Flowgraphs/Realizations of IIR filters with Rational Transfer fns

Direct Form 1.

" " 2.

Cascade ←

Parallel ←

$$H(z) = \frac{\sum_{k=0}^{q} b_k z^{-k}}{1 - \sum_{k=1}^{p} a_k z^{-k}}$$

Fundamental Theorem of Algebra:

[An]y polynomial in <u>one variable</u> can be factored into simple Terms.

Polynomial of degree n, has n real or complex roots. can be factored into n terms.

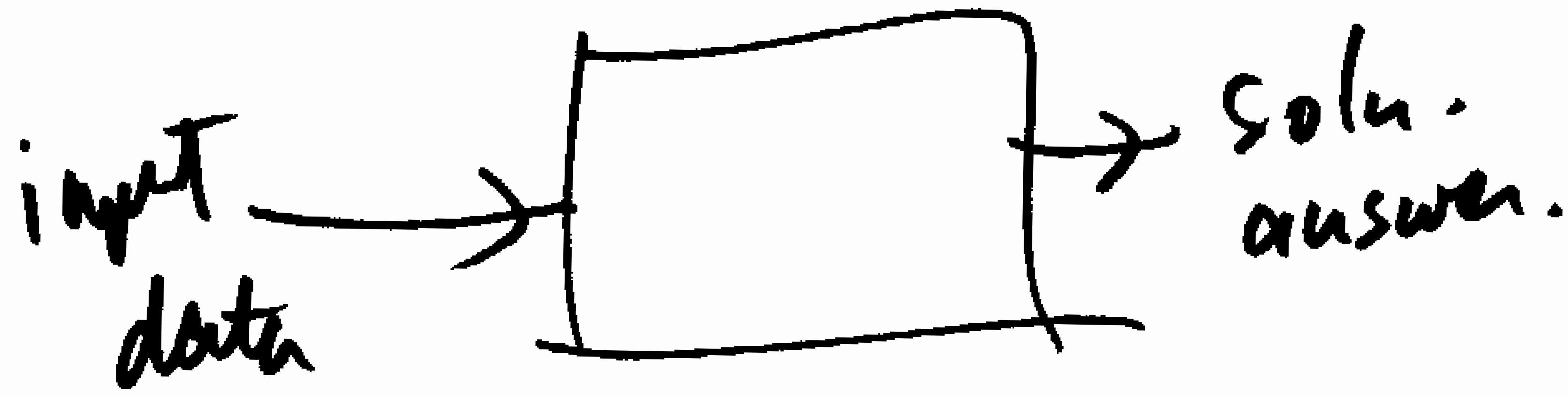
(z) = polynomial in one variable.

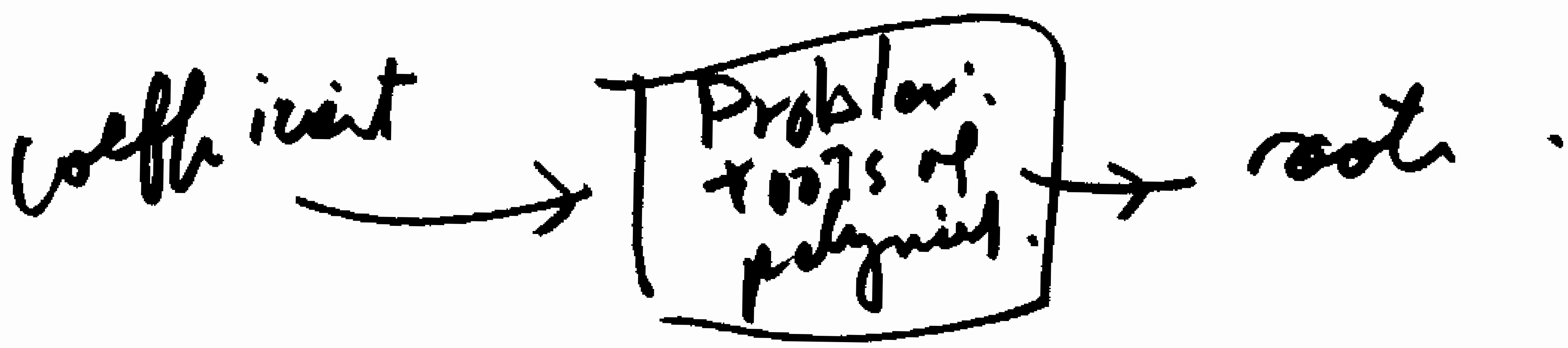
$= \alpha z^3 + \beta z^2 + \gamma z + \delta.$ ← degree 3

$= K(z - z_0)(z - z_1)(z - z_2)$

→ roots.

Problem.





...n is ill conditioned if small perturbations in The input result in large changes in The soln/answer.

me problem are fundamentally ill conditioned.
Condition # for a problem.

