

Nov. 22/03

DCT & its relation to DFT

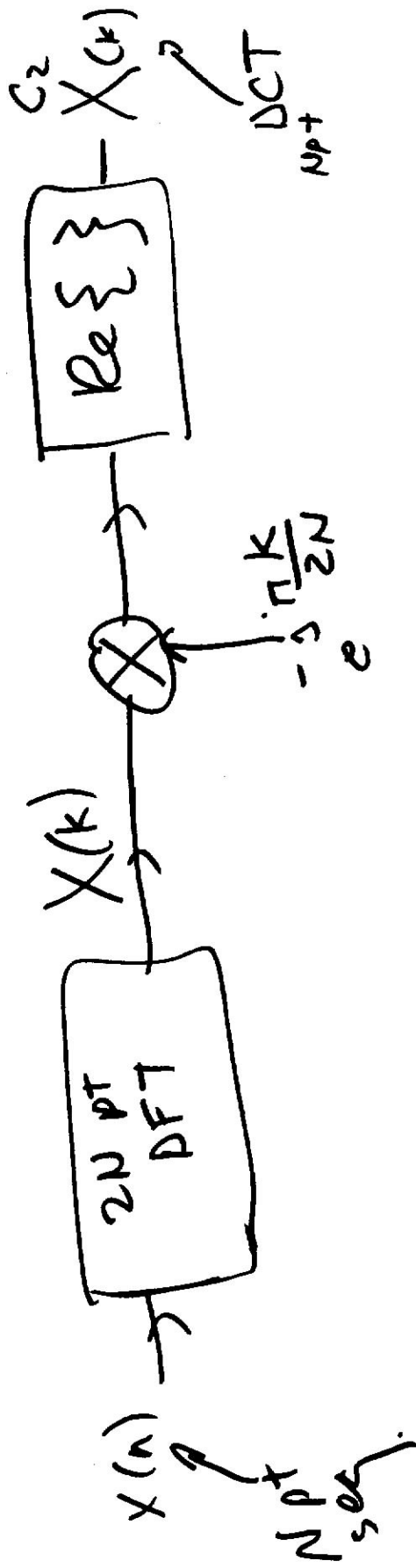
Proposal 1: $x(n)$ N PT sequence, assume it is real
① $x(n)$ N PT DFT $\rightarrow X(k)$
② Take $2N$ PT DFT $\rightarrow X(k)$
③ $2 \operatorname{Re} \{ X(k) e^{-j \frac{\pi k}{2N}} \} \stackrel{C_2}{=} X(k)$

Proof: $X(k) = \sum_{n=0}^{N-1} x(n) e^{-j \frac{2\pi nk}{2N}}$

$$= \sum_{n=0}^{N-1} x(n) e^{-j \frac{\pi k(2n+1)}{2N}}$$

$$X(k) e^{-j \frac{\pi k}{2N}} = \sum_{n=0}^{N-1} x(n) \cos \frac{\pi k(2n+1)}{2N}$$

$$2 \operatorname{Re} \left\{ X(k) e^{-j \frac{\pi k}{2N}} \right\} = 2 \sum_{n=0}^{N-1} x(n) \cos \frac{\pi k(2n+1)}{2N} \stackrel{C_2}{=} X(k) \Rightarrow \text{OED.}$$



