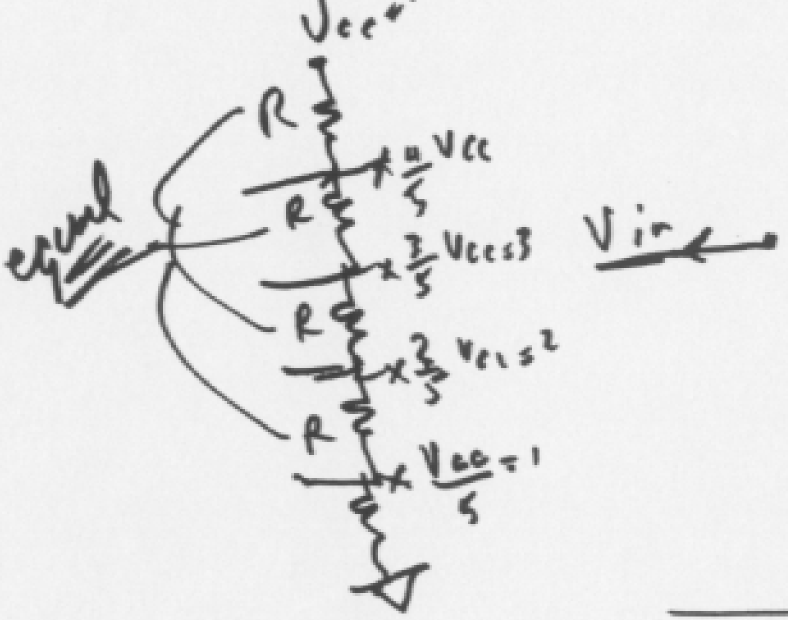
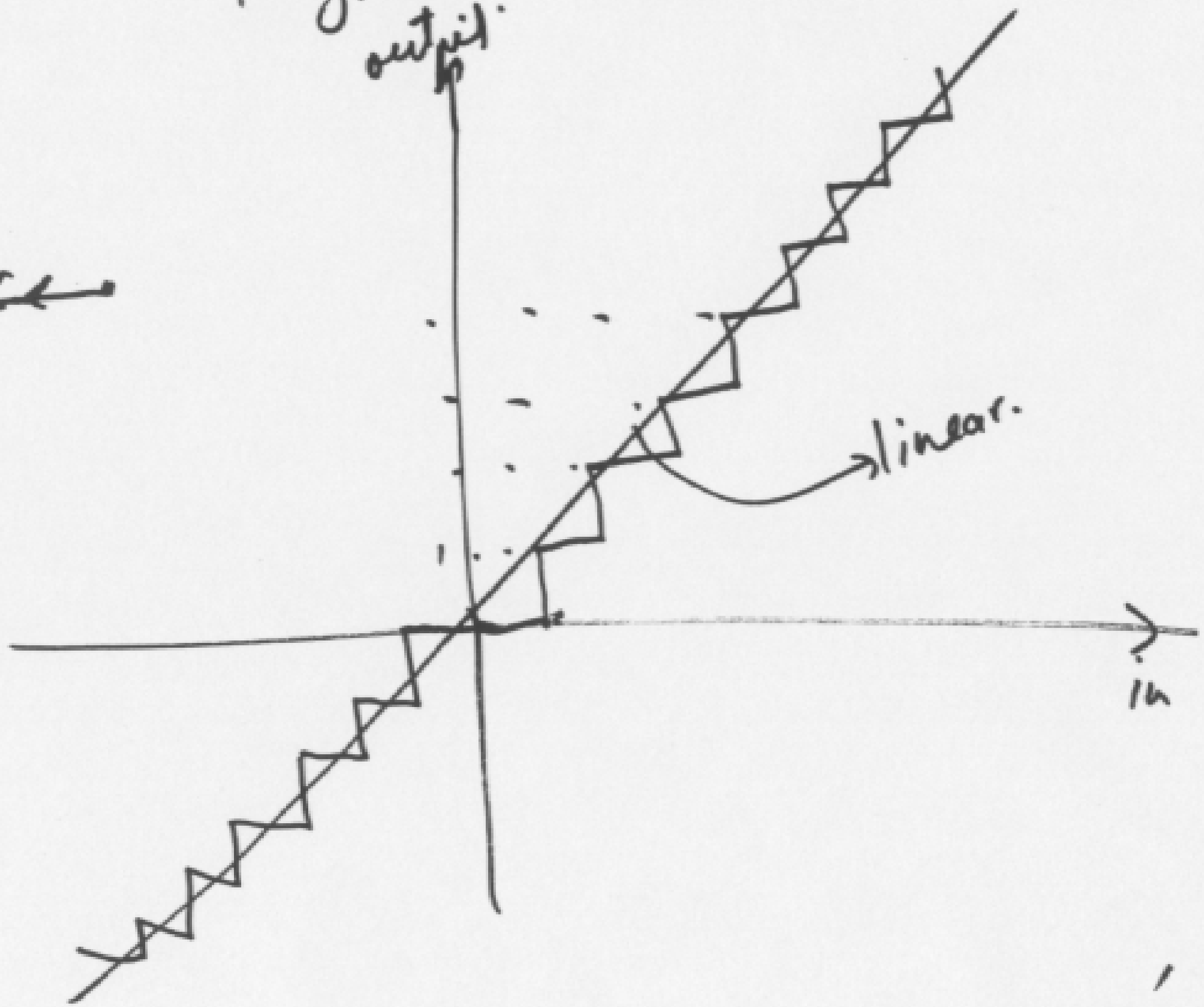
9/3/03

