EE123
Digital Signal Processing

Lecture 10
Time-Dependent FT

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Announcements

- Midterm: 02/22/2016
  - Open everything
  - ... but cheat sheet recommended instead
  - 10am-12pm

- How's the lab going?

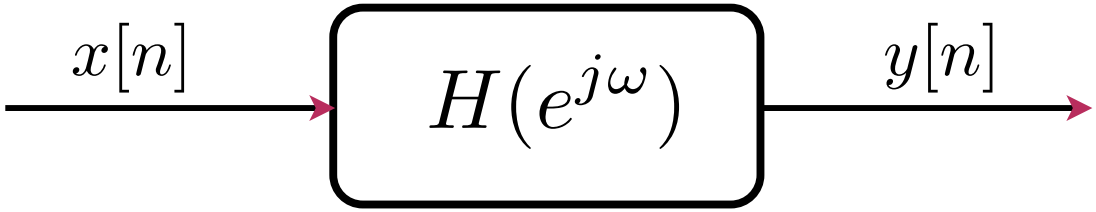
Frequency Analysis with DFT

- Length of window determines spectral resolution
- Type of window determines side-lobe amplitude. (Some windows have better tradeoff between resolution-sidelobe)
- Zero-padding approximates the DTFT better. Does not introduce new information!

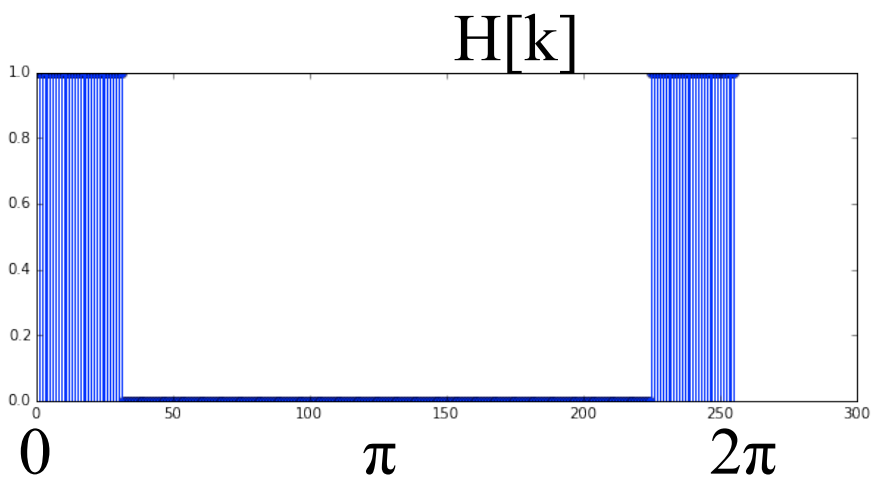
Potential Problems and Solutions

Problem	Possible Solutions
1. Spectral error from aliasing Ch.4	a. Filter signal to reduce frequency content above $\Omega_s/2 = \pi/T$. b. Increase sampling frequency $\Omega_s = 2\pi/T$.
2. Insufficient frequency resolution.	a. Increase L b. Use window having narrow main lobe.
3. Spectral error from leakage	a. Use window having low side lobes. b. Increase L
4. Missing features due to spectral sampling.	a. Increase L, b. Increase N by zero-padding $v[n]$ to length $N > L$.

Subtleties in filtering/processing with DFT

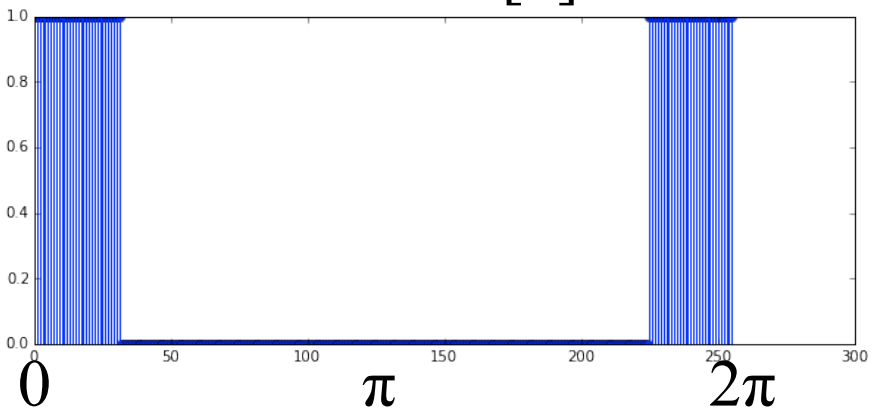


- System is implemented by overlap-and-save
- Filtering using DFT

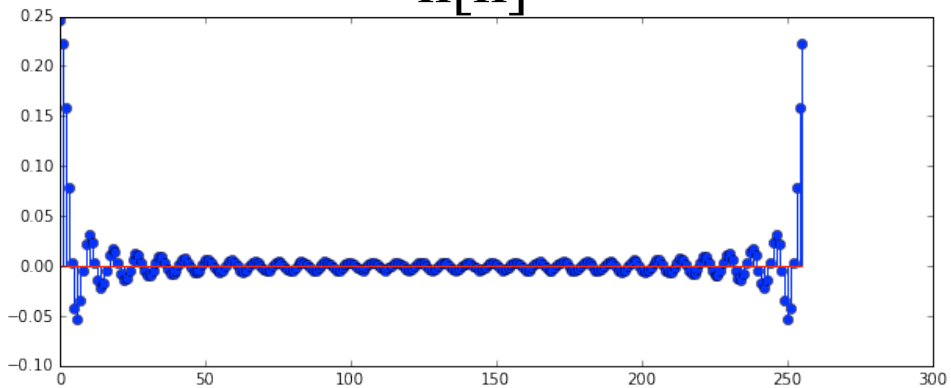


Subtleties in filtering/processing with DFT

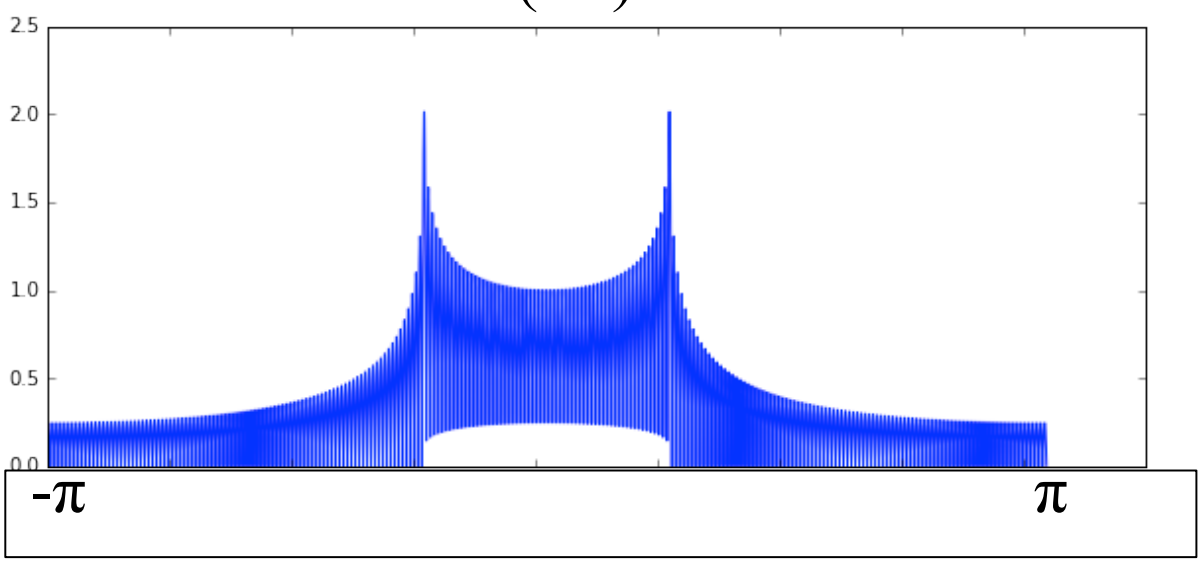
H[k]



h[n]



$H(e^{jw})$



Last Time

- Frequency Analysis with DFT
- Windowing
- Zero-Padding

- Today:
  - Time-Dependent Fourier Transform
  - Heisenberg Boxes

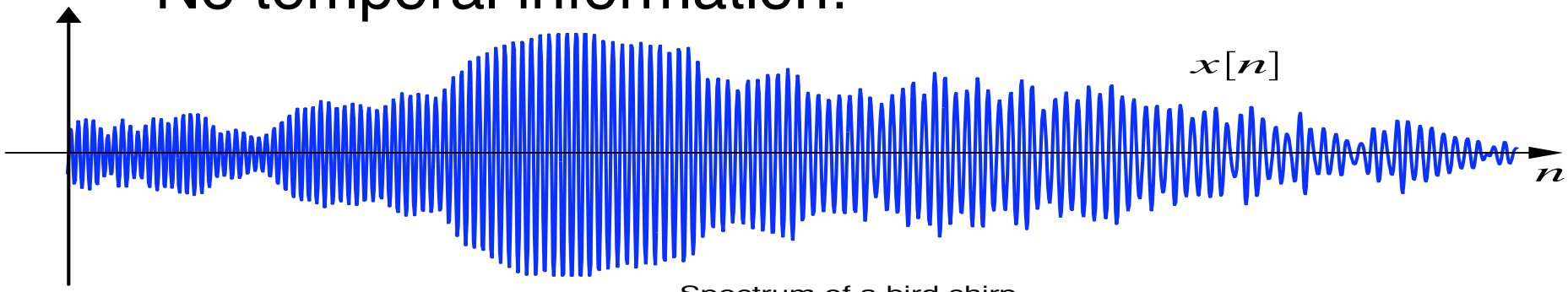
Discrete Transforms (Finite)