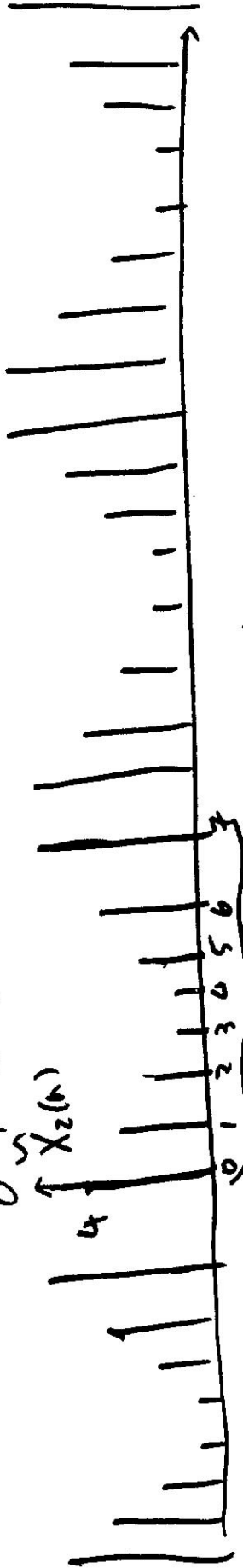
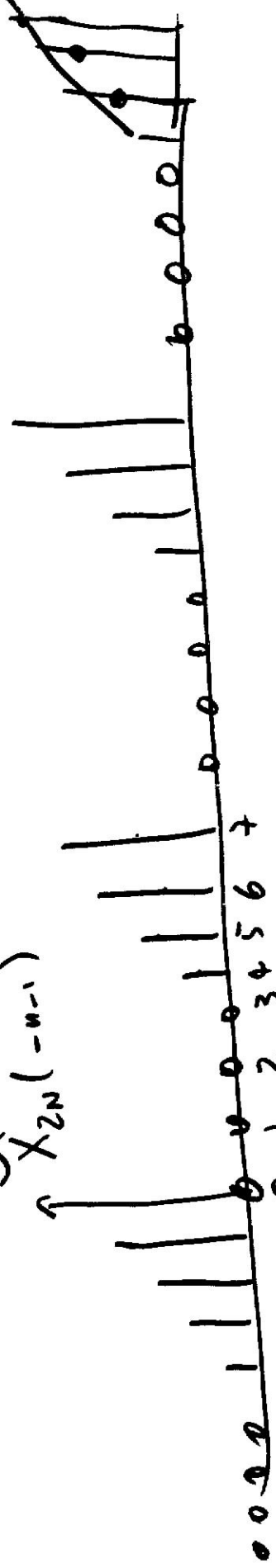
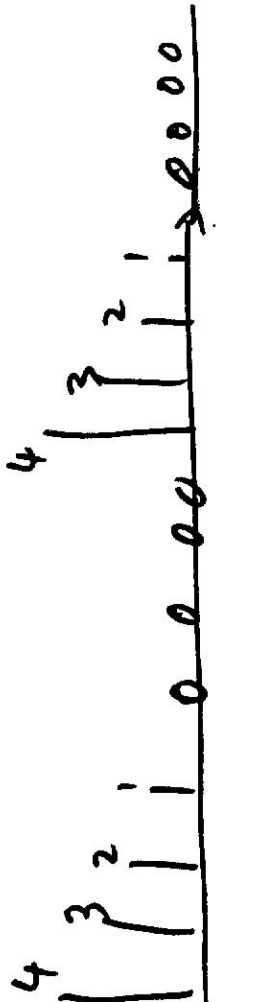
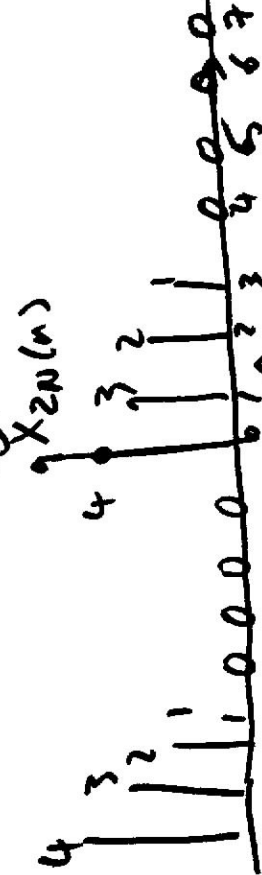
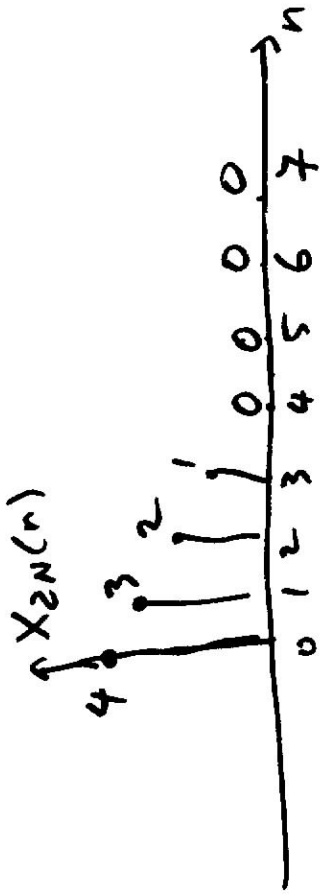
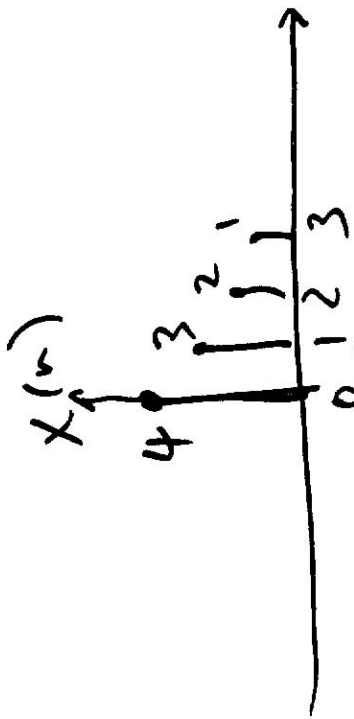
Alter: Relating DFT To DCT:

