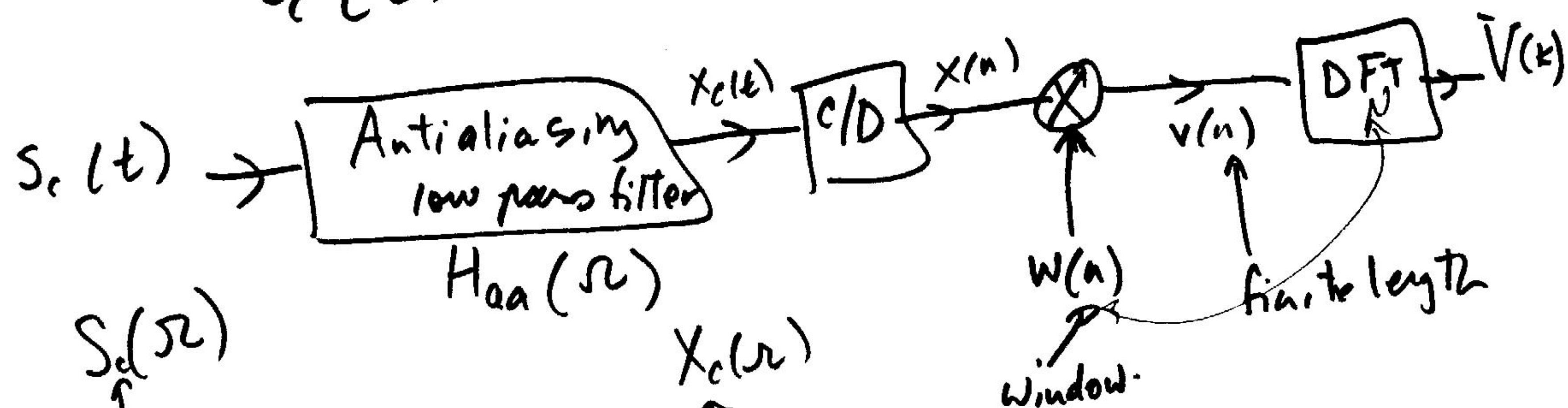
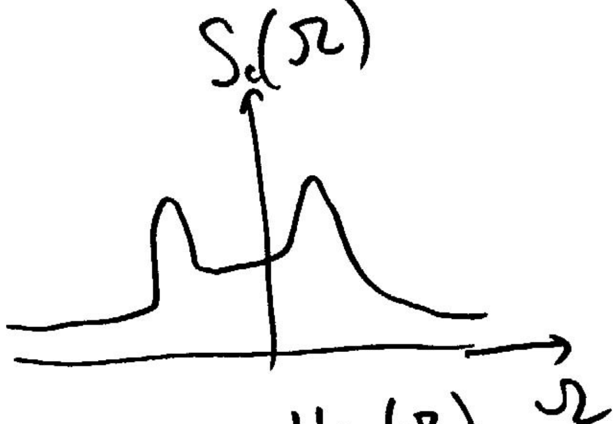
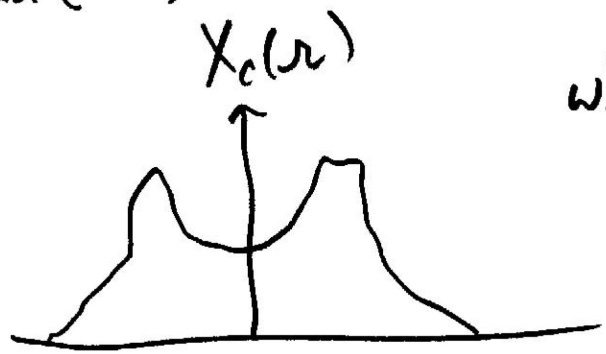
Dec 3, 03

