EECS 120 Signals & Systems
Gastpar

University of California, Berkeley: Spring 2003
March 17, 2003

Midterm Exam

Rules.

- You have two hours to complete this exam.
- There are 100 points for this exam. Points for the individual problems and subproblems are marked in the problem statement.
- The exam is closed-book and closed-notes; calculators, computing and communication devices are *not* permitted.
- However, two handwritten and *not photocopied* double-sided sheets of notes are allowed. These sheets must be handed in with your exam paper.
- Moreover, you receive, together with the exam paper, a copy of Appendix C of the course textbook.
- No form of collaboration between the students is allowed. If you are caught cheating, you may fail the course and face disciplinary consequences.

Please read the following remarks carefully.

- Take into account the points that may be earned for each problem when splitting your time between the problems.
- Do not spend too much time on Problem 1 before having attempted to solve Problems 2 and 3.
- It is recommended that you attempt Problem 4 only after finishing Problems 1, 2, and 3.

Problem	Points earned	out of
Problem 1		20
Problem 2		20
Problem 3		50
Problem 4		10
Total		100

Problem 1 *(Short questions.)* *(20 Points)*

Each of the following is either true or false. If you believe it is true, give a brief argument. If you believe it is false, you can give a brief argument or a counterexample.

(i) (4 Points) The signal $s(t) = \sin(t/1000)$ is a power signal.

(ii) (4 Points) If the system H_1 is linear and the system H_2 is also linear, then the system H defined as $y(t) = H\{x(t)\} = H_2\{H_1\{x(t)\}\}$ is also linear.

(iii) (4 Points) All continuous-time signals $s(t)$ can be expressed as

$$s(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(j\omega)e^{j\omega t}d\omega, \quad \text{where} \quad S(j\omega) = \int_{-\infty}^{\infty} s(t)e^{-j\omega t}dt. \tag{1}$$

(iv) (4 Points) A time-invariant memoryless causal discrete-time system is always linear.

(v) (4 Points) The following is a Fourier transform pair:

$$x(t) = \left|\frac{\sin(t)}{\sqrt{|t|^3}}\right| \quad \overset{FT}{\longleftrightarrow} \quad X(j\omega) = \begin{cases} \sqrt{|\omega|^3}, & |\omega| \le 1 \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

Hint: Do not use too many equations.

Problem 2 *(Evaluation of convolution and Fourier representations.)* *(20 Points)*

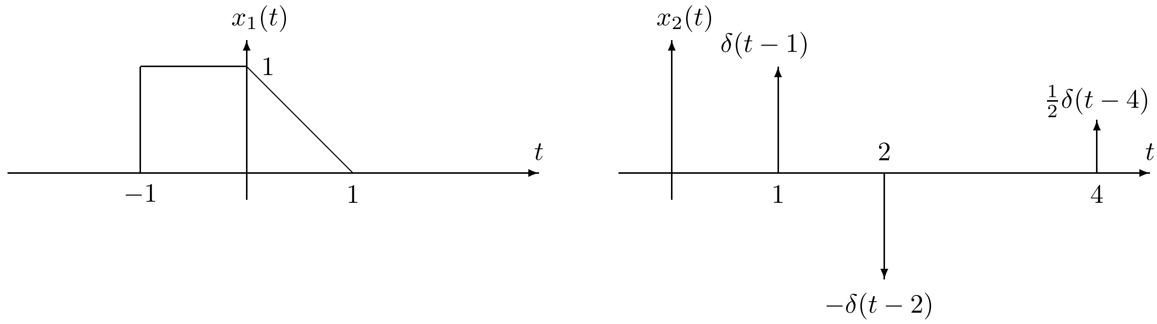
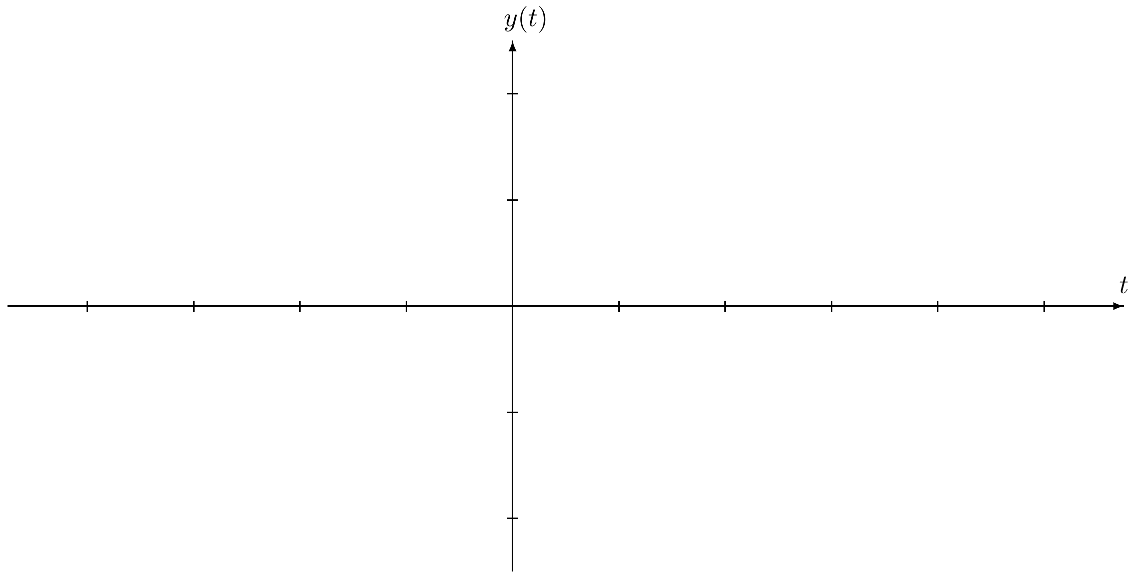


Figure 1: The signals for Problem 2, Part *(i)*.

