EE 123 DIGITAL SIGNAL PROCESSING, Spring 2009
Homework # 7, Due April 2, Thursday

1. Consider the generalized finite-length transform:

$$A[k] = \sum_{n=0}^{N-1} x[n]\phi_k[n]$$

where $\phi_k[n]$, $k = 0, \cdots, N-1$ are orthogonal base sequences.

a) Prove the Parseval Theorem:

$$\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N} \sum_{k=0}^{N-1} a[k]\, |A[k]|^2$$

where

$$a[k] := \frac{N}{\sum_{n=0}^{N-1} \phi_k[n]\phi_k^*[n]}.$$

b) Apply the result of part (a) to obtain Parseval Theorems for DFT, Type-1 DCT, and Type-2 DCT.

2. The MATLAB function `dct` calculates the following normalized variant of the Type-2 DCT:

$$X_{\text{norm}}^{c2} = \sqrt{\frac{2}{N}} B[k] \sum_{n=0}^{N-1} x[n] \cos\left(\frac{\pi k(2n+1)}{2N}\right) \quad k = 0, \cdots, N-1,$$

where

$$B[k] = \begin{cases} \frac{1}{\sqrt{2}} & k = 0 \\ 1 & k = 1, \cdots, N-1. \end{cases}$$

a) Find the synthesis equation based on $X_{\text{norm}}^{c2}$.

b) Obtain the Parseval Theorem for this variant of Type-2 DCT.

c) Use MATLAB to reproduce Figure 8.28 in Oppenheim and Schafer, 2nd ed. (Remember to obtain the *unnormalized* Type-2 DCT coefficients used in the book from the normalized ones that MATLAB calculates.)

d) Write MATLAB programs to calculate the error functions $E^{\text{dft}}[m]$ and $E^{\text{dct}}[m]$ as defined on pages 597-598 of Oppenheim and Schafer, 2nd ed. Then use your programs to reproduce Figure 8.29.

3. Problem 9.26, Oppenheim and Schafer, 2nd ed.

4. Problem 9.27, Oppenheim and Schafer, 2nd ed.

5. Problem 9.29, Oppenheim and Schafer, 2nd ed.