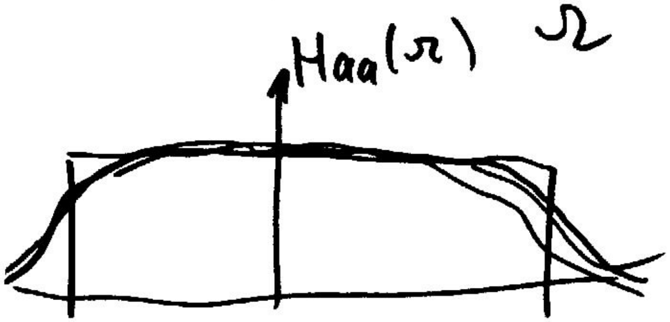
Fourier Analysis of Signals using DFT

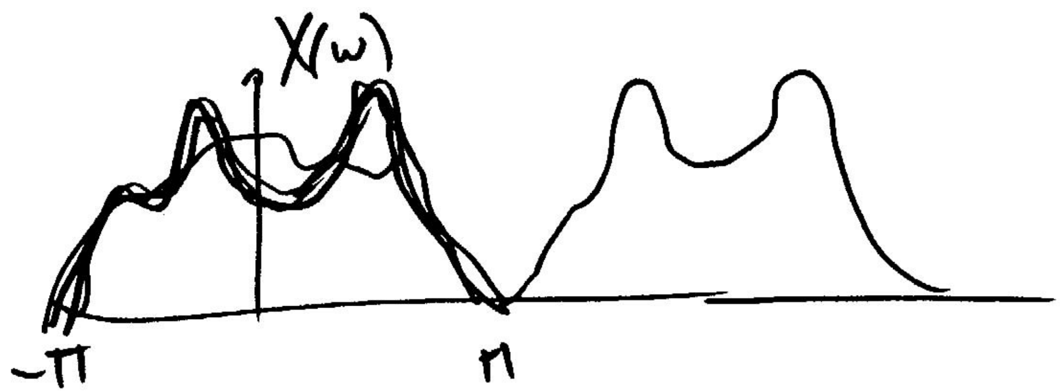
Given analog continuous time signal $s_c(t)$