Tenl

ys. of eqn.

Algorithm

Problem

Alg 1 = Invert The matrix.

Alg 2 - Gaussian Elimination

pivoting

w/o pivoting

terrible

Alg 3 Gauss Siedel.

Alg 4 QR decomposition

extremely good

ax

in

if problem is well condition,
applying ill condition alg. can make answer be sensitive to input.

…damental Thm of Algebra does not hold in 2, 3, or higher # of variable.

$$P(z_1, z_2) = \alpha z_1^2 + \beta z_2^2 + \gamma z_1 z_2 + \delta + \epsilon z_1 + \eta z_2$$

most polynomials in 2 or more variables are non factorable. EE225B

→ Cannot check stability IIR filters in 2D easily.

→ enables you to factor denominator

$$H(z) = \frac{P(z)}{Q(z)}$$ → poles.

⟹ Check stability easily

coeffts on polynomial in one variable.
re real, then roots are either real or
They are complex conjugates.

If z_0 is root complex then z_0^* is also a root.

y deg 2 $\longrightarrow$ $\alpha z^2 + \beta z + \gamma$

γ, α, β real

Both real.

A pair of complex conjugate pole.

ly of deg 3 $\longrightarrow$ $\alpha z^3 + \beta z^2 + \gamma z + \delta$

coeff real

sts
ul.

one real
2 complex conjugate
pair.

ly of deg 4 :

① 4 real roots
② 2 real roots
one complex con
③ 2 pairs of
complex conjugate
pair.

Polynomial with real coeff is of odd deg.
$\Rightarrow$ it always has a real root.

For a polynomial real coeff,
I can factor it this way:

$$\text{real coeff} = \prod_k (1 - c_k z^{-1}) \prod_k (1 - d_k z^{-1})(1 - d_k^* z^{-1})$$

real.

d_k = complex

$$\frac{\sum_{k=0}^{q} b_k z^{-k}}{1 - \sum_{k=1}^{p} a_k z^{-k}}$$

real

complex

$$= A \frac{\prod_k (1 - e_k z^{-1}) \prod_k (1 - f_k z^{-1})(1 - f_k^* z^{-1})}{\prod_k (1 - c_k z^{-1}) \prod_k (1 - d_k z^{-1})(1 - d_k^* z^{-1})}$$

real. complex

$$-: (1-d_k z^{-1})(1-d_k^* z^{-1})$$

$$= 1 - 2\,\mathrm{Re}[d_k]\, z^{-1} + |d_k|^2 z^{-2}$$

polynomial with $\underline{\underline{\text{all}}}$ real coeffs

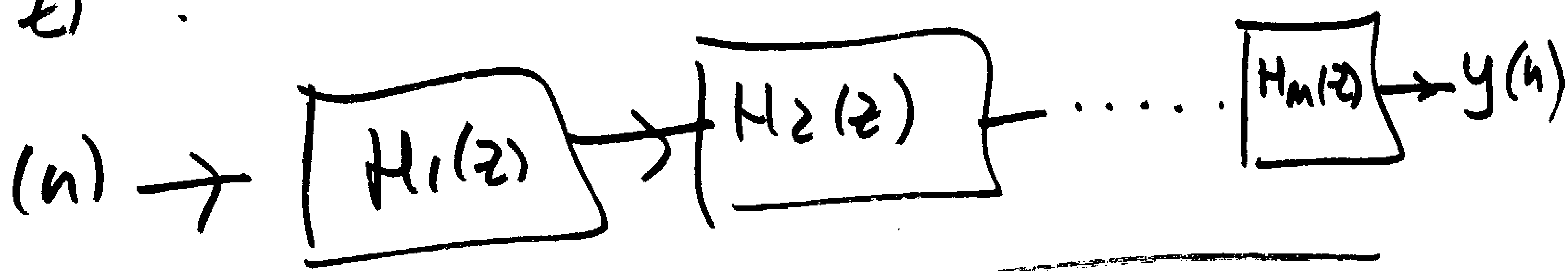
$$= 1 + \underbrace{\beta_{1k}}_{\text{real}} z^{-1} + \underbrace{\beta_{2k}}_{\text{real}} z^{-2}$$