Sampling

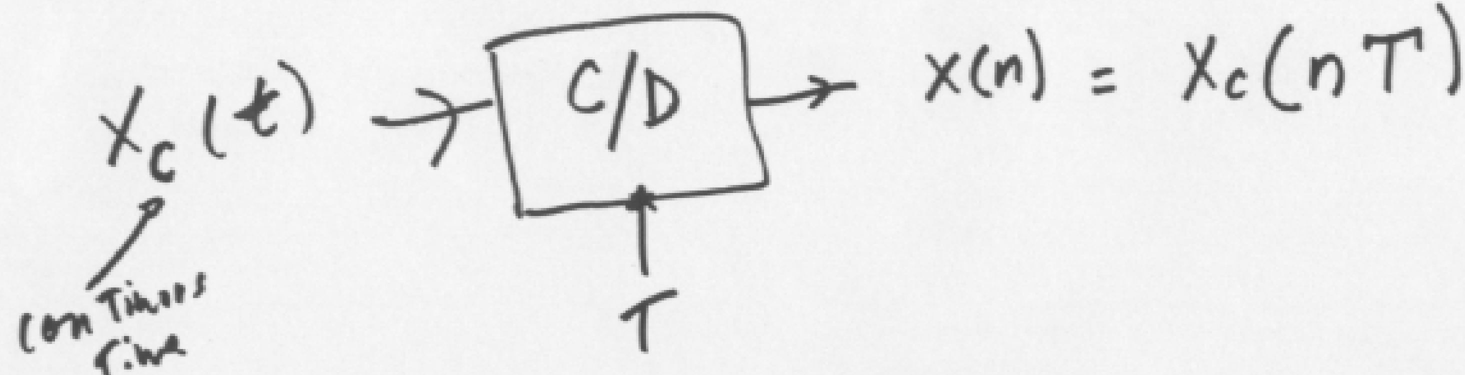
A/D Analog to
Digital Conversions.