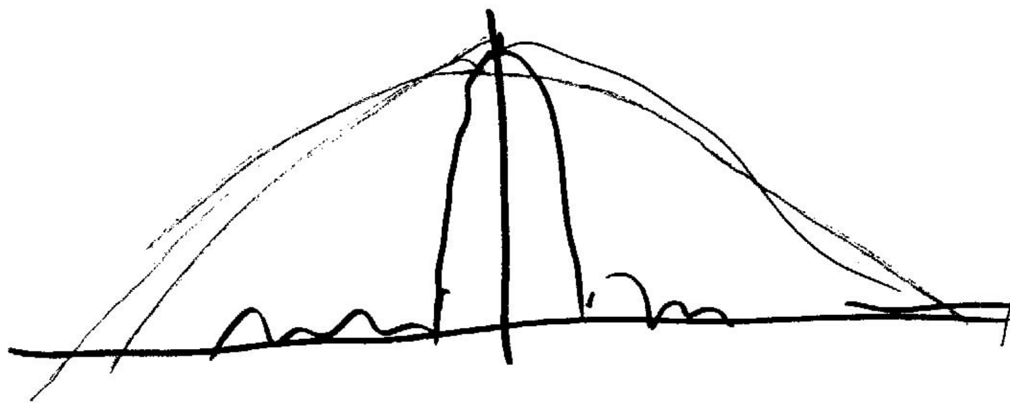


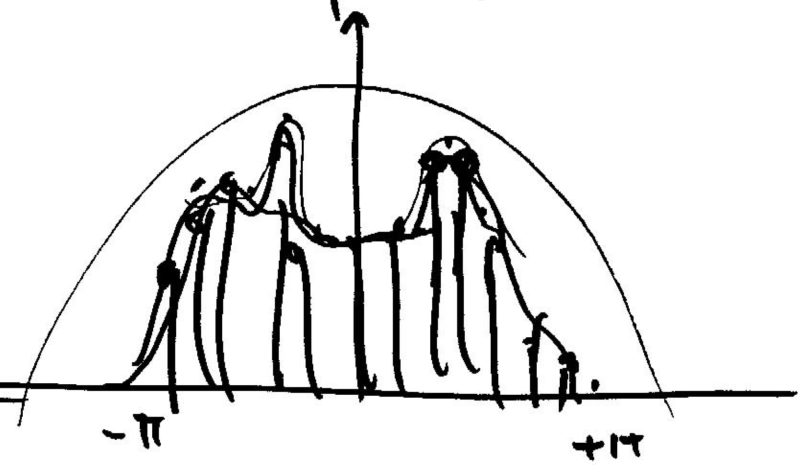






$\text{F.T.}\{w(n)\} = |W(\omega)|$





Sum of 2 sinusoids

- $S_c(t) = A_0 \cos(\Omega_0 t + \theta_0) + A_1 \cos(\Omega_1 t + \theta_1)$

  Sample, no aliasing, no quantization:

- $x(n) = A_0 \cos(\omega_0 n + \theta_0) + A_1 \cos(\omega_1 n + \theta_1)$

  $\omega_0 = \Omega_0 T \qquad \omega_1 = \Omega_1 T$

  Window $x(n)$ with window fn $w(n)$

- $v(n) = A_0 w(n) \cos(\omega_0 n + \theta_0) + A_1 w(n) \cos(\omega_1 n + \theta_1)$

  $$v(n) = \frac{A_0}{2} w(n) e^{j\theta_0} e^{j\omega_0 n} + \frac{A_0}{2} w(n) e^{-j\theta_0} e^{-j\omega_0 n} + \frac{A_1}{2} w(n) e^{j\theta_1} e^{j\omega_1 n} + \frac{A_1}{2} w(n) e^{-j\theta_1} e^{-j\omega_1 n}$$

- $V(\omega) =$

$$V(\omega) = \frac{A_0}{2} e^{j\theta_0} W(\omega - \omega_0) + \frac{A_0}{2} e^{-j\theta_0} W(\omega + \omega_0)$$
$$+ \frac{A_1}{2} e^{j\theta_1} W(\omega - \omega_1) + \frac{A_1}{2} e^{j\theta_1} W(\omega + \omega_1)$$

__Plan__ $A_0 = 1$ $A_1 = 0.75$

sampling rate $1/T = 10$ kHz.

$\theta_0 = \theta_1 = 0$

length of rectangular $= 64$

__Exp__

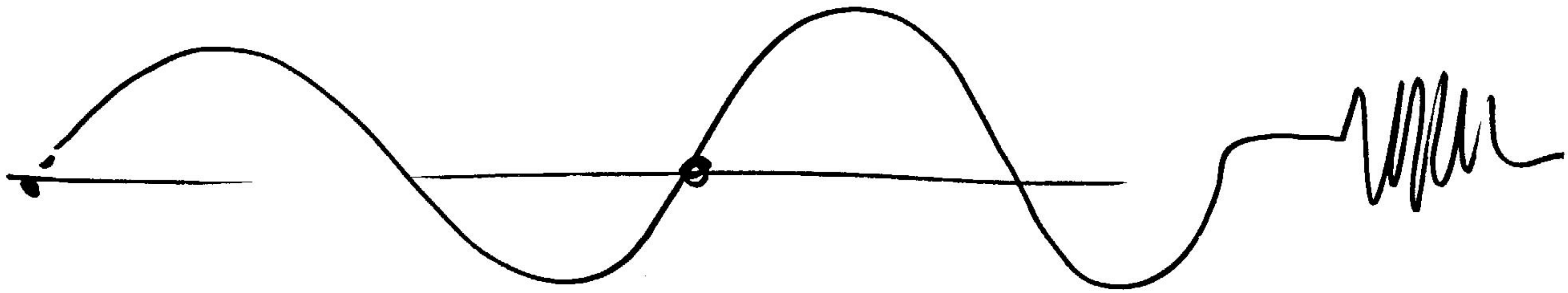
Distance $\Omega_1 - \Omega_0$ progressively smaller.

look at $|V(\omega)|$

The short Term Fourier Trafom.