$$z) = A \prod_k H_k(z)$$

where

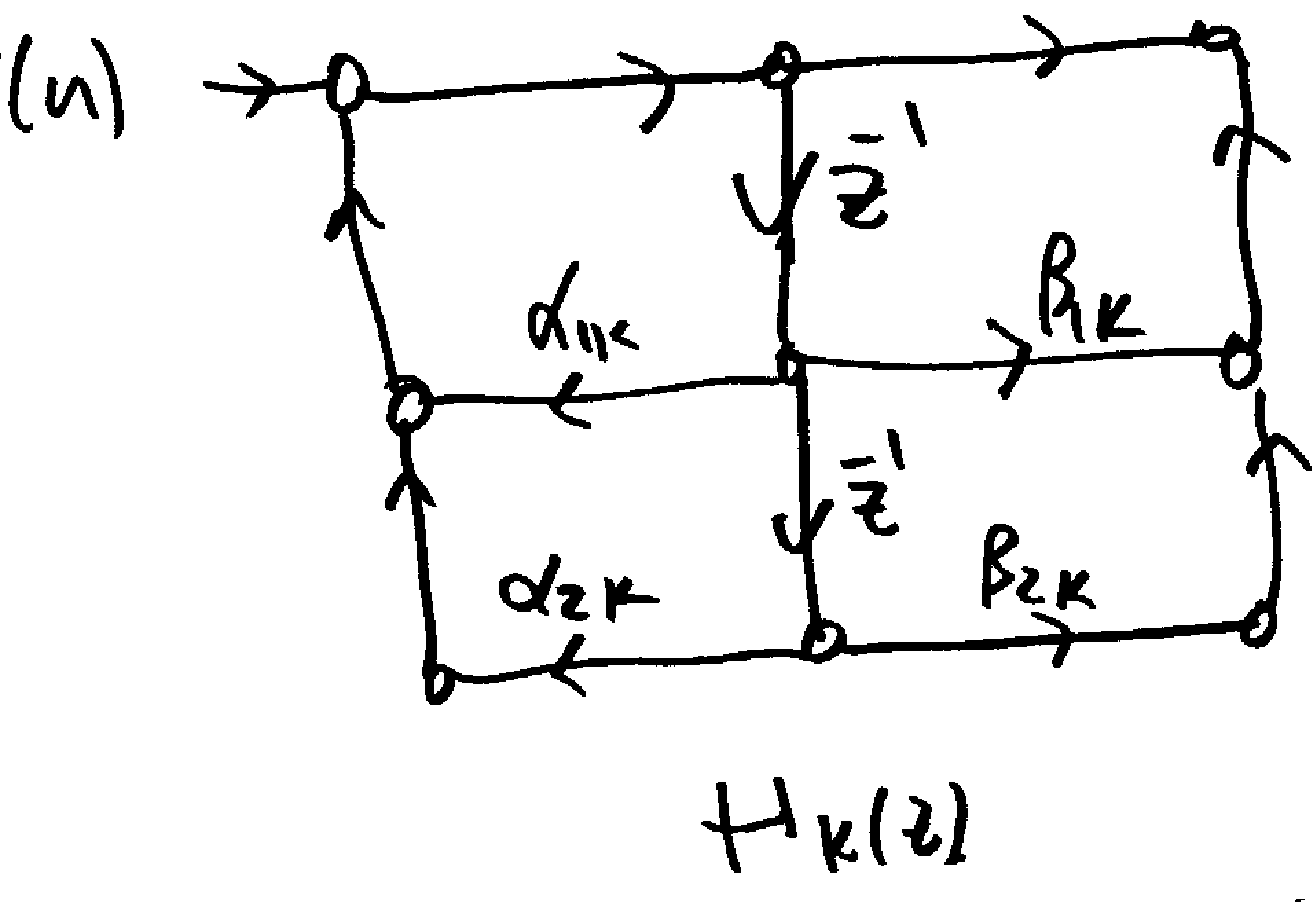
$$H_k(z) = \frac{1 + \overset{\text{real}}{\beta_{1k}} z^{-1} + \overset{\text{real}}{\beta_{2k}} z^{-2}}{1 - \underset{\text{real}}{\alpha_{1k}} z^{-1} - \underset{\text{real}}{\alpha_{2k}} z^{-2}}$$

z) :



Cascade.

How to implement $H_k(z)$.