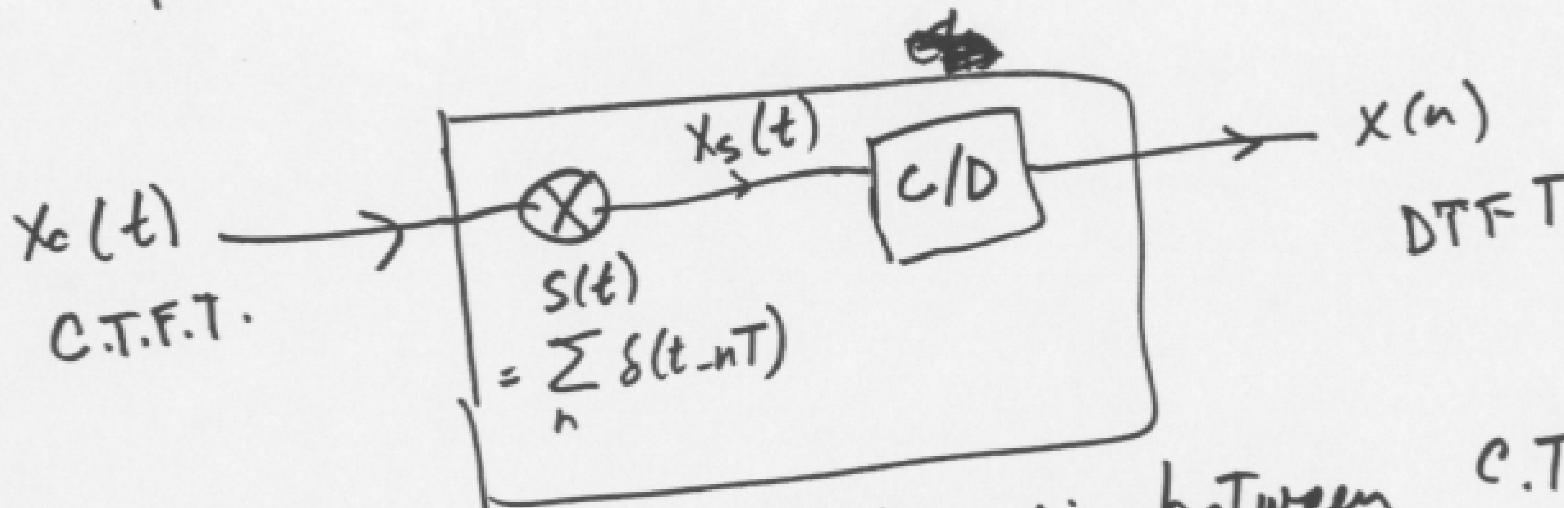
How to sample:





- $\Sigma\Delta$ Convertors.