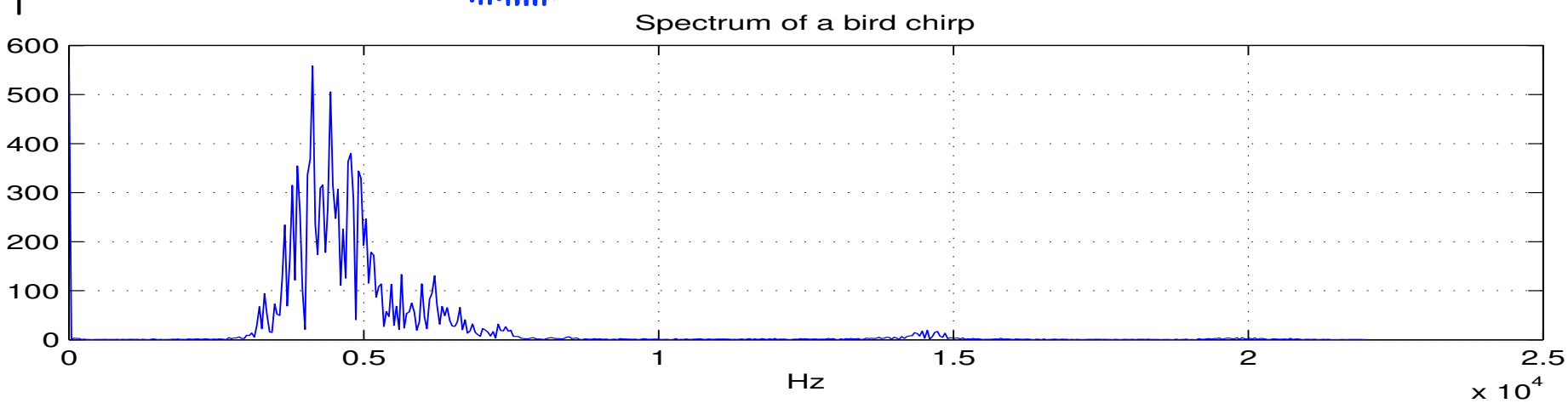
- DFT is only one out of a LARGE class of transforms
- Used for:
  - Analysis
  - Compression
  - Denoising
  - Detection
  - Recognition
  - Approximation (Sparse)

Sparse representation has been one of the hottest research topics in the last 15 years in sp

Example of spectral analysis

- Spectrum of a bird chirping
  - Interesting,.... but...
  - Does not tell the whole story
  - No temporal information!





Time Dependent Fourier Transform

- To get temporal information, use part of the signal around every time point

$$X[n,\omega) = \sum_{m=-\infty}^{\infty} x[n+m]w[m]e^{-j\omega m}$$

*Also called Short-time Fourier Transform (STFT)

- Mapping from 1D ⇒ 2D, n discrete, *w* cont.
- Simply slide a window and compute DTFT

Time Dependent Fourier Transform

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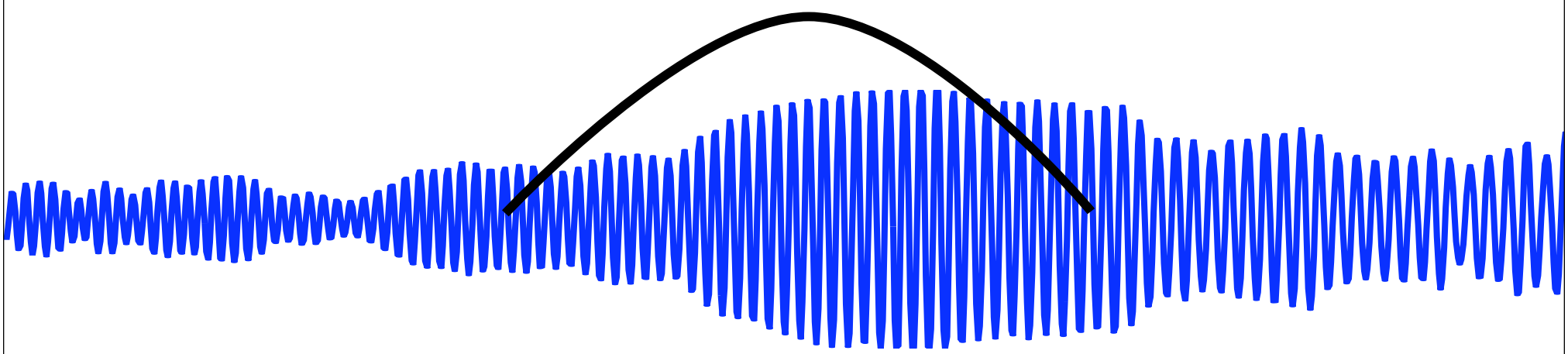
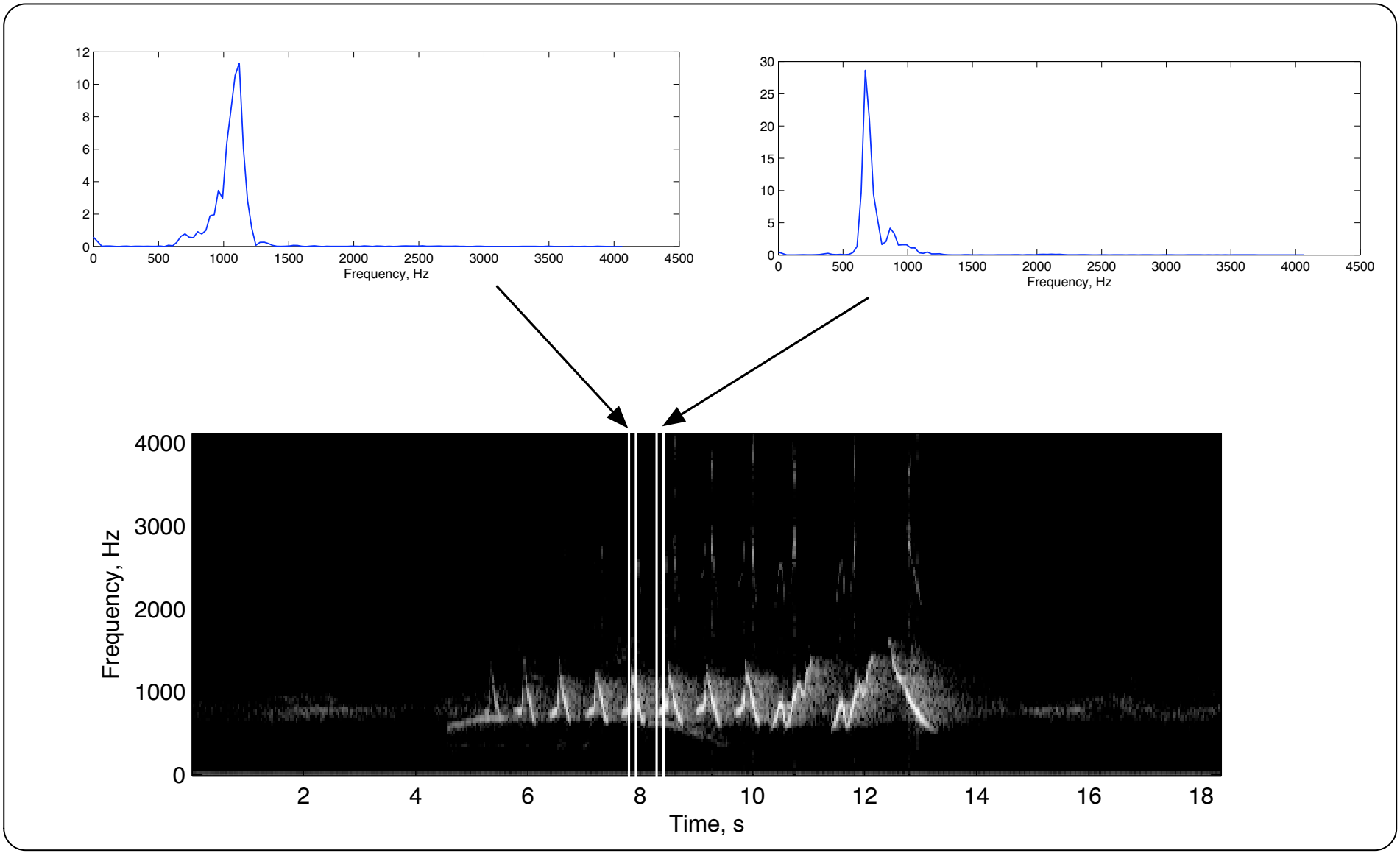