C.T. short Term F.T.

$$\underset{\text{of } x(t)}{STFT}\{\tau, f\} = \int_{-\infty}^{+\infty} x(t)\, g^{*}(t-\tau)\, e^{-j2\pi f t}\, dt$$



<u>Problem</u> duration of g:

if extent g is Too large $\rightarrow$ lose Temporal resolution

If extent g is Too small $\rightarrow$ lose spectral/freq. resolution

$$g(t) \longrightarrow G(f) \quad \text{window fn:}$$

$$(\Delta f)^2 = \frac{\int f^2 |G(f)|^2 \, df}{\int |G(f)|^2 \, df}$$

for a given window $g(t) \Longleftrightarrow G(f)$,
Two sinusoids are discriminated if they are
more than Δf apart.

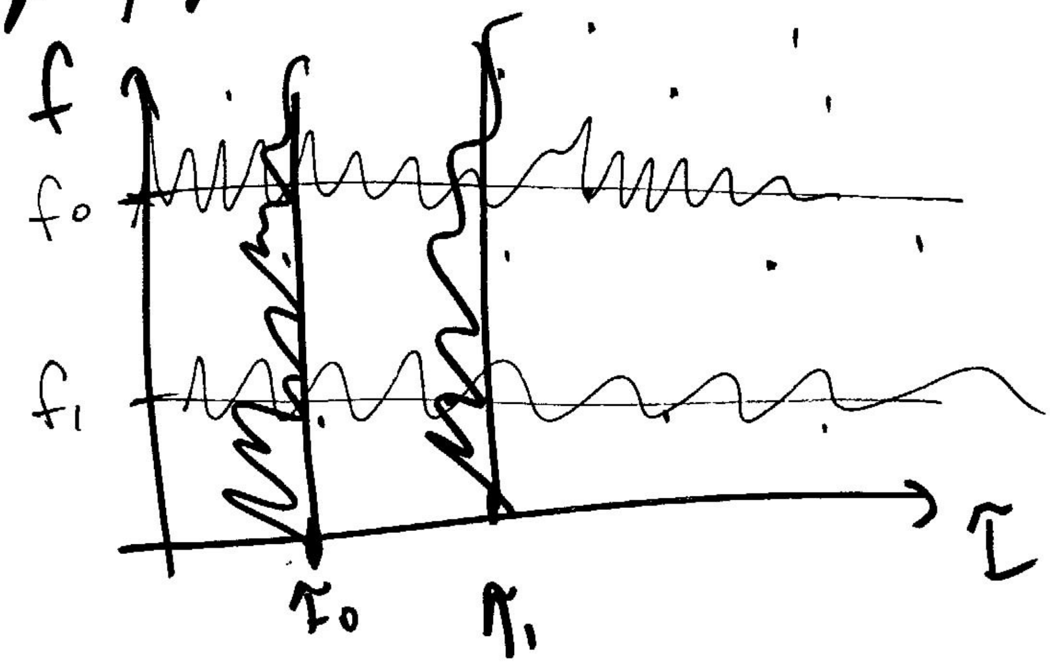
$$(\Delta t)^2 = \frac{\int t^2 |g(t)|^2 \, dt}{\int |g(t)|^2 \, dt}$$

for a given $g(t)$, Two pulses can be discriminated if there are more than Δt apart.

