Oct 5, 2005

# Realizations of IIR filters with Rational Transfer function

$$H(z) = \frac{\sum_{k=0}^{q} b_k z^{-k}}{1 - \sum_{k=1}^{p} a_k z^{-k}}$$

1. Direct Form 1
2. " " 2
3. Cascade structure
4. Parallel structure

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# Fundamental Thm of Algebra

Any polynomial in <u>one variable</u> can be factored into simple (1st order) terms.

Polynomial of degree $n$, has $n$ real or complex roots. Can be factored into $n$ terms.

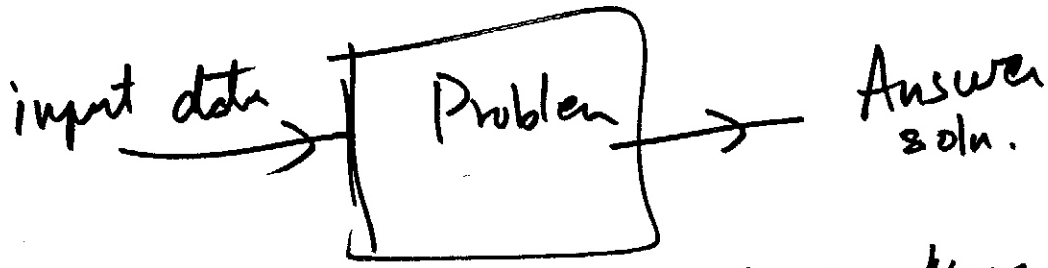
<u>Ex</u> 3rd order (degree) polynomial.

$$p(z) = \alpha z^3 + \beta z^2 + \gamma z + \delta$$

$$= K \underbrace{(z - z_0)}_{\text{root 1}} \underbrace{(z - z_1)}_{\text{root 2}} \underbrace{(z - z_2)}_{\text{root \#3}}$$

ILL Conditioned → Problem
ILL Conditioned → Algorithm.

Problem:




Ex: linear syst of Eqns. 2 equations in 2 unknowns

$$\left\{\begin{array}{l} 5x + 3y = 2 \\ 9x + 20y = 15 \end{array}\right\} \quad x, y.$$

$$A \quad x = b$$

ALG: Many diff. algorithms can be used to solve the same problem

1. Cramer's rule.
2. Gaussian elimination → GOOD with pivot; → BAD without
3. Invert matrix. ← ALG BAD
4. QR Decomposition
5. Gauss Seidl. 3

Problem is ill conditioned: if small perturbation in the input results in large change in the output.

well conditioned prob $\begin{bmatrix} 5 & 3 \\ 9 & 20 \end{bmatrix}$ $\begin{bmatrix} 2 \\ 15 \end{bmatrix}$

only dependent on the data in the problem itself not on the Alg used to solve it

Terribly

$\begin{bmatrix} 5 & 3 \\ 10 & 6 \end{bmatrix}$ $\begin{bmatrix} 5 & 3 \\ 10 & 5.9999 \end{bmatrix}$

det = 0 ill cond. | many soln

a little ill cond.

Condition # of a problem: Ratio between Relative change in output over relative change in input.

Condition # of problem is large $\Longrightarrow$ ill conditioned

" " " " " small (close to 1) $\Longrightarrow$ well conditioned

- Problem itself can be <u>inherently</u> either ill conditioned or well conditioned

~~If problem ill conditioned → good or bad~~

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<u>Condition # of the Alg</u>

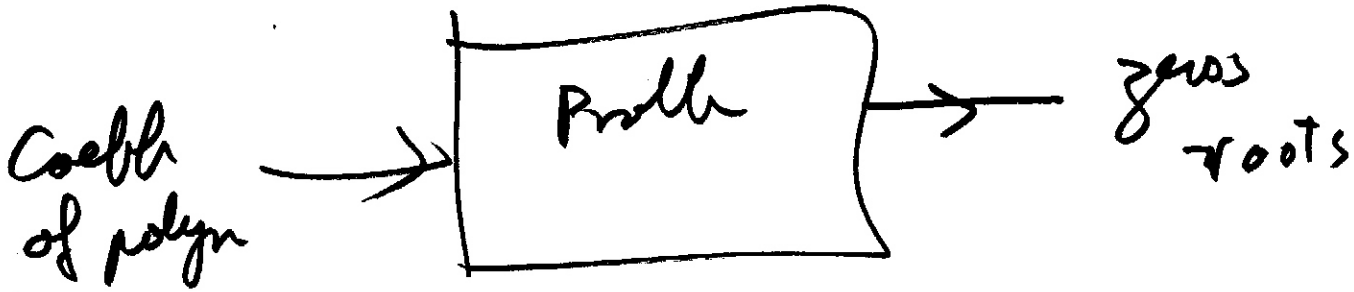
relative change in output over relative change in the input using that <u>alg</u> to compute input/output.

If problem is <u>well conditioned</u>, an ill conditioned alg could give <u>Bad</u> results.

But a <u>well Conditioned Alg</u> give good results

~~If Pr~~

<u>Statement</u>: Problem of finding the roots of a polynomial is ill conditioned.




$$\alpha z^3 + \beta z^2 + \gamma z + \delta$$

- small perturbation in $\alpha, \beta, \gamma, \delta$ change the roots a lot.

- Problem is worse for higher deg. polynomials.

$$H(z) = \frac{P(z)}{Q(z)}$$

$\Longrightarrow$ Direct Form 1 & 2 since they use coeff of poly in their implementation.

The zeros and poles of the system are very sensitive to round off error.

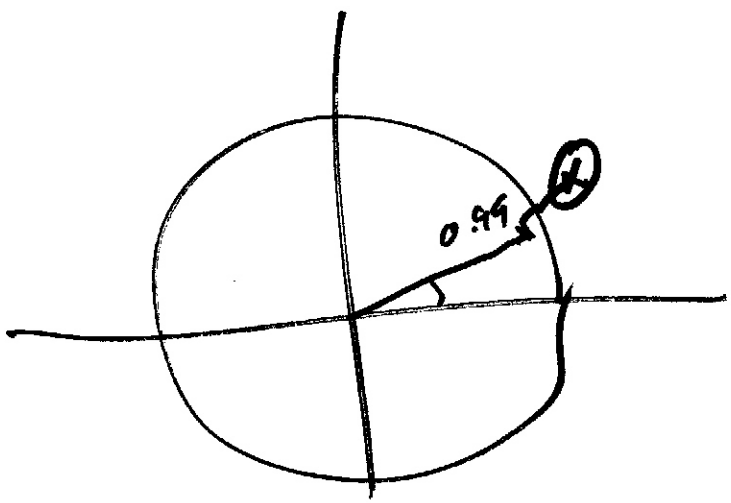
→ Finite precision in coeff representation

→ Finite precision in calculation.

↓

motivation

Cascade + Parallel.



Fundamental Thm of Algebra <u>does not</u> hold in 2 or 3 or higher # of variables..

$$P(z_1, z_2) = \alpha z_1^2 + \beta z_2^2 + \gamma z_1 z_2 + \delta + \epsilon z_2 + \eta z_2$$

<u>Thm</u>: set of factorable polynomials in 2 or more variable is of measure zero in the set of polynomials.

$\Longrightarrow$ <u>Implication</u>: (1) cannot check stability of 2D. IIR filters easily.

(2) Cannot come up with cascade structure for 2D-IIR.