Spectrogram



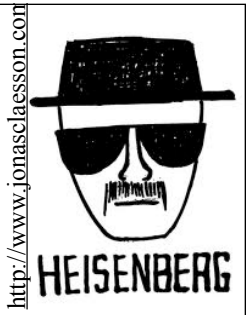
Discrete Time Dependent FT

$$X_r[k] = \sum_{m=0}^{L-1} x[rR+m]w[m]e^{-j2\pi km/N}$$

- L - Window length
- R - Jump of samples
- N - DFT length

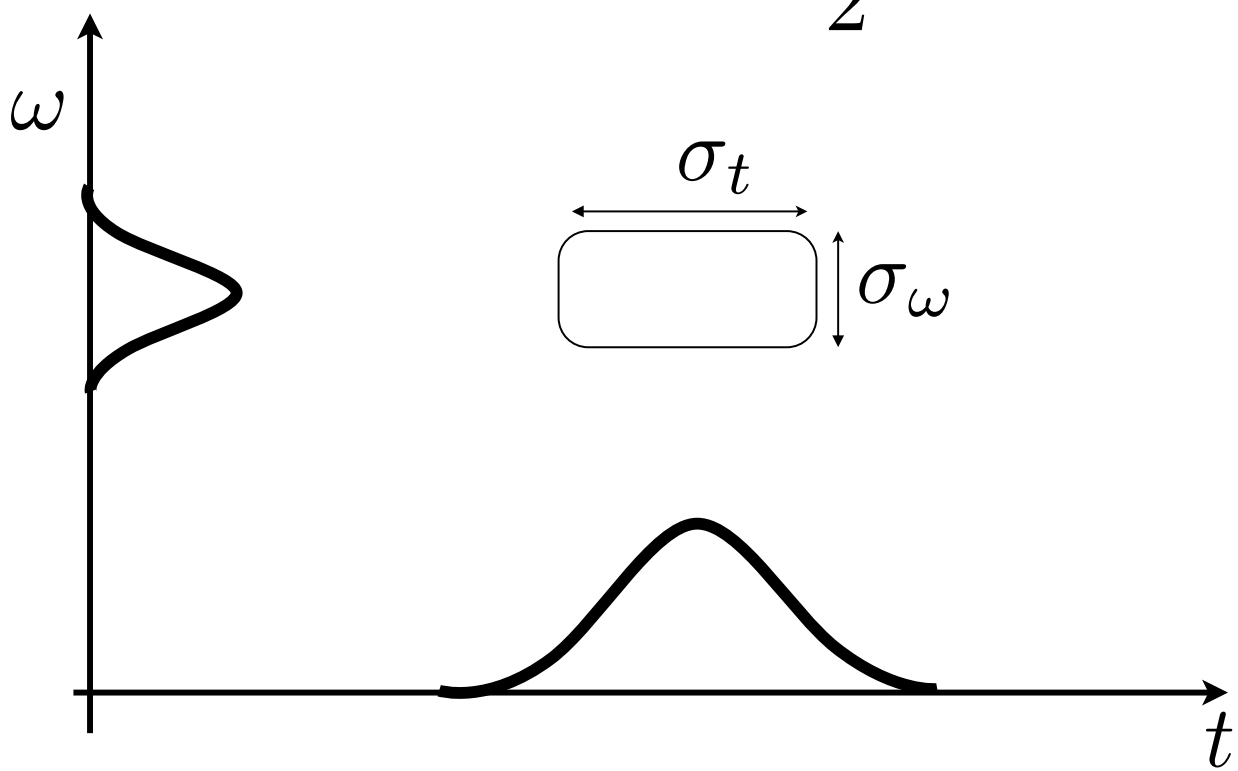
- Tradeoff between time and frequency resolution

Heisenberg Boxes



- Time-Frequency uncertainty principle

$$\sigma_t \cdot \sigma_\omega \geq \frac{1}{2}$$



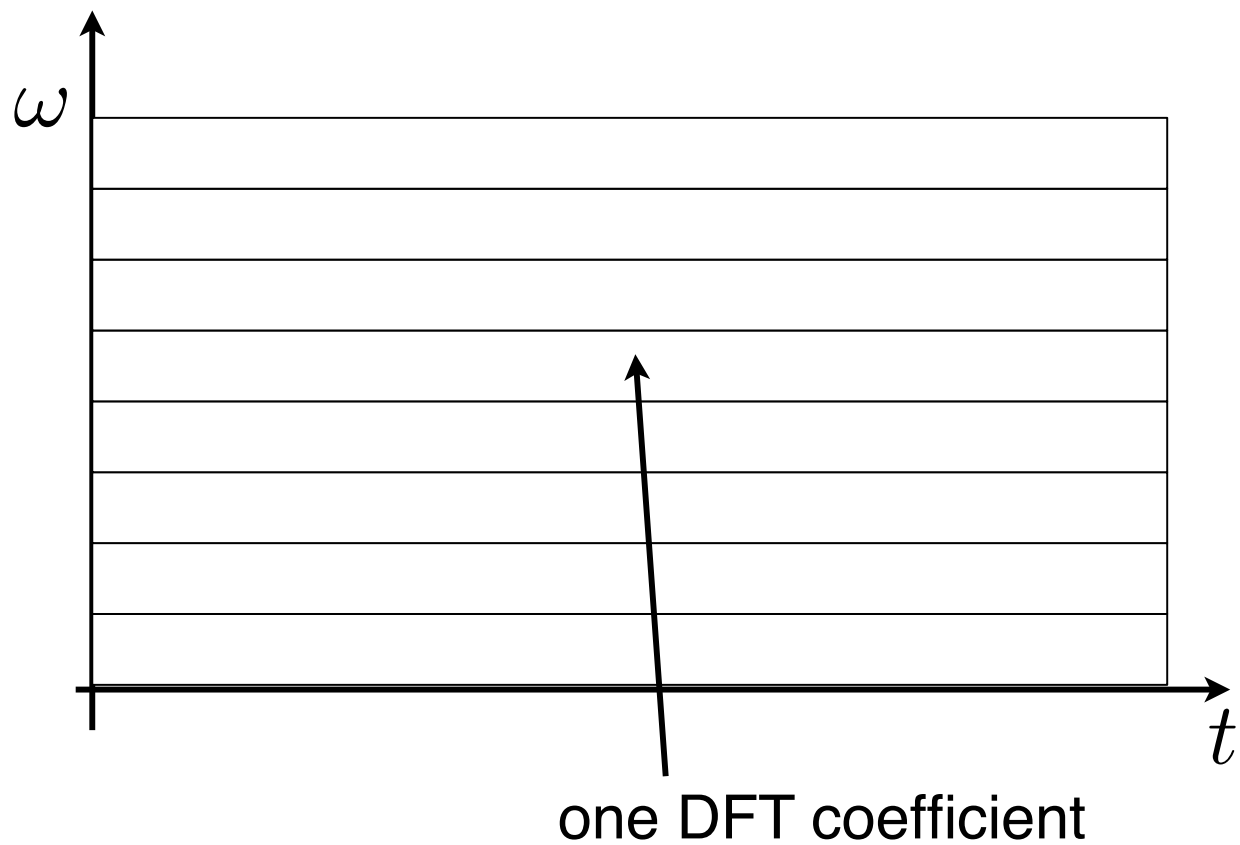
DFT

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N}$$

$$\Delta\omega = \frac{2\pi}{N}$$

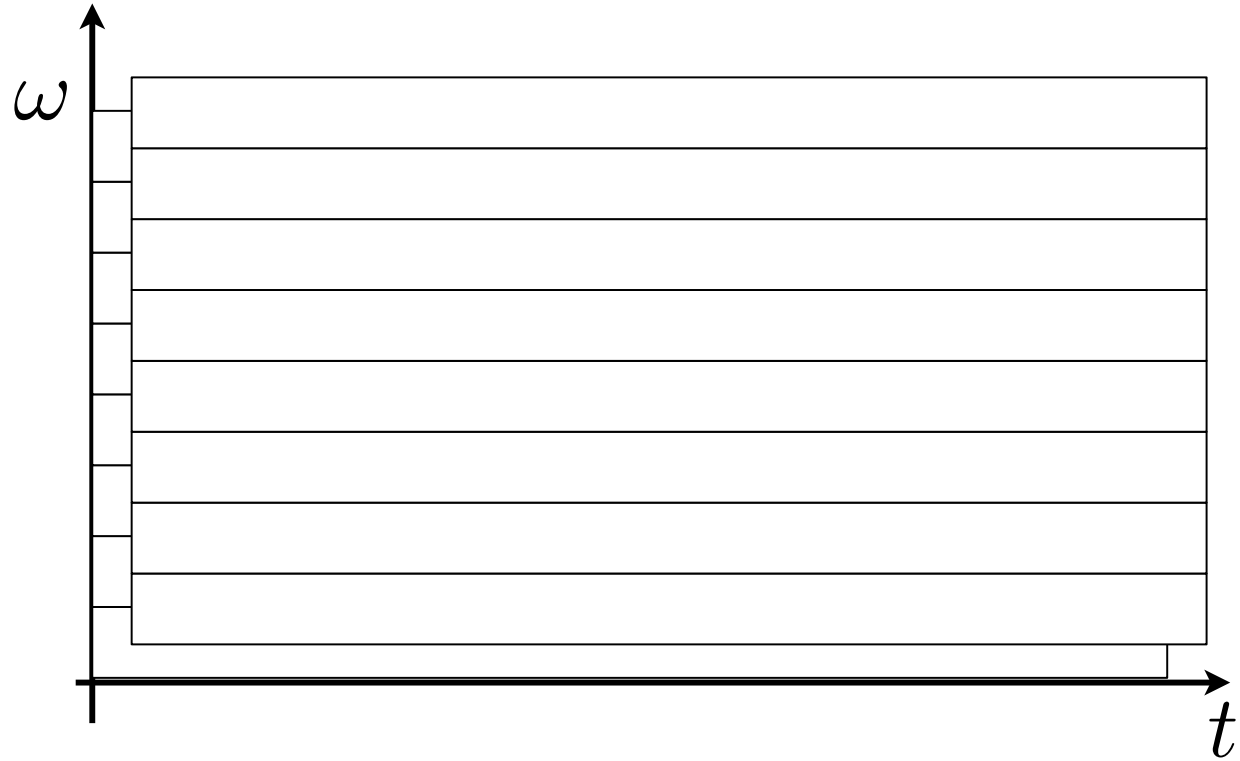
$$\Delta t = N$$

$$\Delta\omega \cdot \Delta t = 2\pi$$



DFT

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N}$$



$$\Delta\omega = \frac{2\pi}{N}$$

$$\Delta t = N$$

$$\Delta\omega \cdot \Delta t = 2\pi$$

Question: What is the effect of zero-padding?
Answer: Overlapped Tiling!