$H_k(z)$

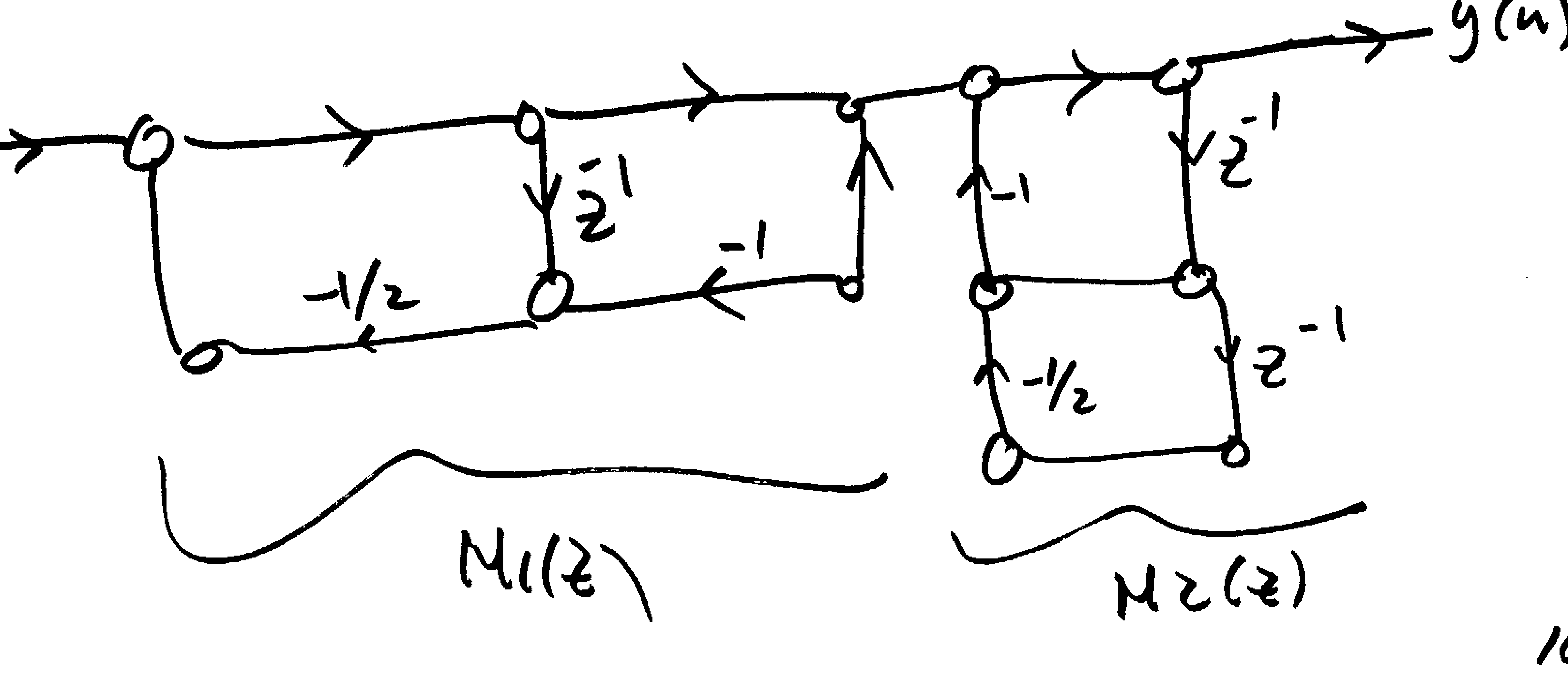
Direct form 2 implementation $H_k(z)$

$$H_k(z) = \frac{1 + \beta_{1k} z^{-1} + \beta_{2k} z^{-2}}{1 - \alpha_{1k} z^{-1} - \alpha_{2k} z^{-2}}$$