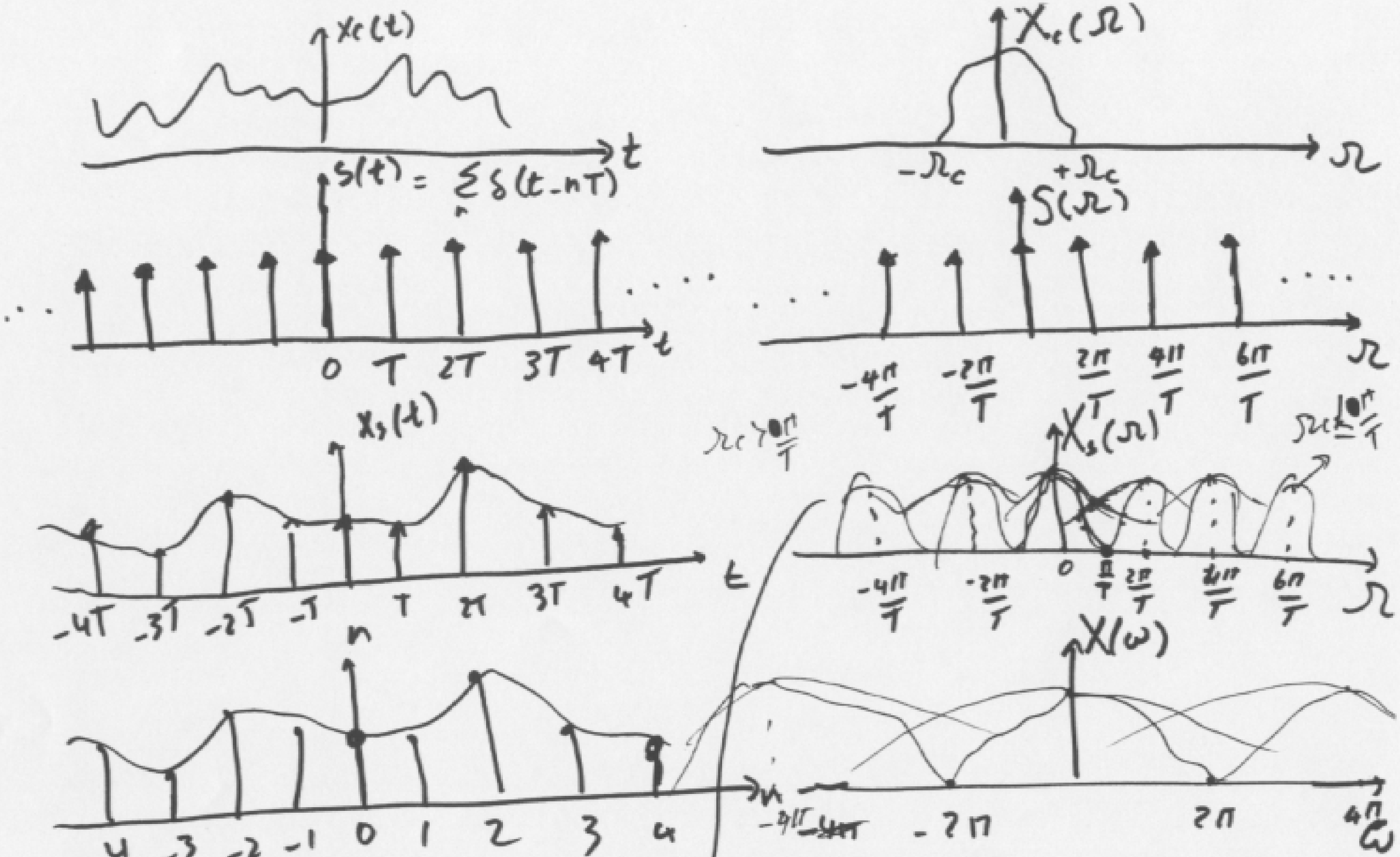


Q: what is the relationship between C.T.F.T of $x_c(t)$ and DTFT of $x(n)$?

$$X_c(\Omega) = \text{C.T.F.T.}\{x_c(t)\} = \int_{-\infty}^{+\infty} x_c(t)\, e^{-j\Omega t}\, dt$$

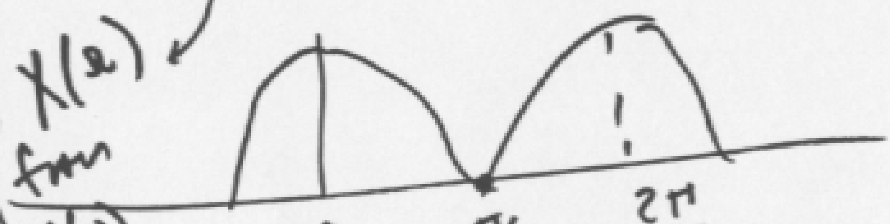
$$X(\omega) = \text{D.T.F.T.}\{x(n)\} = \sum_{n=-\infty}^{+\infty} x(n)\, e^{-j\omega n}$$

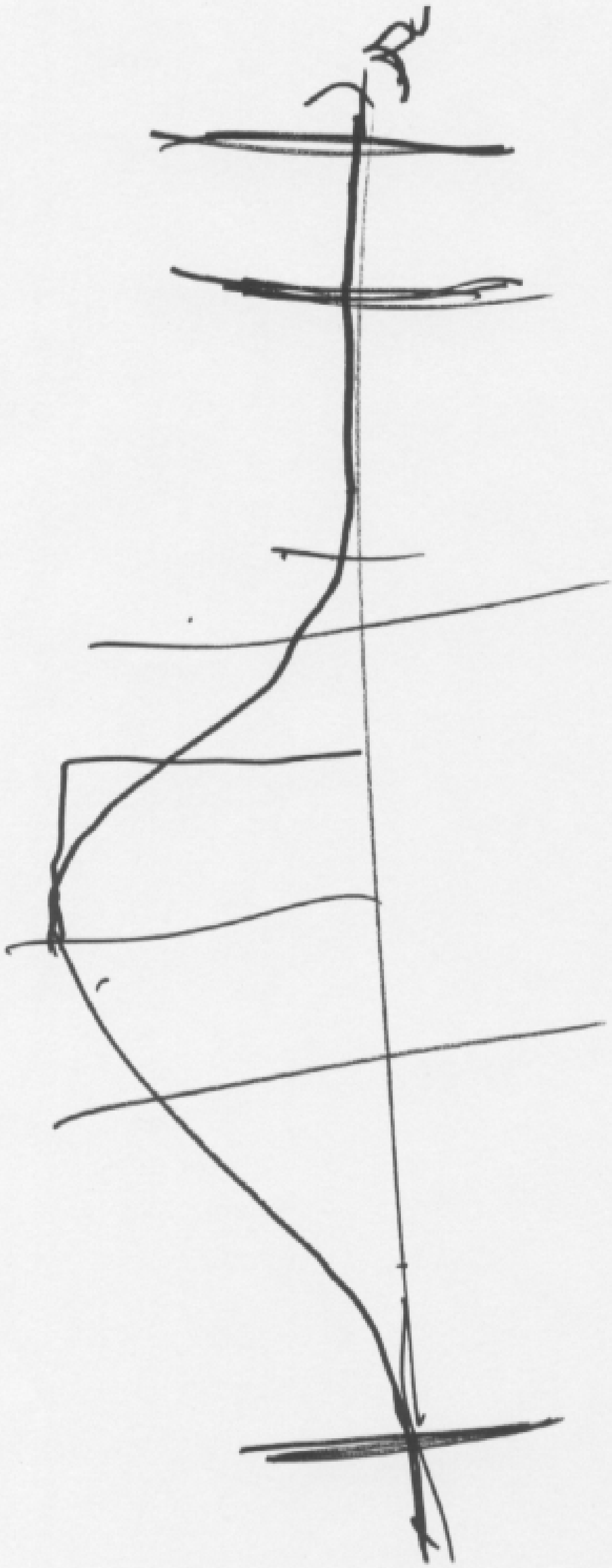
$$X(\omega) = \frac{1}{T} \sum_{K=-\infty}^{+\infty} X_c\left(\frac{\omega - 2\pi k}{T}\right)$$