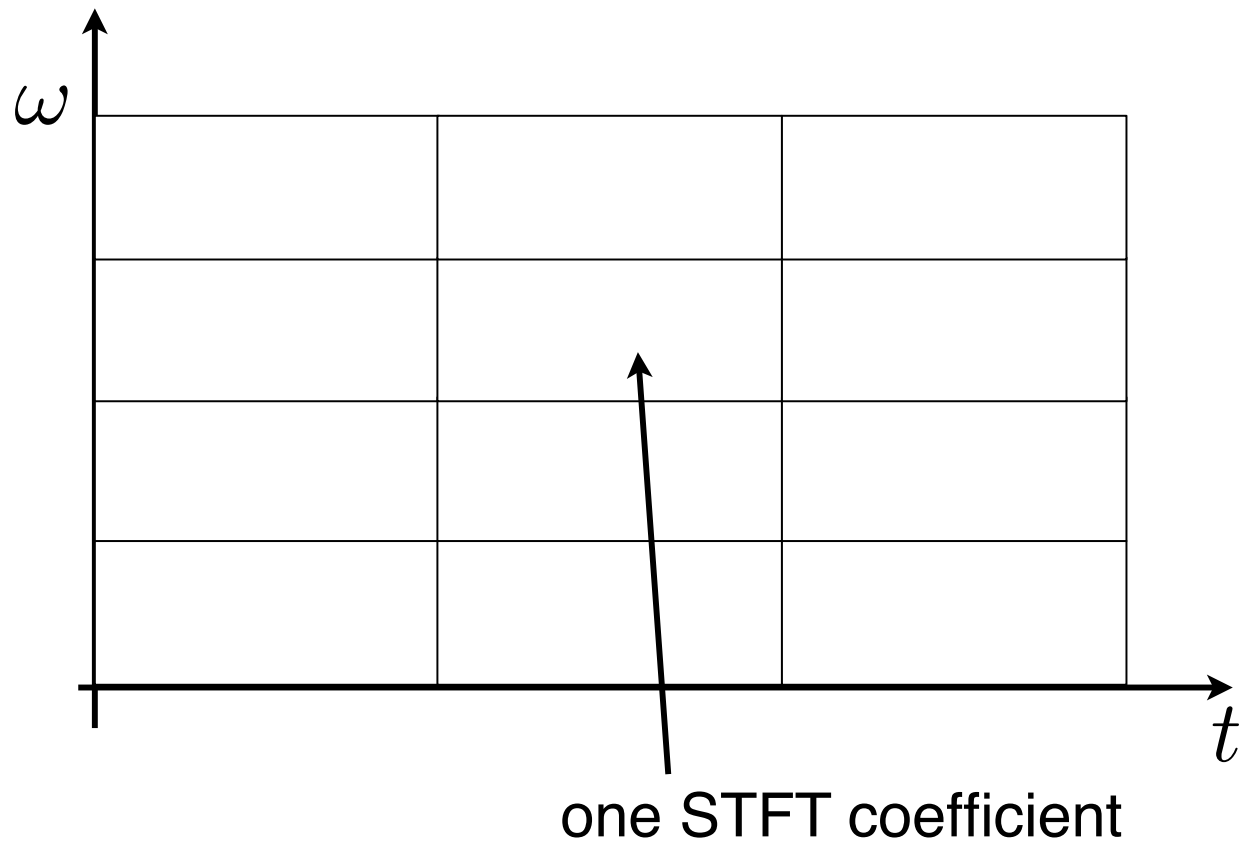
Discrete STFT

optional

$$X[r,k] = \sum_{m=0}^{L-1} x[rR + m]w[m]e^{-j2\pi km/N}$$

$$\Delta\omega = \frac{2\pi}{L}$$

$$\Delta t = L$$



Discrete STFT

optional

$$X[r,k] = \sum_{m=0}^{L-1} x[rR + m]w[m]e^{-j2\pi km/N}$$

$$\Delta\omega = \frac{2\pi}{L}$$

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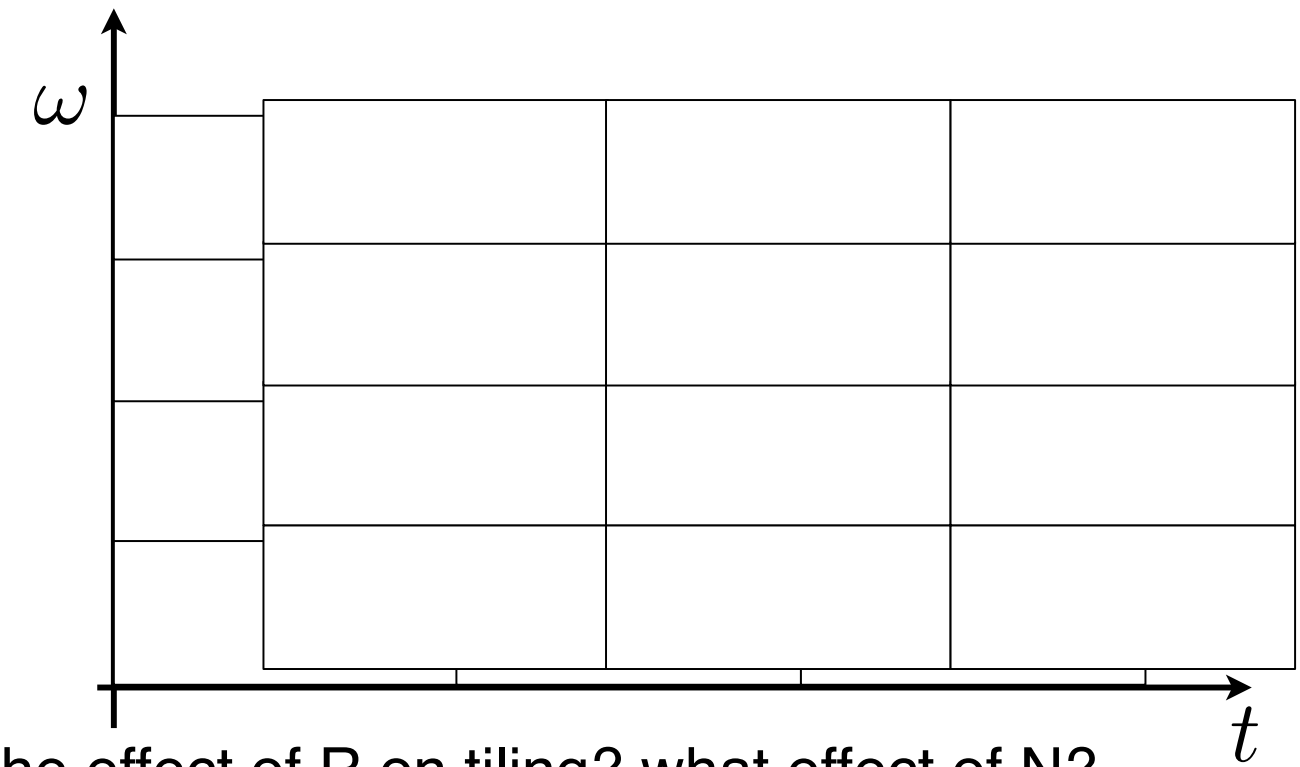
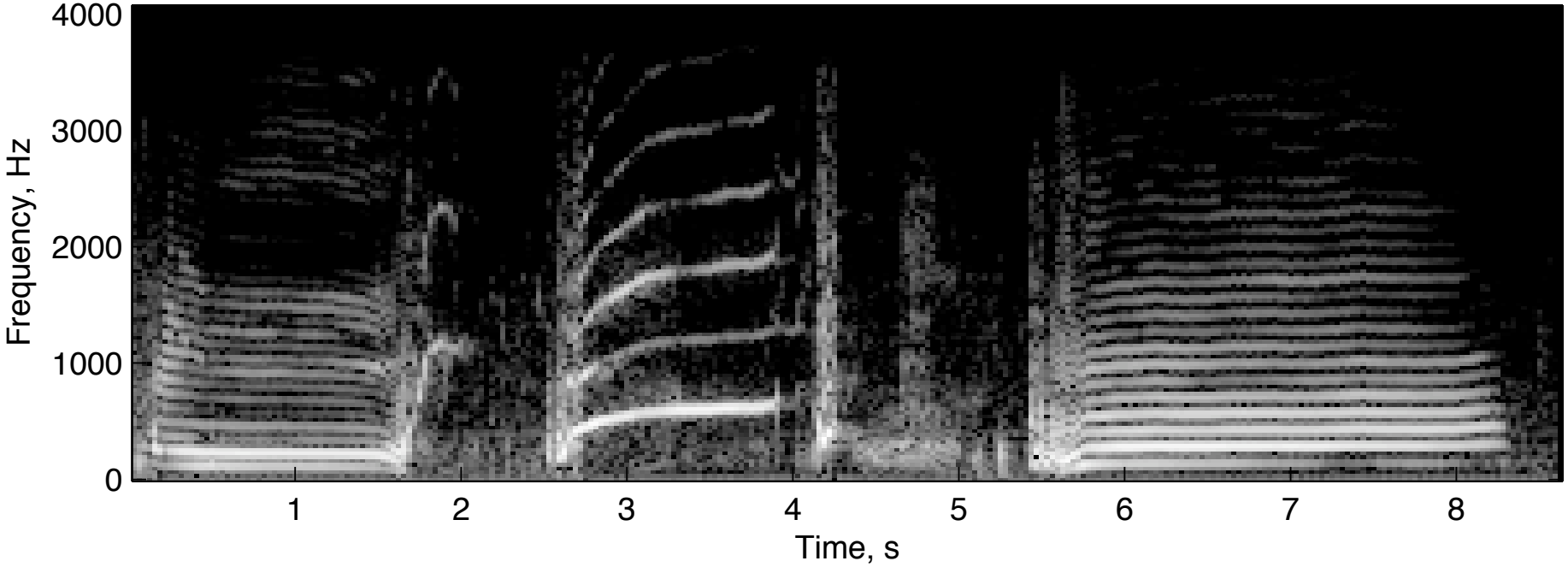


Question: What is the effect of R on tiling? what effect of N?
Answer: Overlapping in time of frequency or both!