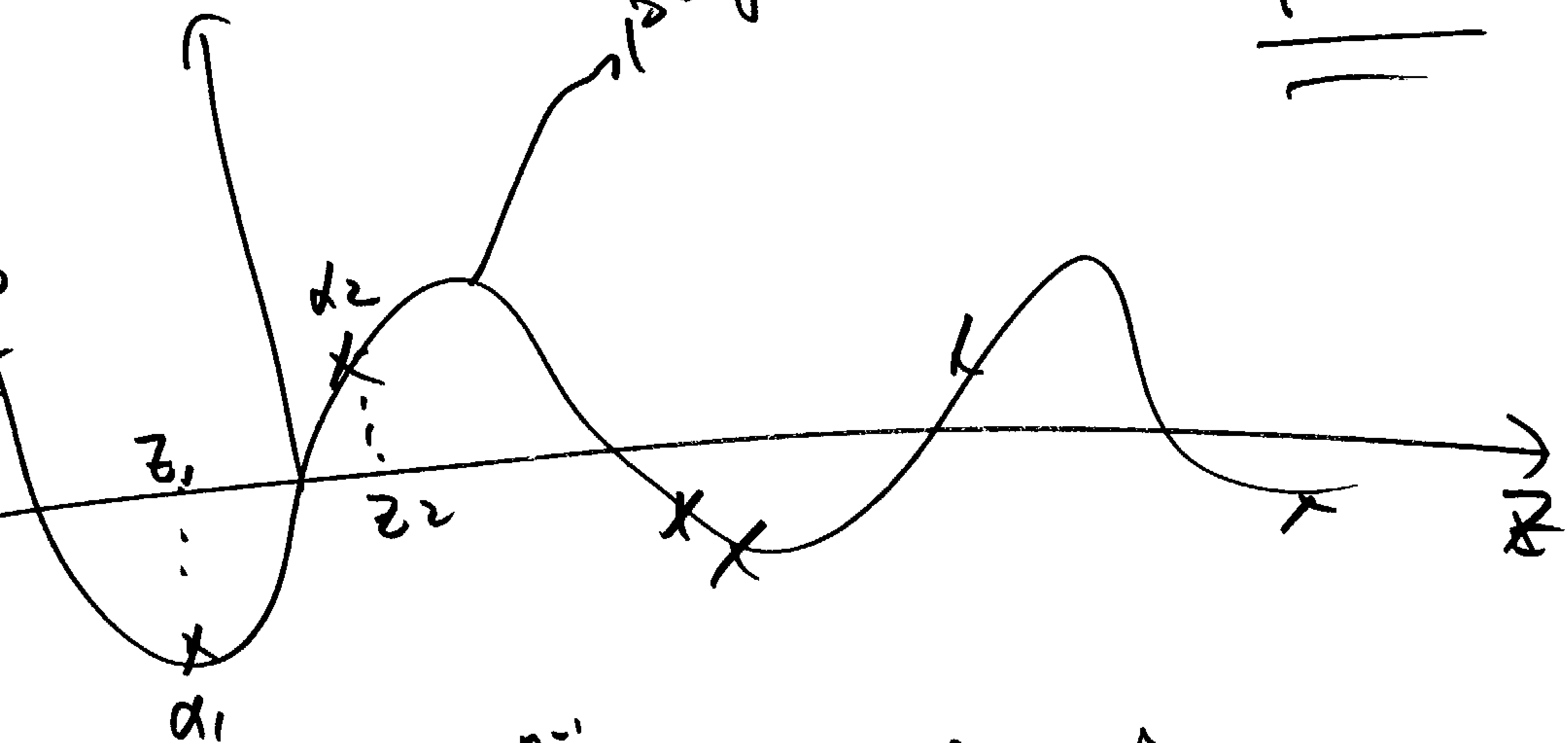
$$H(z) = \frac{1 - z^{-1}}{\left(1 + \frac{1}{2} z^{-1}\right)\left(1 + z^{-1} + \frac{1}{2} z^{-2}\right)}$$

$$z) = \frac{1 - z^{-1}}{\left(1 + \frac{1}{2} z^{-1}\right)} \quad \frac{1}{\left(1 + z^{-1} + \frac{1}{2} z^{-2}\right)}$$