(i) (10 Points) Consider the two signals shown in Figure 1. Note that the signal $x_2(t)$ consists of three impulse functions,

$$x_2(t) \quad = \quad \delta(t-1) - \delta(t-2) + \frac{1}{2}\delta(t-4). \tag{3}$$

Sketch the convolution of the two signals, that is, sketch the signal $y(t) = (x_1 * x_2)(t)$ in the figure below. Label the axes carefully.



(ii) (10 Points) The spectrum of the continuous-time signal $x(t)$ is shown in Figure 2. Determine the signal $x(t)$.

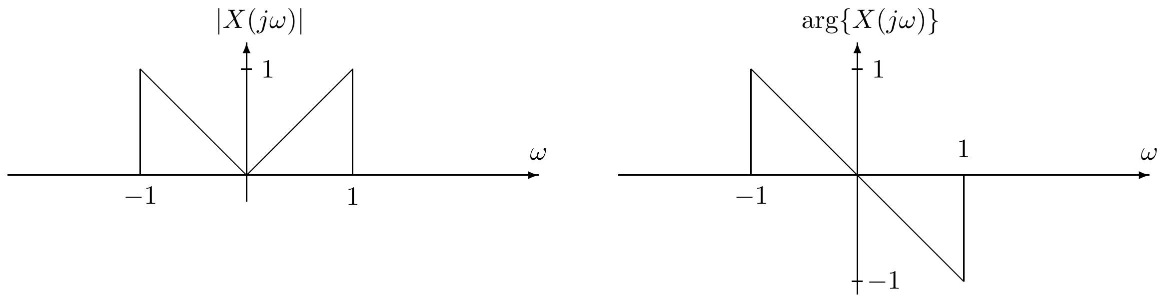


Figure 2: The spectrum for Problem 2, Part *(ii)*.

Problem 3 *(Analysis of a causal discrete-time system.)* *(50 Points)*

Note: The parts of this problem are quite independent. If you do not manage to solve one part, please read on.

The following recursion equation describes a causal discrete-time system:

$$y[n] \quad = \quad x[n] + \alpha x[n-1] + \beta y[n-1]. \tag{4}$$

(i) (10 Points) Draw a system diagram involving only delay elements, multiplications, and addition. Is the system linear? Is it time-invariant? Give a brief justification of your answer.

(ii) (15 Points) Suppose that the input is $x[n] = u[n]$ and the initial condition is $y[-1] = 0$. Compute the output $y[n = 0]$. For $n \geq 1$, we can rewrite the difference equation as

$$y[n] \quad = \quad 1 + \alpha + \beta y[n-1]. \tag{5}$$

Find $y[n]$ for $n \geq 1$ by solving this difference equation. The $y[n]$ you have found is the step response $s[n]$ of the system. For $n \geq 1$, the impulse response $h[n]$ can be found by taking differences,

$$h[n] \quad = \quad s[n] - s[n-1], \text{ for } n \geq 1. \tag{6}$$

For $n = 0$, the impulse response is simply $h[n = 0] = 1$. Hence, you can write the impulse response as

$$h[n] \quad = \quad \delta[n] + (s[n] - s[n-1])u[n-1] \tag{7}$$

Explain why $h[n] = 0$ for $n < 0$. Find the frequency response $H(e^{j\Omega})$ of the system by taking the discrete-time Fourier transform (DTFT) of $h[n]$.

(iii) (15 Points) Use the direct method that we have discussed in class and in the homework problems to find the frequency response $H(e^{j\Omega})$, i.e., assume that $x[n] = e^{j\omega n}$ and that $y[n] = H(e^{j\Omega})x[n]$. Then, use the frequency response $H(e^{j\Omega})$ to find the impulse response $h[n]$ of the system.

(iv) (10 Points) For what values of α and β is the system stable? Justify your answer. Sketch the magnitude of $H(e^{j\Omega})$ for $\alpha = \beta = 1/2$. Is the system behavior rather low-pass or rather high-pass? Give a brief justification. *Hint:* Evaluate your expression numerically for $\Omega = 0, \pi/2, \pi, 3\pi/2, 2\pi$.

Problem 4 *(Multiplication of Polynomials.)* *(10 Points)*

Multiplying two polynomials is a cumbersome task. For example, if

$$f(x) \quad = \quad 3x^2 + x + 2, \tag{8}$$

and

$$g(x) \quad = \quad x + 4, \tag{9}$$

then we find

$$h(x) \quad = \quad f(x)g(x) = 3x^3 + 13x^2 + 6x + 8. \tag{10}$$

However, we can define the coefficient signals for each polynomial, as follows:

$$f[n] \quad = \quad 3\delta[n-2] + \delta[n-1] + 2\delta[n], \text{ and} \tag{11}$$

$$g[n] \quad = \quad \delta[n-1] + 4\delta[n]. \tag{12}$$

Then, the coefficient signal $h[n]$ of the polynomial $h(x)$ is given by $h[n] = (f * g)[n]$.

(i) (5 Points) Sketch the coefficient signals $f[n]$ and $g[n]$ versus n. Evaluate the convolution and confirm that this indeed gives the coefficient signal $h[n]$ of the polynomial $h(x)$.

(ii) (5 Points) For the polynomials

$$a(x) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n x^n, \text{ and} \tag{13}$$

$$b(x) = \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n x^n, \tag{14}$$

find their product, i.e., find the polynomial $c(x) = a(x)b(x)$.