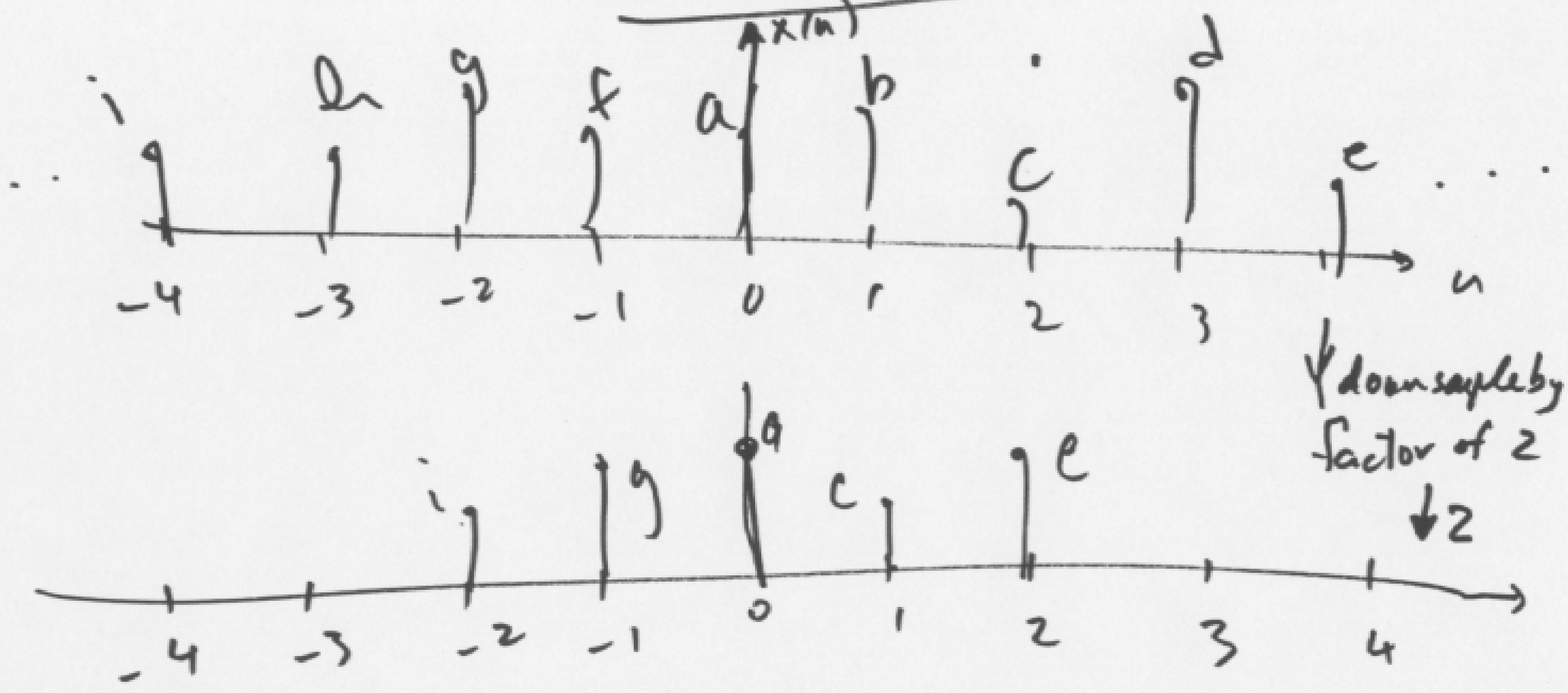
If $\boxed{\Omega_c < \frac{\pi}{T}} \Rightarrow$ recover $X(\Omega)$ exactly from $X(\omega)$