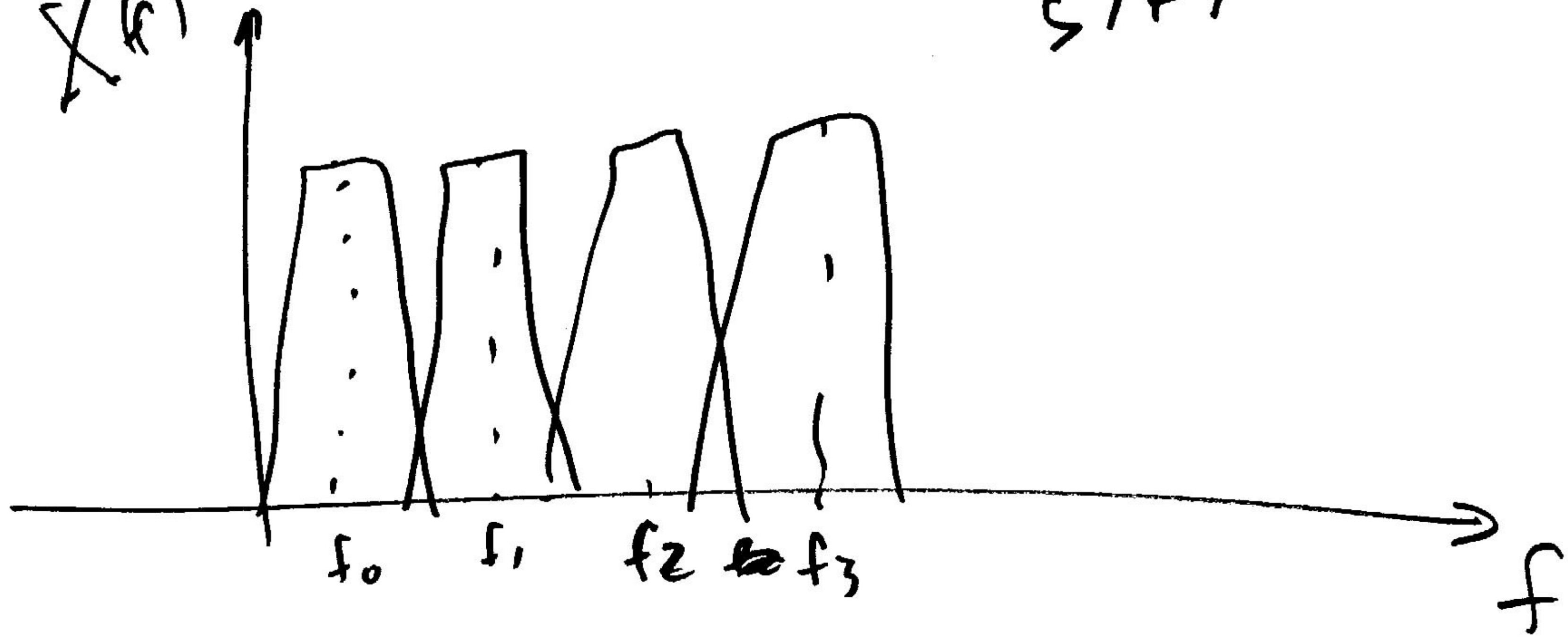
Uncertainty principle

Time-bandwidth product $\Delta t\, \Delta f \geq \frac{1}{4\pi}$

$\Longrightarrow$ Once you pick $g(t)$, Temporal / Spectral resolution is fixed for the entire Time-frequency plane