$$H(z) = H_1(z) \qquad H_2(z)$$



polynomial of degree n $\underline{\underline{p(z)}}$



$p(z) = \beta_n z^n + \beta_{n-1} z^{n-1} + \cdots\cdots \beta_1 z + \beta_0$

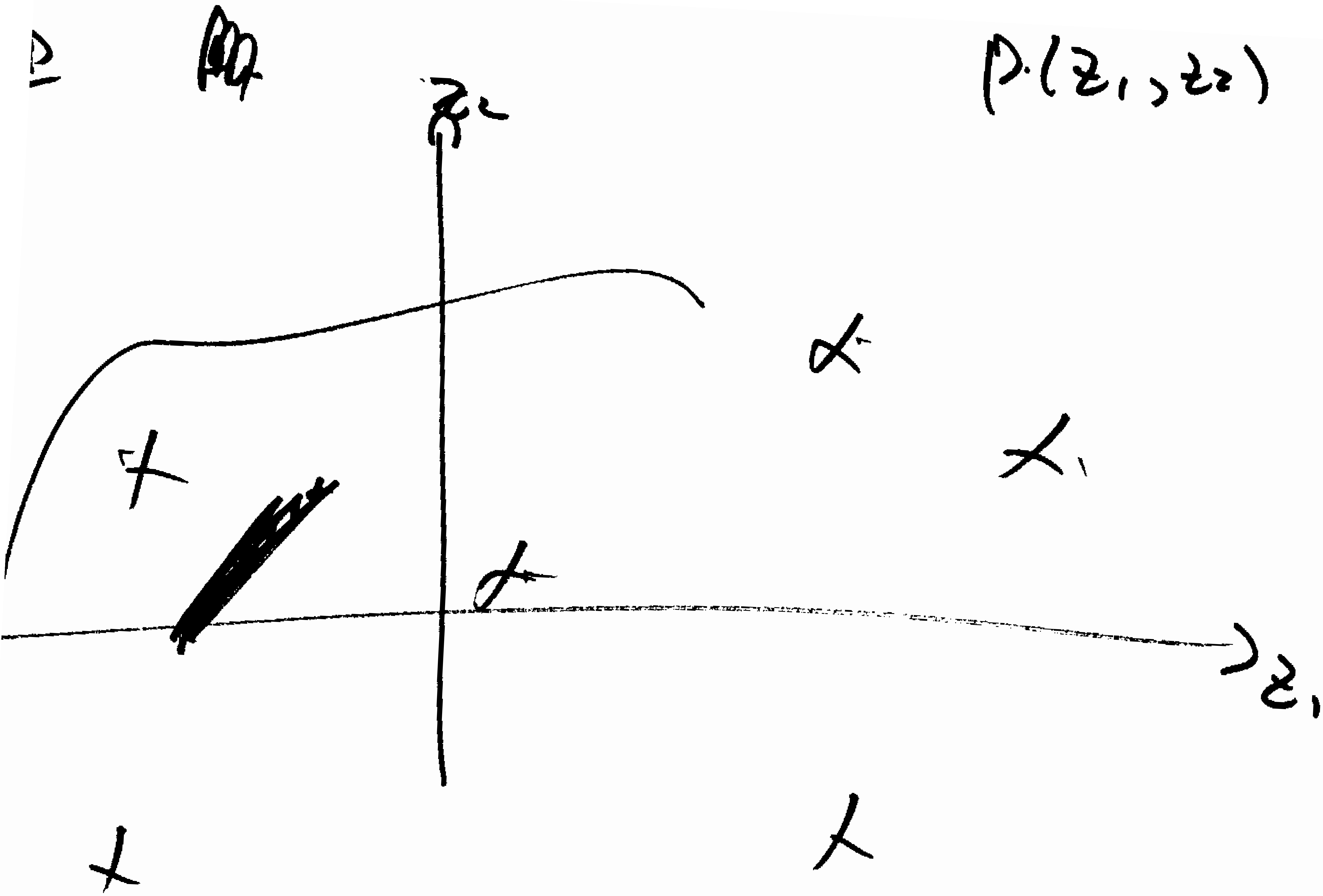
VRN $\leftarrow A\vec{x} = \vec{b}$

Fundamental Thm Alg. $\Longrightarrow$ A is never singular.

$$\begin{bmatrix} z_0^n & z_0^{n-1} & z_0^{n-2} & \cdots & z_0^2 & z_0 & 1 \\ z_1^n & z_1^{n-1} & z_1^{n-2} & \cdots & & z_1 & 1 \\ \vdots & & & & & & \\ z_n^n & z_n^{n-1} & & \cdots & & z_n & 1 \end{bmatrix}$$

$\rightarrow$ Van der Monde.

$p(z_1, z_2)$



$$\ldots_1, z_2) = \alpha z_1^2 + \beta z_2^2 + \gamma z_1 z_2 + \epsilon z_1 + \eta z_2 + \delta$$