no aliasing $\Rightarrow$





Down Sampling

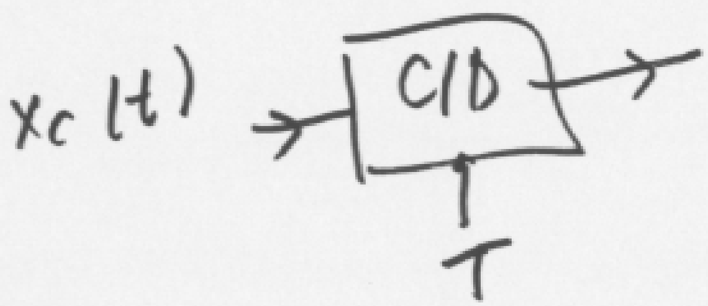


Intuitively: keep one sample throwing away N

$N=2$.

$x_c(t) =$ continuous Time Signal.

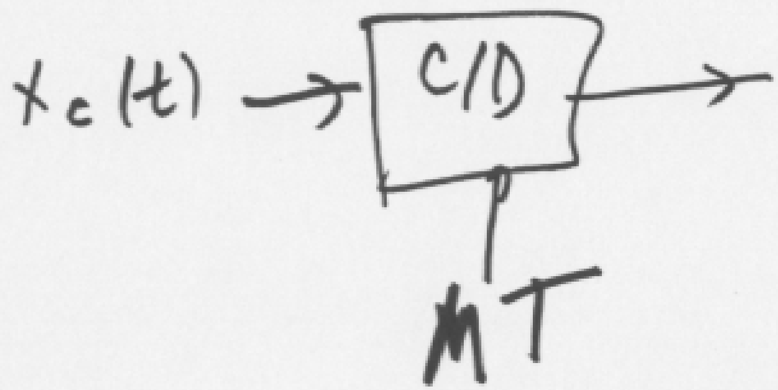
sample it at 2 rates.

① 

$x(n) = x_c(nT)$

discrete Time Signal.

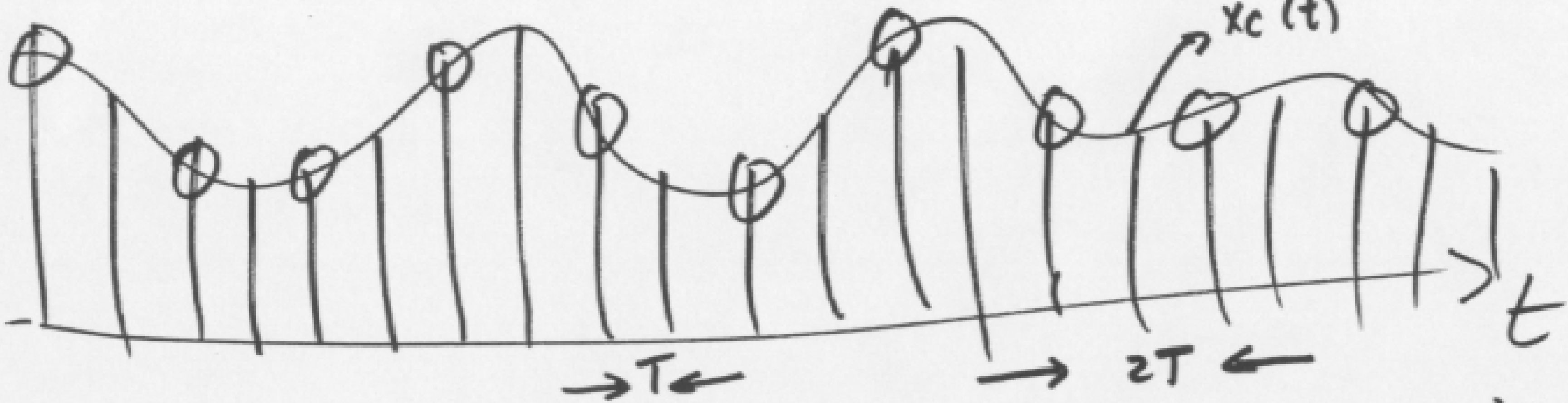
continuous time signal.