STFT



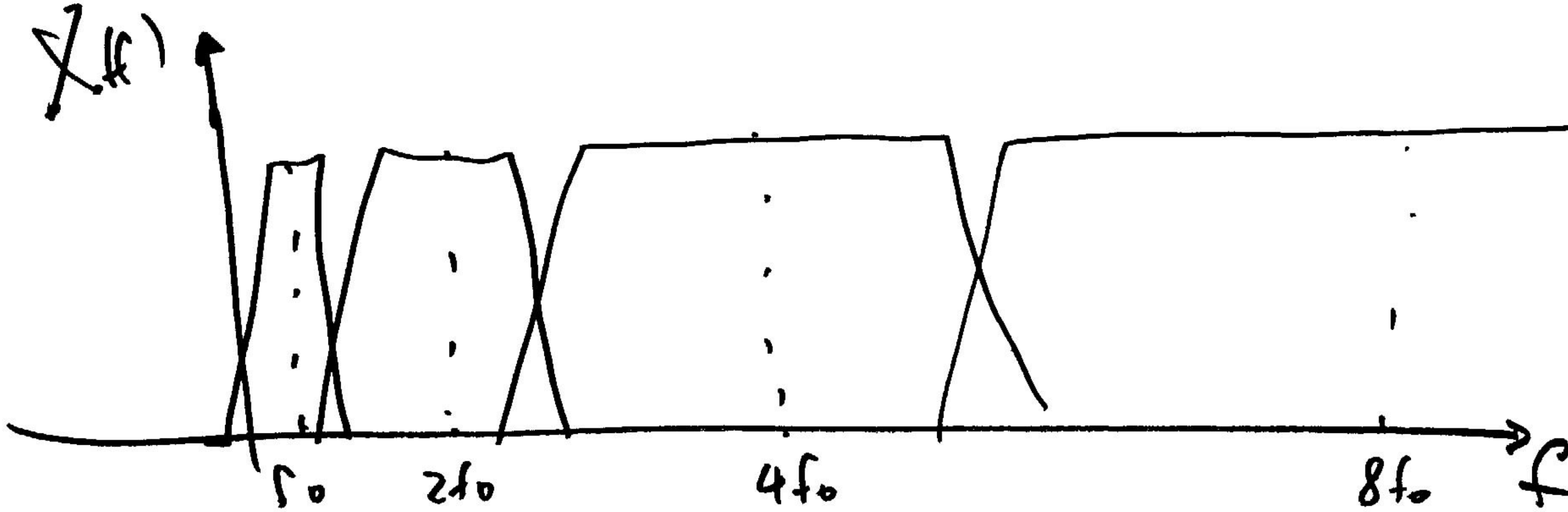
Continuous Time Wavelet Transform

Basic idea: vary Δt, Δf in T-f plane to get around STTF resolution problem

$$\frac{\Delta f}{f} \approx c$$

Δf proportional to f

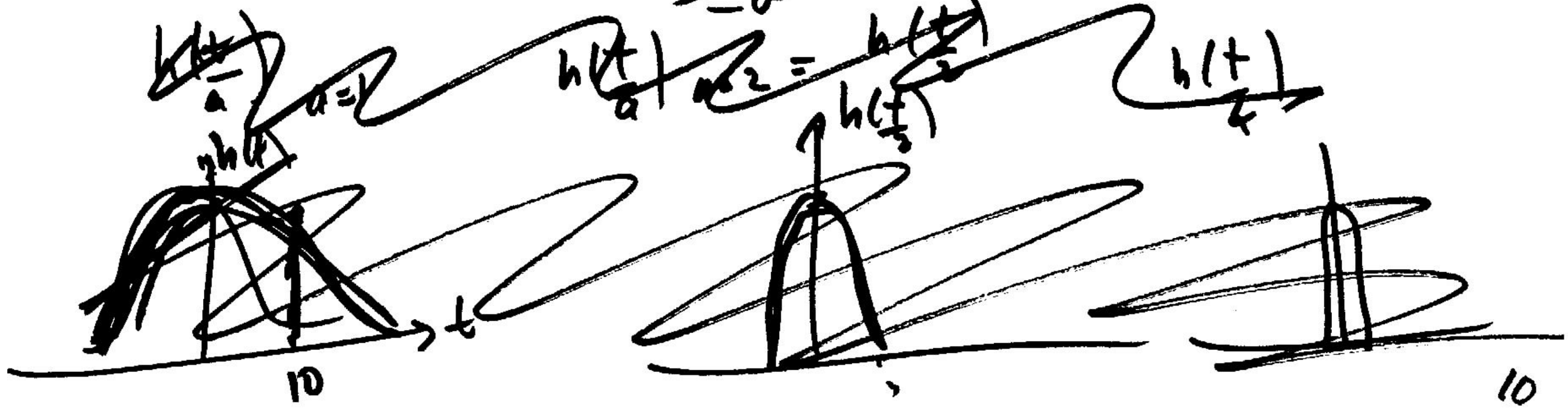