Proposed:

1. Start with N pt real sequence $x(n)$.
2. Pad it with N zeros $\rightarrow X_{2N}(n)$
3. Form a periodic sequence $X_2(n) = X_{2N}(n) + X_{2N}(-n-1)$

4. Take $2N$ PT DFT of one period of $X_2(n) \rightarrow X_2(k)$

5. Relate $X_2(k)$ to $X_1^{(L)}(k)$.



2NPT i.e. 8PT DFT.

Q How is $X_2(k)$ related to $X^{C_2}(k)$?

Let $X(k) = 2N$ pt DFT of $x(n)$.

~~$X_2(k) =$~~

$$\begin{aligned}
 X_2(k) &= X(k) + e^{j\frac{2\pi k}{2N}} X^*(k) \\
 &= e^{j\frac{\pi k}{2N}} \left[X(k) e^{-j\frac{\pi k}{2N}} + X^*(k) e^{j\frac{\pi k}{2N}} \right] \\
 X_2(k) &= e^{j\frac{\pi k}{2N}} \{ 2 \operatorname{Re} \{ X(k) e^{-j\frac{\pi k}{2N}} \} \}
 \end{aligned}$$

$X^{C_2}(k) \leftarrow$ proposed

$$X_2(k) e^{-j\frac{\pi k}{2N}} = X^{C_2}(k)$$

Fig 8-29

FFT = Fast Fourier Transform

+ merely a way of computing DFT faster.

not a new Transform.

Decimation in Time.

+ Two flavors

Decimation in freq.

+ Motivation

Real life sys \rightarrow LTI \rightarrow Convolution \rightarrow DFT \rightarrow FFT

DFT: $X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N}$ $0 \leq k < N$

Direct Computation :

For each k :

N values of k

\Rightarrow # add N^2
multi N^2

$N(N-1) \approx N^2$

$O(N^2)$

$(N-1)$ adds (complex)
 N complex multi

FFT

$N \log_2 N$

$N=10^6$

Direct comput $O(10^{12})$

FFT $\Rightarrow 10^6 \log_2 10^6 \approx 2 \times 10^7$

5 orders of magnitude