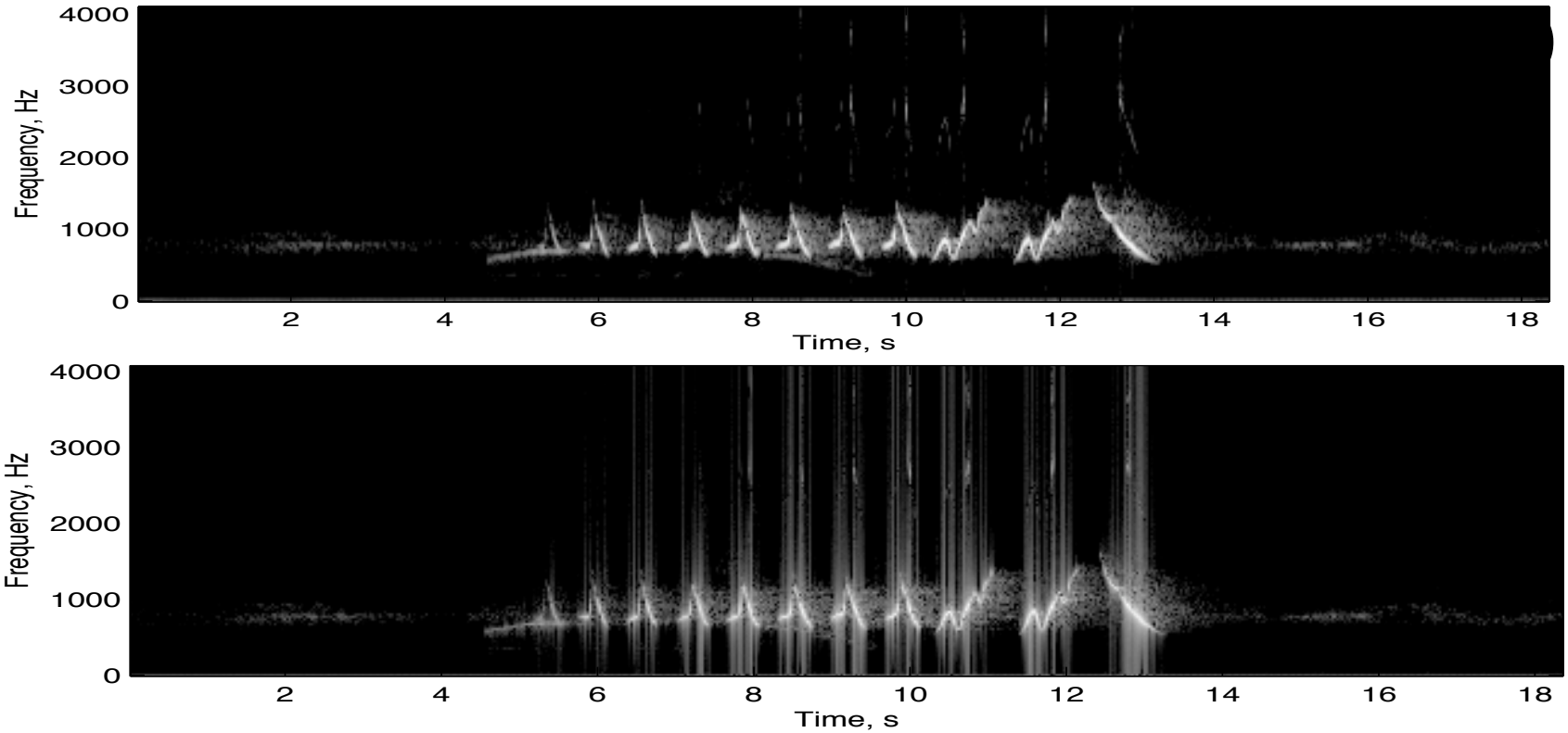
Applications

- Time Frequency Analysis

Spectrogram of Orca whale



Spectrogram



- What is the difference between the spectrograms?

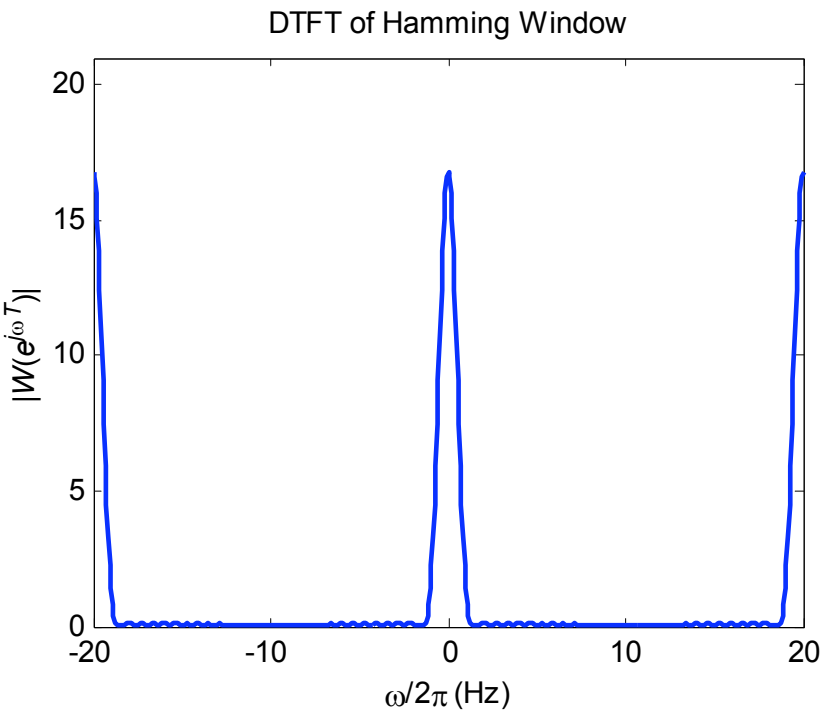
a) Window size B<A

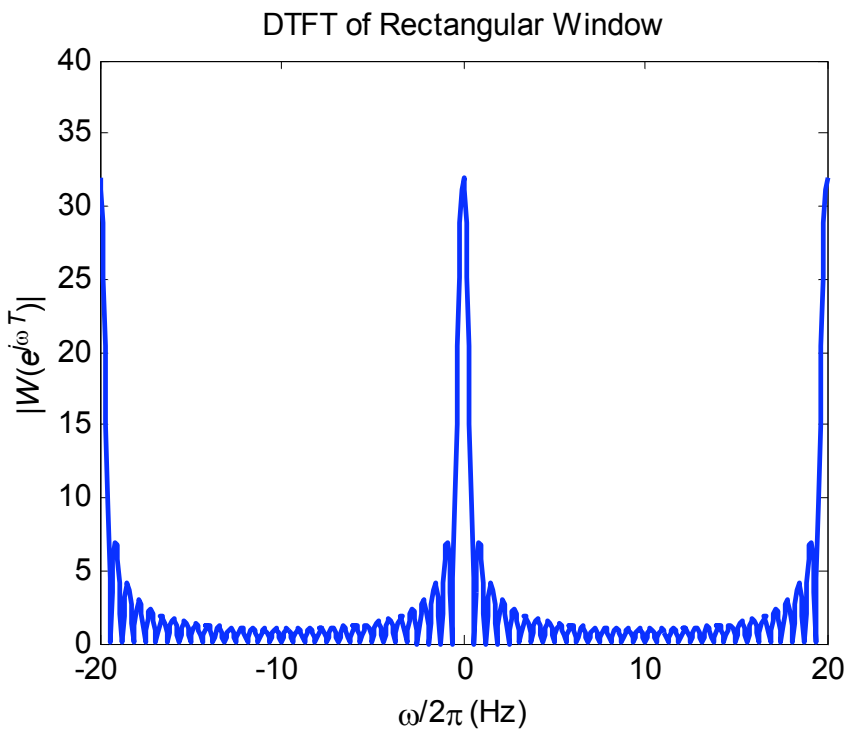
b) Window size B>A

c) Window type is different

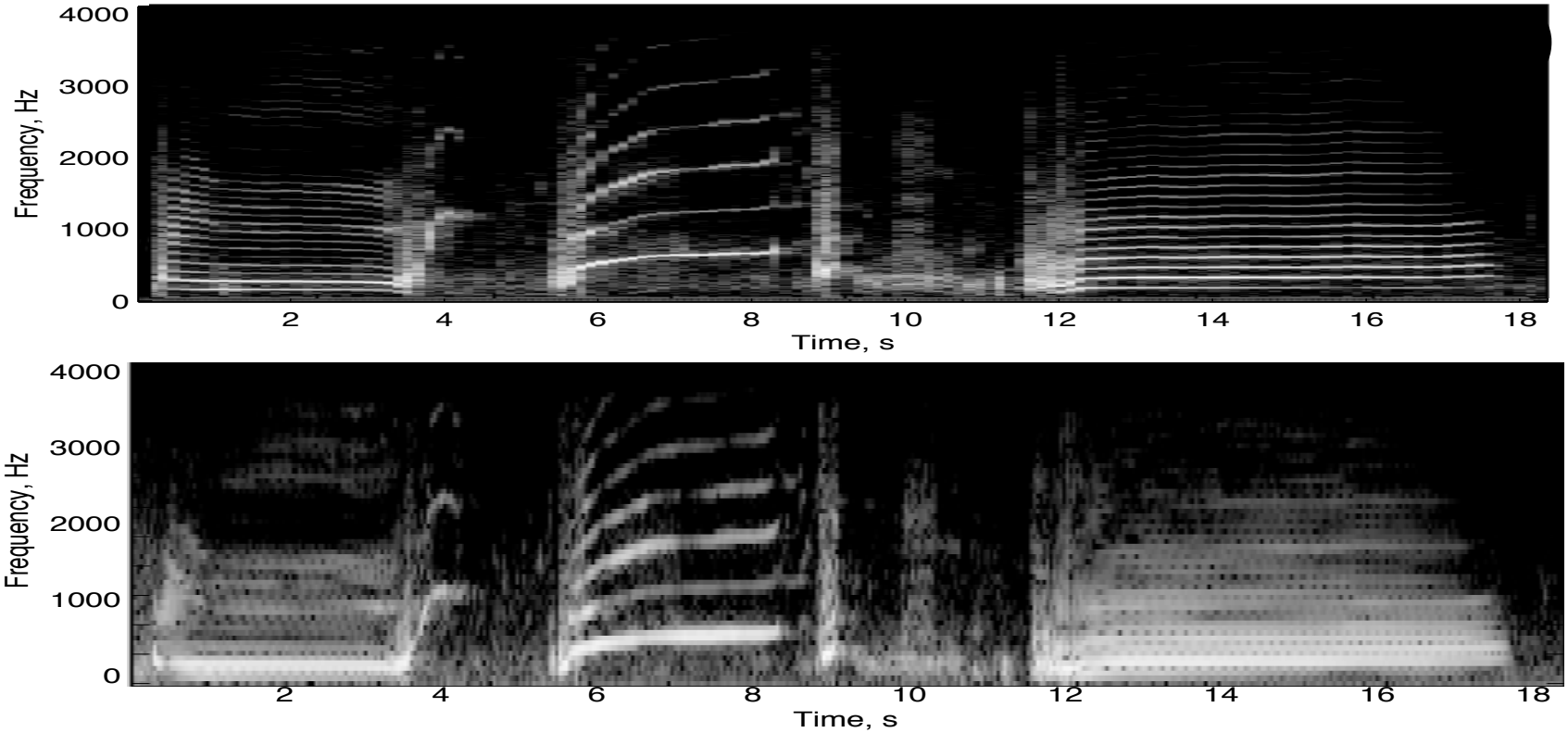
d) (A) uses overlapping window

Sidelobes of Hann vs rectangular window





Spectrogram



- What is the difference between the spectrograms?