② 

$x_d(n) = x_c(n\,MT)$

$x_d(n)$ is downsampled version of $x(n)$ by a factor of M.

Relate DTFT of $x(n)$ to DTFT of $x_d(n)$



$$X(\omega) = \text{D.T.F.T.}\{x(n)\} = \frac{1}{T} \sum_{k=-\infty}^{+\infty} X_c\left(\frac{\omega}{T} - \frac{2\pi k}{T} \right)$$

$$X_d(\omega) = \text{D.T.F.T.}\{x_d(n)\} = \frac{1}{MT} \sum_{r=-\infty}^{+\infty} X_c\left(\frac{\omega}{MT} - \frac{2\pi r}{MT} \right)$$

change of variable. $r = i + kM$

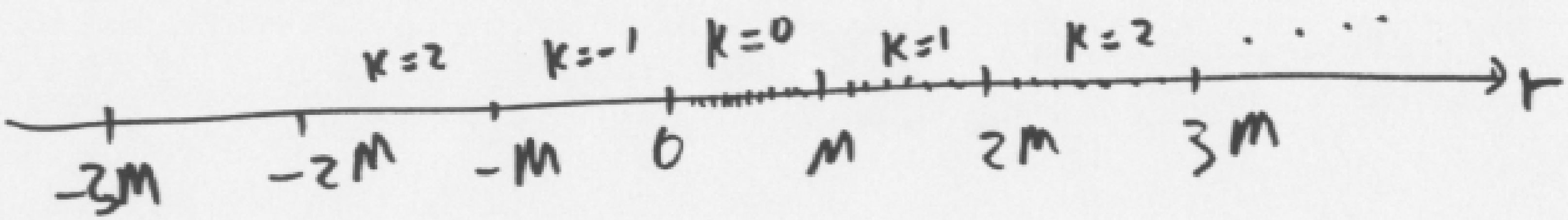
$-\infty < k < \infty$

$0 \leq i \leq M-1$

$$\left.\begin{array}{l} k=0 \\ i: 0 \rightarrow M-1 \end{array}\right\} \longrightarrow r: \; 0 \longrightarrow M-1$$

$$\left.\begin{array}{l} k=1 \\ i: 0 \rightarrow M-1 \end{array}\right\} \longrightarrow r: \; M \longrightarrow 2M-1$$

$$\left.\begin{array}{l} k=2 \\ i: 0 \rightarrow M-1 \end{array}\right\} \longrightarrow r: \; 2M \longrightarrow 3M-1$$



$$X_d(\omega) = \frac{1}{M} \sum_{i=0}^{M-1} \frac{1}{T} \sum_{k=-\infty}^{+\infty} X_c\left(\frac{\omega}{MT} - \frac{2\pi k}{T} - \frac{2\pi i}{MT}\right)$$

$$X_d(\omega) = \frac{1}{M} \sum_{i=0}^{M-1} \frac{1}{T} \sum_{k=-\infty}^{+\infty} X_c\left(\frac{\omega - 2\pi i}{MT} - \frac{2\pi k}{T}\right)$$

