in terms of the time domain signals.