Homework 4
Due: Thursday, February 17, 2005, at 11:30am

Reading OWN Chapters 3 and 4.

Please write your section day and time on the upper left of the front page of your homework. This will help us return your homeworks.

You may work in (small) groups to do the homework, but each person must write up their own answers. Note that working together does not mean dividing up the problems and sharing answers later.

For any Matlab problems, submit computer generated plots only. **No code is required!**

Problem 0 *(Study for Quiz 1 - Wednesday, Feb. 16th.)*

Problem 1 *(The Discrete Time Fourier Series - Matlab.)*

The Discrete Time Fourier Series (DTFS), a.k.a. Discrete Fourier Transform (DFT), is an extremely important tool in signal processing and communications. It is essentially the only one of the four Fourier transforms that can be computed *exactly* for any signal. Matlab is particularly good about dealing with the DFT. We will gain some experience with the DFT in this problem. Note that if you want to learn more about the DFT, you should take EE 123 (Digital Signal Processing).

(a) The DFT can be viewed as a change of basis for complex vectors of length N. The equation in MATLAB for the length N DFT of a signal (with period N) is

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi k}{N}n}$$

Note that this is not the definition in OWN. There is a difference in the scaling factor of $\frac{1}{N}$. Matlab puts this in the inverse equation, while OWN puts it in the transform equation.

If x is an N-dimensional complex vector and X is the (Matlab) DFT of x (X is also an N-dimensional complex vector), find the N by N matrix D such that $X = Dx$.

(b) Verify that the columns of D are orthogonal to each other (that is, their dot products are 0). Now what is the DFT really doing to your signal? This notion can be extended to the other Fourier transforms, but it is easiest to see in this case.

(c) Now let's get into the Matlab part of the problem. Create a new m-file. Matlab's function for the DFT is called `'fft'` (Fast Fourier Transform). It computes the same transform, just faster.

Let's look at some DFTs of length 16 signals.

```
x1 = [1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0];
x2 = [1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0];
x3 = sin(3*pi*(0:15)/8); x4 = exp(j*14*pi*(0:15)/16);
```

Compute the FFT of these signals using `'fft'` as well as the matrix multiplication you derived in part a. Be careful because Matlab starts indexing at 1 rather than 0. Verify that they give you the same answers and plot the absolute value of the FFTs next to the signals. Use the command `'subplot'` to put all the plots in one figure.

Do the FFTs of the signals (particularly signals 2, 3 and 4) make sense to you?

(d) Now we'll explore the relationship between the DFT and DTFT. Type in the following.

```
h = [zeros(1,4) ones(1,8) zeros(1,4)];
[H,W] = freqz(h,1, 512);
figure; plot(W, abs(H));
```

The function `'freqz'` is evaluating the DTFT of the signal `h` at 512 points in the interval $[0, \pi]$. Now, in the same plot, use `'stem'` to plot the first 9 absolute values of the fft of h. The axis should be `(0:8)*pi/8'`. The reason we are using only the first nine values of the fft is because `freqz` evaluates the DTFT only for nonnegative frequencies.

What do you see? Use this intuition to explain the relationship between the DTFT and the DFT (you should get an explicit formula). Don't forget to hand in the plot.

Problem 2 *(Continuous Time Fourier Transform Properties)*

The properties of the CTFT are easy to derive and very useful. If you can't remember all the properties, you should be able to derive them. Derive the following properties from Table 4.1 (Pg. 328) in OWN.

(a) Frequency Shifting

(b) Time and Frequency Scaling

(c) Multiplication

(d) Conjugate Symmetry for Real Signals

Of course, if you have time, go through all the properties and derive them on your own.

Problem 3 *(Time-domain filters.)*

While playing with your new electric thermometer you notice that the measurements are jumping all over the place. Looking around, you realize it probably has something to do with the electrical engineer sitting next to you building an unlicensed wireless transmitter. Not to be outnerded, you crack open your thermometer and peek inside.

You discover that your thermometer is generating 9 measurements every second. Unfortunately, thanks to the 2.4 GHz signal blasting out of your classmate's transmitter, there is a lot of noise in your system. Realizing that the temperature measurements can basically be treated as a DC signal in comparison to your classmate's 2.4 GHz signal, you decide to build a filter to cancel out his interference. Model your noisy temperature as

$$x[n] = (A + B\cos(2\pi * \frac{2.4 * 10^9}{9} * n))u[n]$$

where A represents the temperature measurement you would like to recover, B represents the strength of your classmate's signal, and $u[n]$ represents the fact that you turn on your thermometer at time 0.

(a) You decide this is model is too complicated. Instead, you decide to use the model:

$$x[n] = (A + B\cos(\frac{4\pi n}{3}))u[n]$$

Write a couple sentences explaining why this new model is just as good.

(b) You decide to use a moving-average filter, one of the simplest low-pass filters:

$$h_1[n] = \frac{1}{3}(u[n] - u[n-3])$$

You feed your thermometer's signal into the filter and wait a little while. Let $y_1[n]$ denote the output of your filter. For large n, what is the value of $y_1[n]$? Plot $y_1[n]$ for $-5 \le n \le 20$.

(c) You decide to try a slightly different moving-average filter:

$$h_2[n] = \frac{1}{9}(u[n] - u[n-9])$$

You feed your thermometer's signal into the filter and wait a little while. Let $y_2[n]$ denote the output of your filter. For large n, what is the value of $y_2[n]$? Plot $y_2[n]$ for $-5 \le n \le 20$. Do you see any problems that might arise for a filter with a very long impulse response? Explain, briefly.

(d) Your classmate sees what you're doing and decides to turn your thermometer into a small wireless receiver. Find a simple (only two non-zero values in the impulse response) filter that he can use to remove your DC signal from $x[n]$. You have just designed a high-pass filter.

Problem 4 *(Continuous time Fourier series.)*

(a) OWN 3.22a: figures a,d,e

(b) OWN 3.25