$$\left[X(\omega) \right]_{\omega \leftarrow \frac{\omega - 2\pi i}{M}}$$

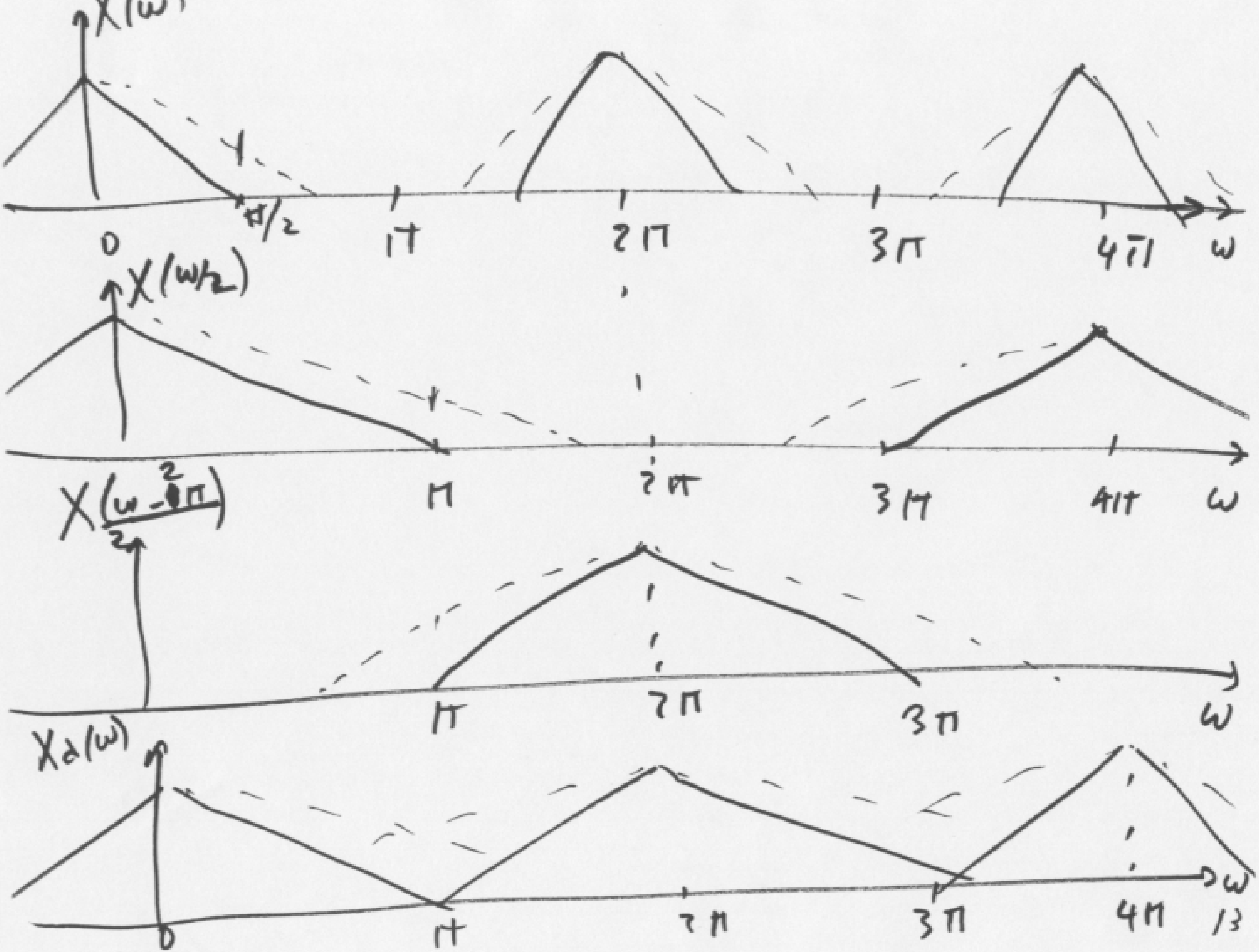
$$= X\left(\frac{\omega - 2\pi i}{M} \right)$$

$$\boxed{X_d(\omega) = \frac{1}{M} \sum_{i=0}^{M-1} X\left(\frac{\omega - 2\pi i}{M} \right)}$$

Relationship between DTFT of $x(n)$ and $x_d(n)$.

Let $M = 2$ $\quad X_d(\omega) = \frac{1}{2} \sum_{i=0}^{1} X\left(\frac{\omega}{2} - \frac{2\pi i}{2} \right)$

$$X_d(\omega) = \frac{1}{2}\left[X\left(\frac{\omega}{2}\right) + X\left(\frac{\omega}{2} - \pi\right) \right]$$



X(ω)
0
π/2
π
2π
3π
4π
ω
X(ω/2)
π
2π
3π
4π
ω
X((ω−2π)/2)
π
2π
3π
ω
Xd(ω)
0
π
2π
3π
4π
ω
13