a) Window size B<A

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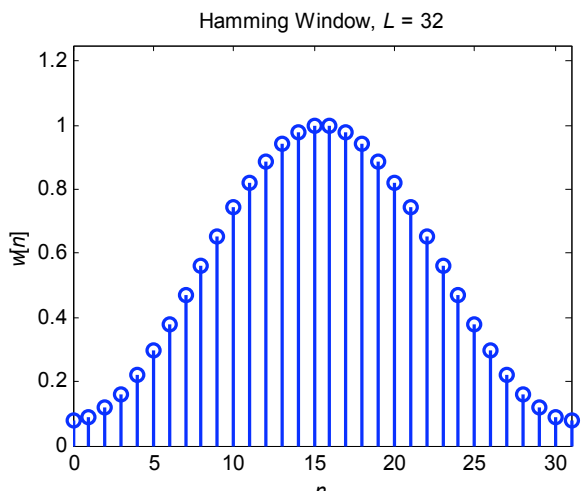
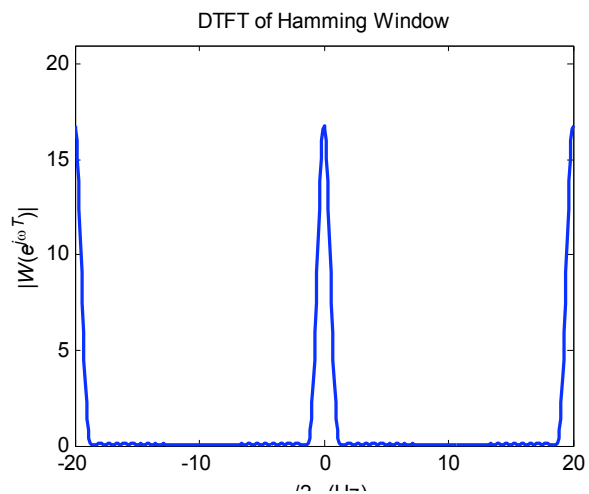
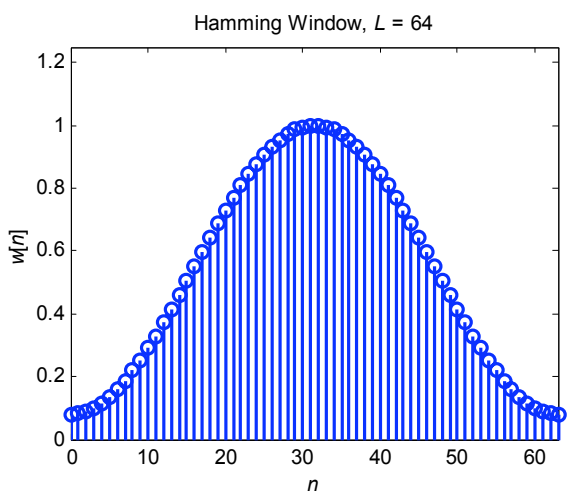
Spectrogram