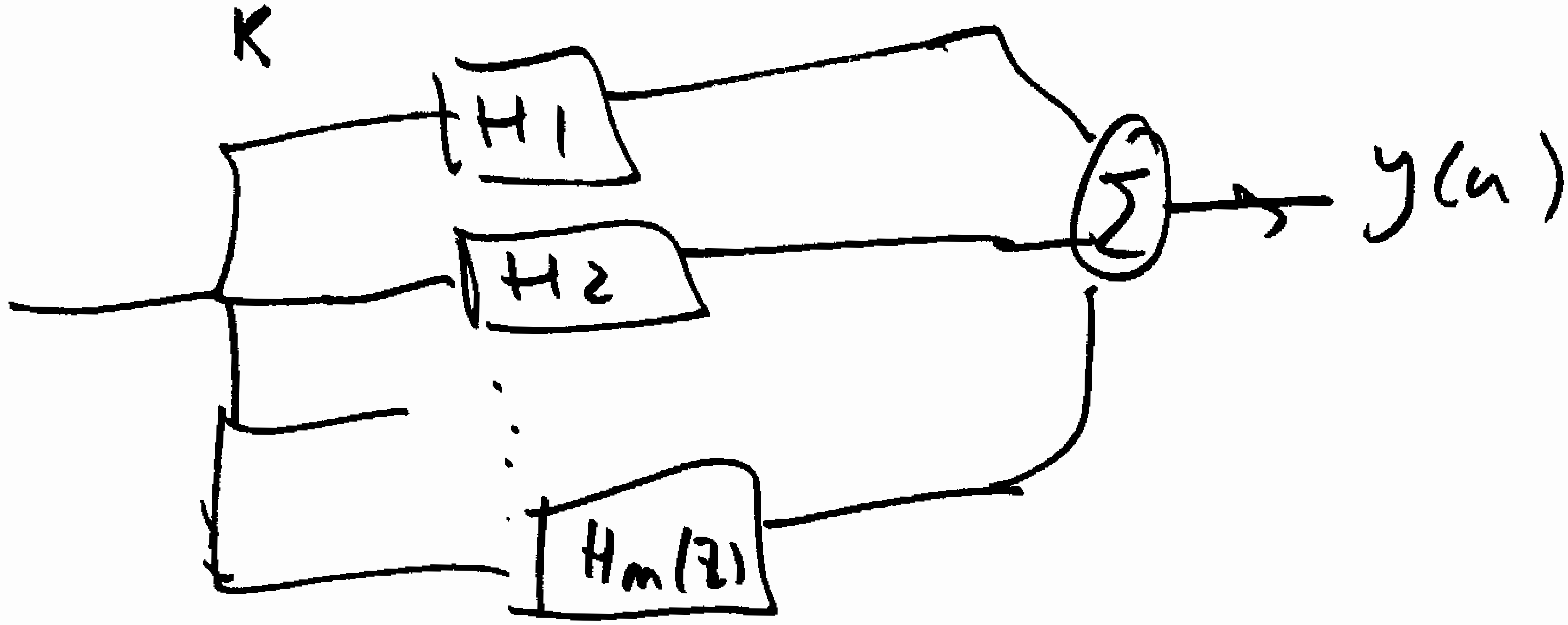
Parallel implementāt

$$\frac{\sum_{k=0}^{q} b_k z^{-k}}{1 - \sum_{k=1}^{p} a_k z^{-k}}$$

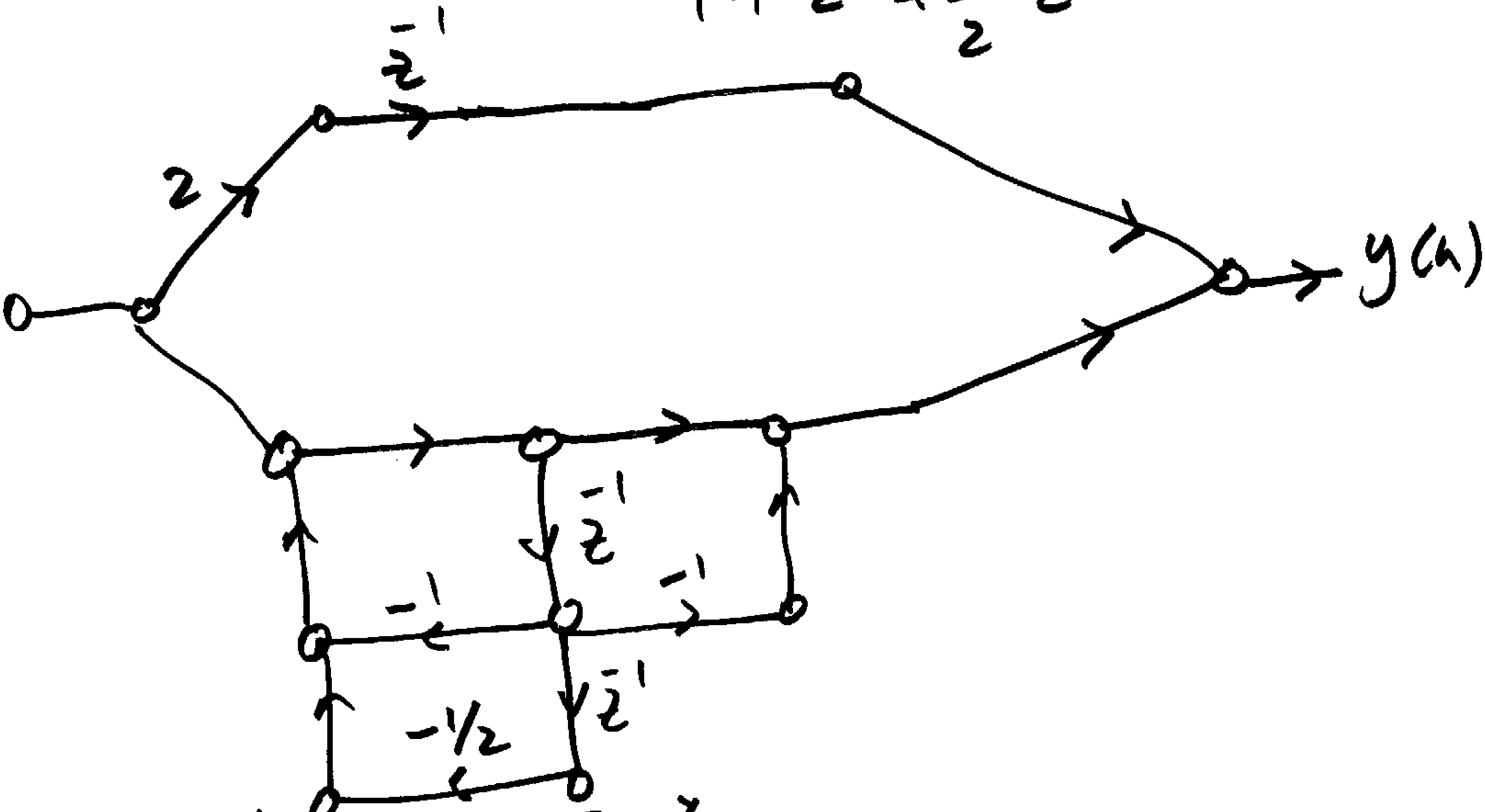
$$= \sum_k A_k z^{-k} + \sum_k \frac{B_k}{1 - g_k z^{-1}} + \sum_k \frac{C_k + D_k z^{-1}}{1 - h_{1k} z^{-1} - h_{2k} z^{-2}}$$

real real real.