Δt proportional to $\hat{f}$

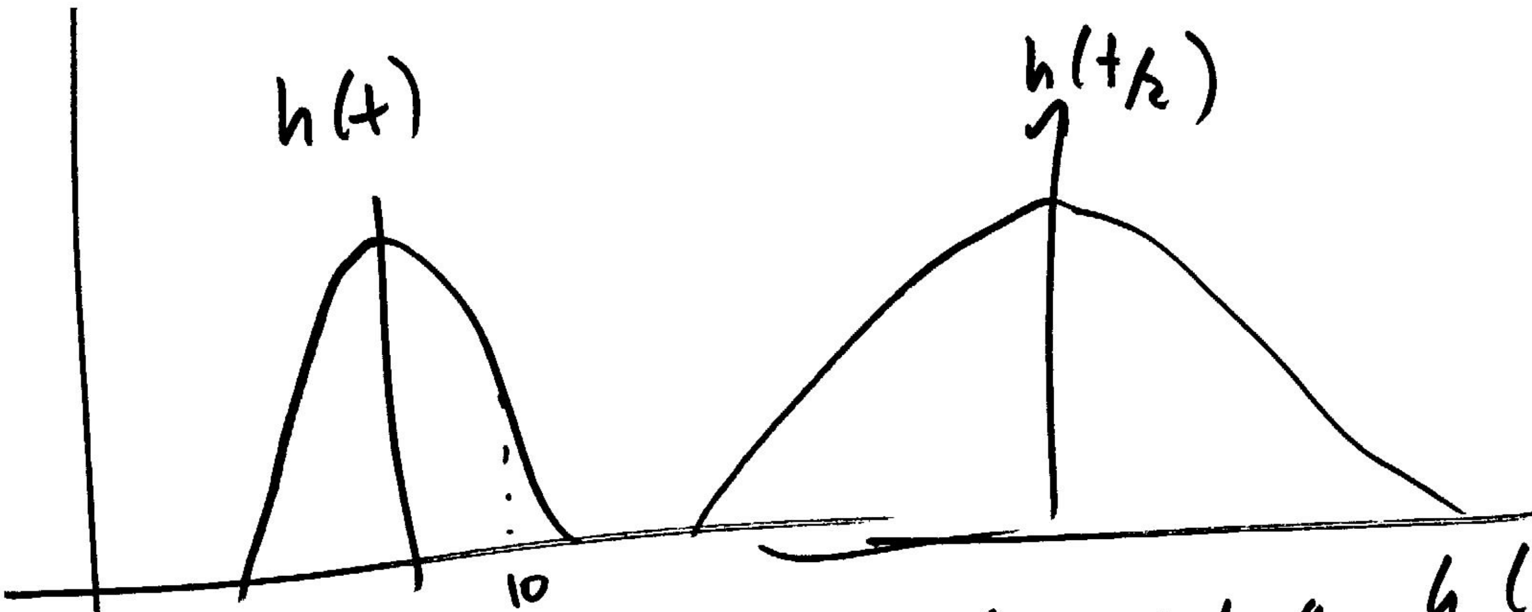


C.T.W.T. analysis & synthesis.

scale. $a \propto \frac{1}{f}$

$$CWT_x(\tau, a) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} x(t)\, h^*\left(\frac{t-\tau}{a}\right) dt$$