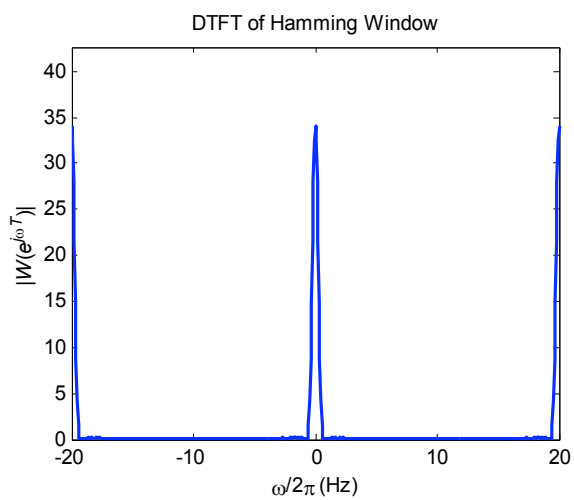
Hamming Window, $L = 32$





Hamming Window, $L = 64$



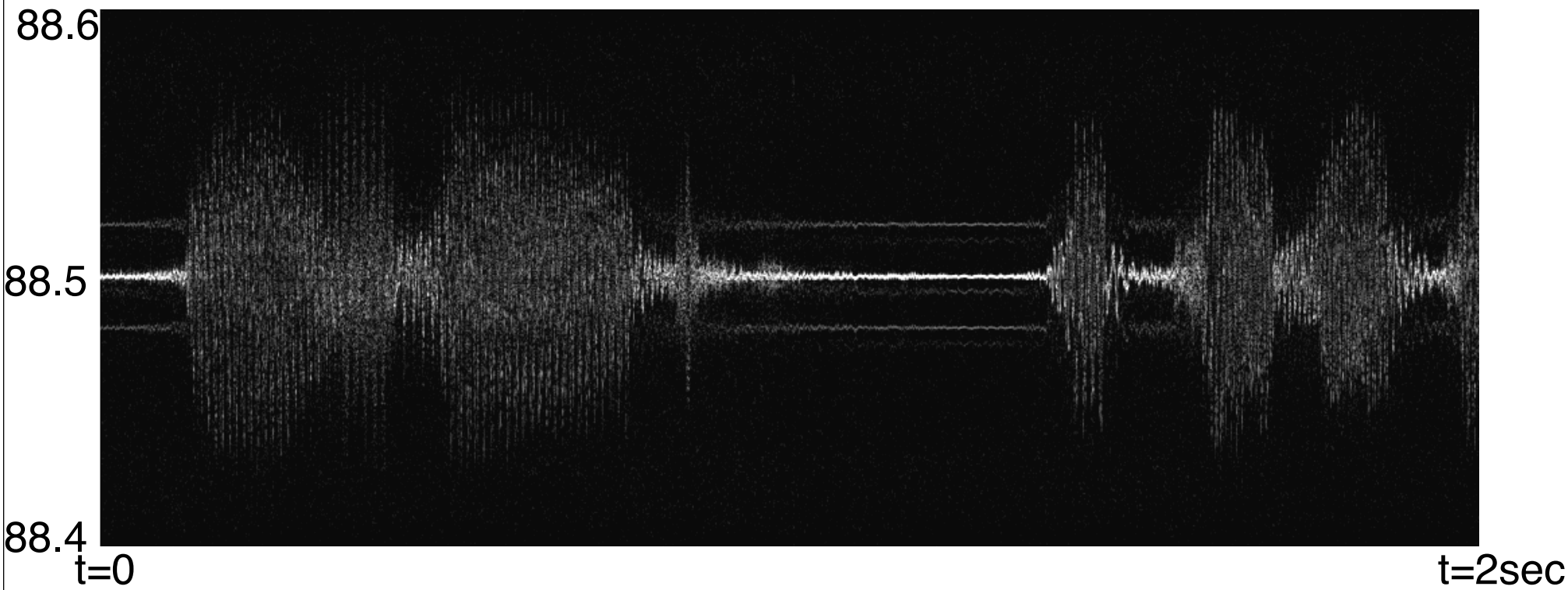


Spectrogram of FM

$$y_c(t) = A\cos\left(2\pi f_c t + 2\pi\Delta f \int_0^t x(\tau)d\tau\right)$$

$$y[n] = y(nT) = A\exp\left(j2\pi\Delta f \int_0^{nT} x(\tau)d\tau\right)$$

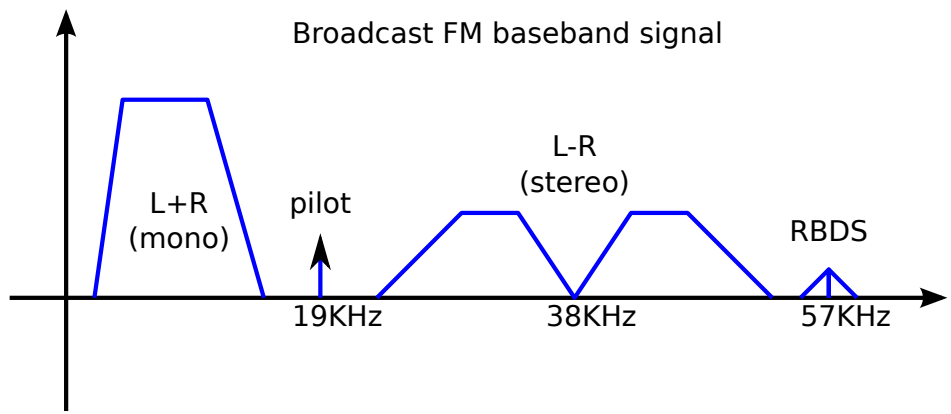
Spectrogram of FM radio



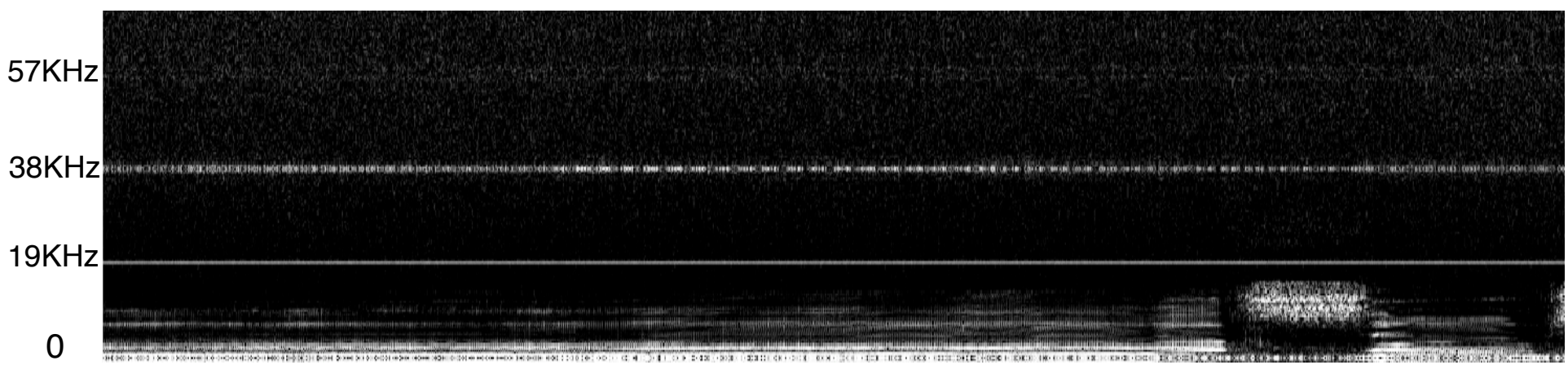
Spectrogram of FM radio Baseband

$$y[n] = y(nT) = A \exp\left(j2\pi\Delta f \int_0^{nT} x(\tau)d\tau \right)$$

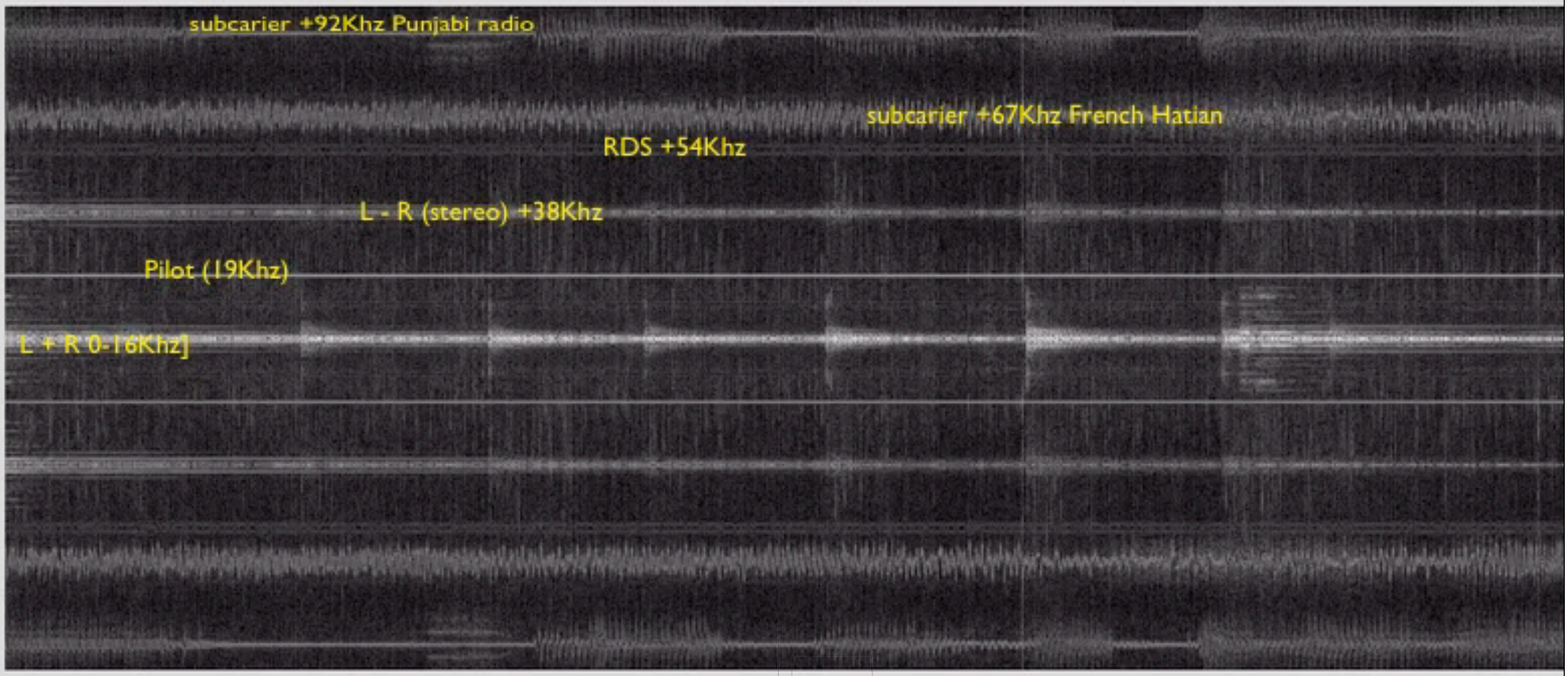
$$x(t) = \underbrace{(L+R)}_{\text{mono}} + \underbrace{0.1 \cdot \cos(2\pi f_p t)}_{\text{pilot}} + \underbrace{(L-R)\cos(2\pi(2f_p)t)}_{\text{stereo}} + \underbrace{0.05 \cdot \text{RBDS}(t)\cos(2\pi(3f_p)t)}_{\text{digital RBDS}}.$$

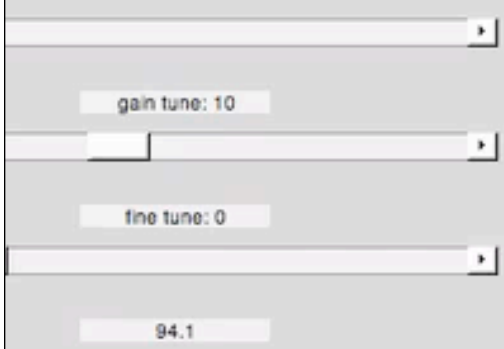


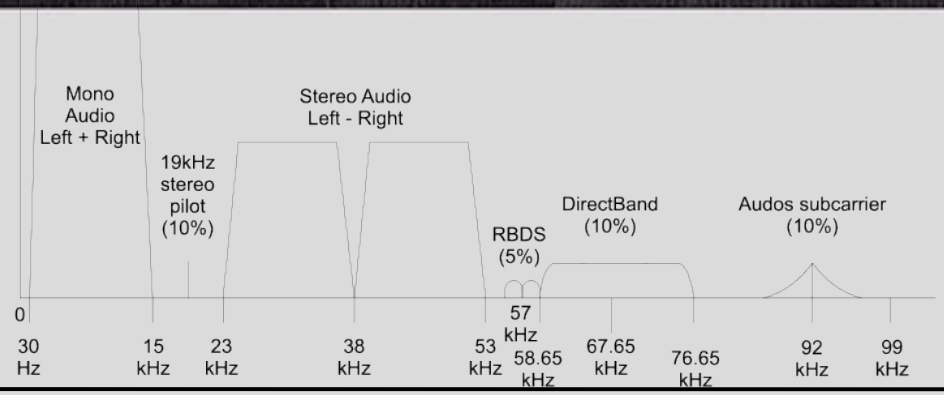
Spectrogram of **Demodulated** FM radio (Adele on 96.5 MHz)



Subcarrier FM radio (Hidden Radio Stations)



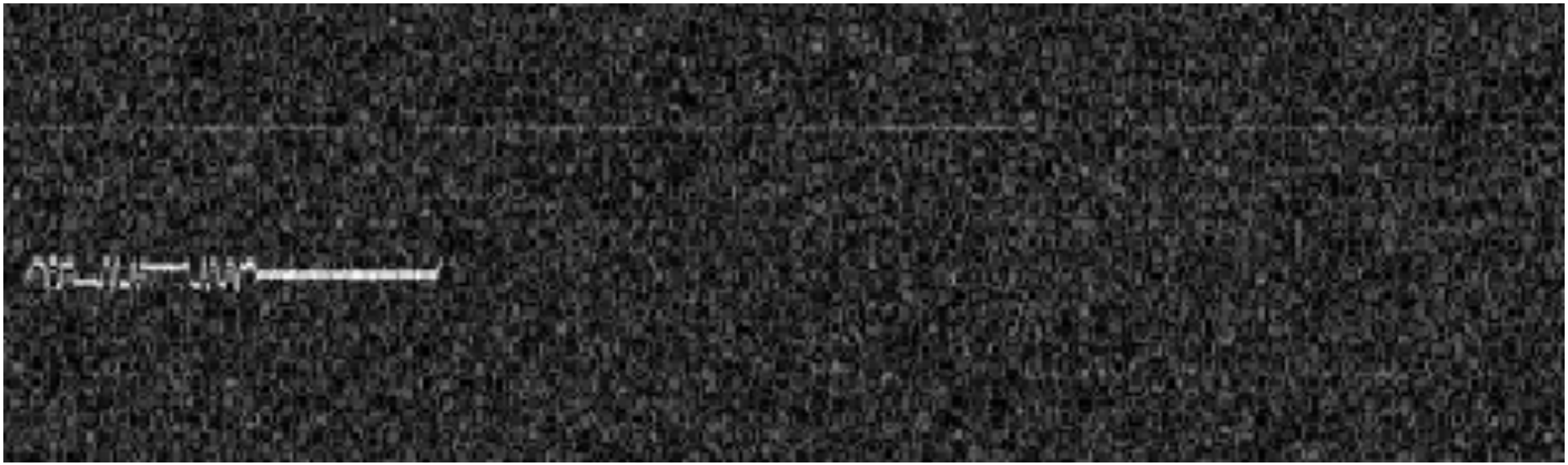




Applications

- Time Frequency Analysis

Spectrogram of digital communications - Frequency Shift Keying



t=0 t=1sec

STFT Reconstruction

$$x[rR+m]w_L[m] = \frac{1}{N}\sum_{k=0}^{N-1} X[n,k]e^{j2\pi km/N}$$

- For non-overlapping windows, R=L :

$$x[n] = \frac{x[n-rL]}{w_L[n-rL]}$$

$$rL \leq n \leq (r+1)R-1$$

- What is the problem?