$$= \sum_K H_k(z)$$

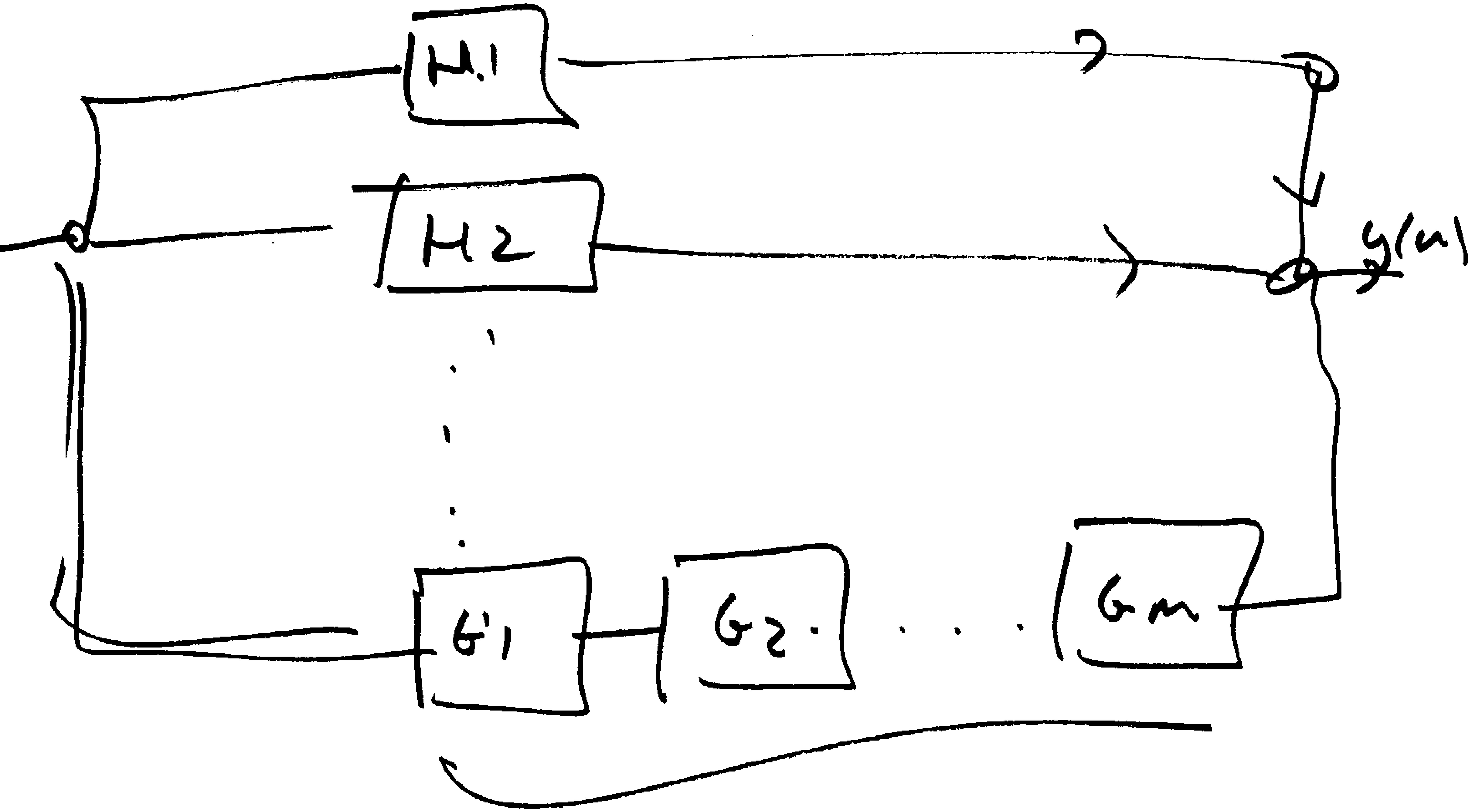


$) = 2z^{-1} + \dfrac{1-z^{-1}}{1+z^{-1}+\frac{1}{2}z^{-2}}$



Cascade/parallel → Trellis

$) = \sum_k H_k(z) + \prod_K G_k(z)$


H1
H2
y(n)
G1
G2
Gm