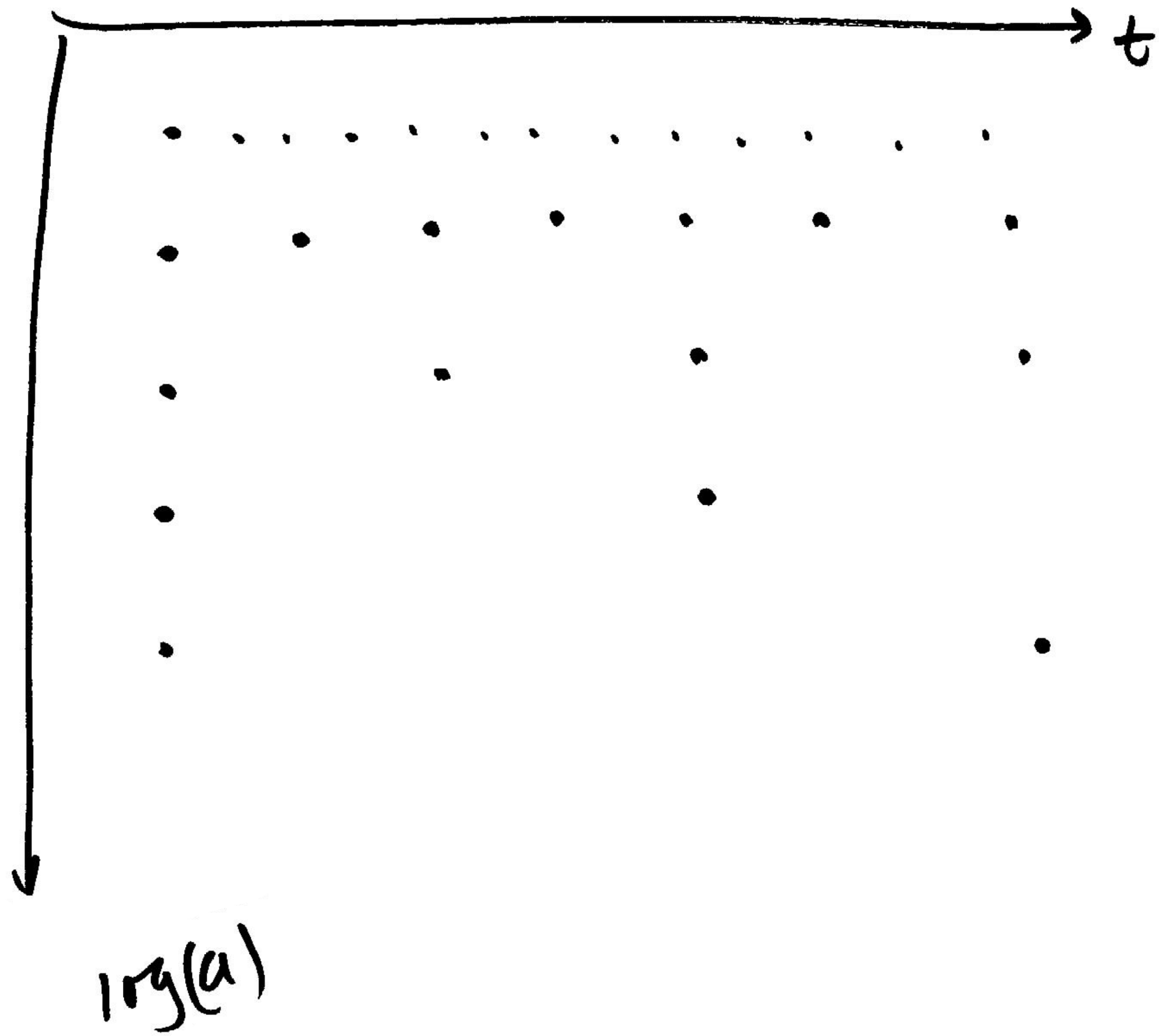
as $a \uparrow$, duration of "window" $h\left(\frac{t}{a}\right)$ goes up

$\Rightarrow$ I can do better frequency analy

$$x(t) = C \iint_{a>0} CWT_x(\tau, a) \frac{1}{\sqrt{a}} h\left(\frac{t-\tau}{a}\right) da \, d\tau$$

Scalogram (spectrogram)
CWT STFT

C.T.W.T.



sampling $h_{j,k}(t) = a_0^{-j/2} \, h(a_0^{-j} t - kT)$

wavelet coefficient.

$$c_{j,k} = \int x(t) \, h^*_{j,k}(t) \, dt$$

orthogonal wavelet base.

$$\int h_{j,k}(t) \, h^*_{j,k}(t) \, dt = \begin{cases} 1 & j = j', \; k = k' \\ 0 & \text{otherwise} \end{cases}$$

$$x(t) = \sum_{j,k} c_{jk} \, h_{j,k}(t)$$