STFT Reconstruction

$$x[rR+m]w_L[m] = \frac{1}{N}\sum_{k=0}^{N-1} X[n,k]e^{j2\pi km/N}$$

- For non-overlapping windows, R=L :

$$x[n] = \frac{x[n-rL]}{w_L[n-rL]}$$

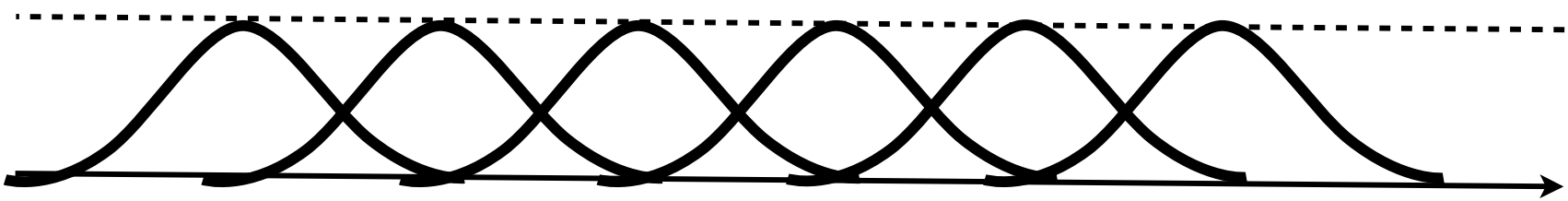
$$rL \leq n \leq (r+1)R-1$$

- For stable reconstruction must overlap window 50% (at least)

STFT Reconstruction

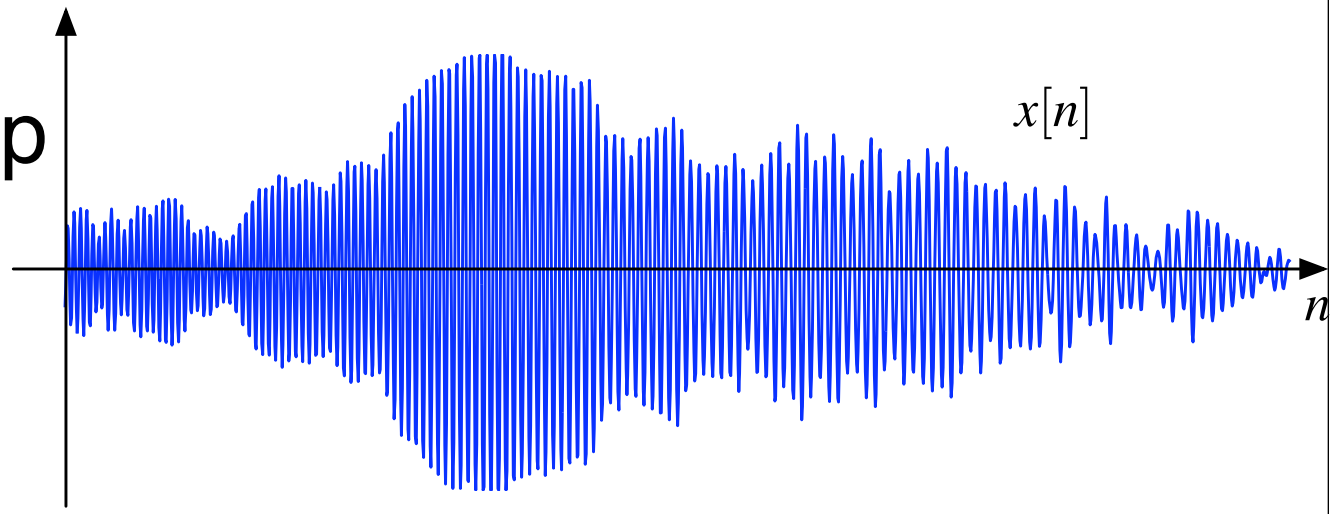
- For stable reconstruction must overlap window 50% (at least)

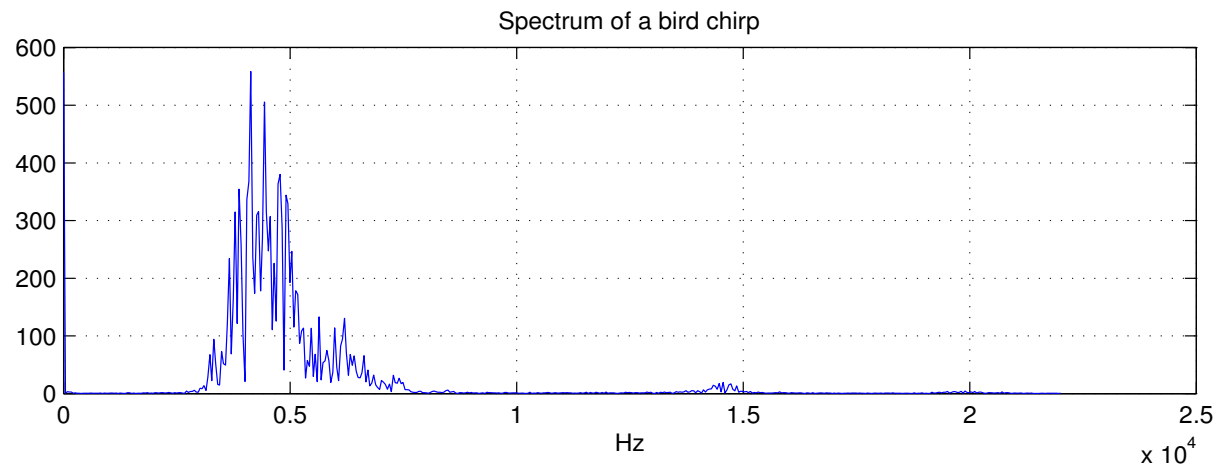
- For Hann, Bartlett reconstruct with overlap and add. No division!



Applications

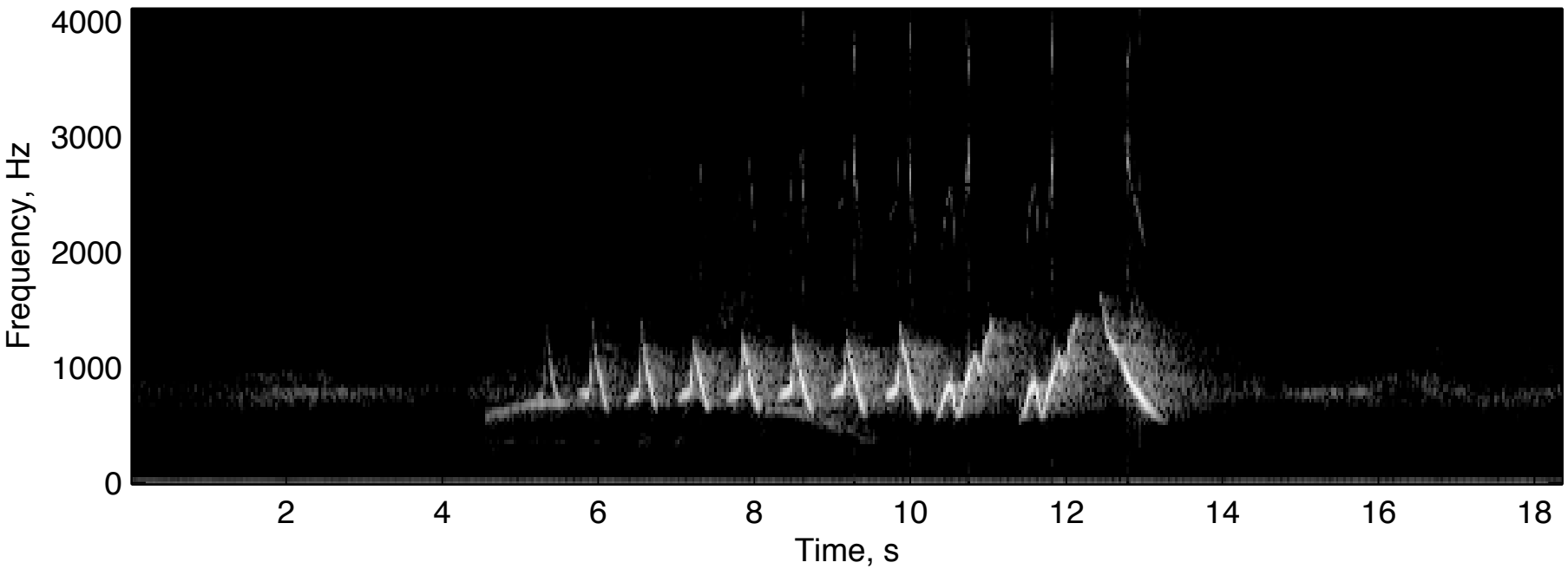
- Noise removal
- Recall bird chirp





Application

- Denoising of Sparse spectrograms



- Spectrum is sparse! can implement adaptive filter, or just threshold!

Limitations of Discrete STFT

- Need overlapping ⇒ Not orthogonal
- Computationally intensive O(MN log N)
- Same size Heisenberg boxes

From STFT to Wavelets

- Basic Idea:
  - low-freq changes slowly - fast tracking unimportant
  - Fast tracking of high-freq is important in many apps.
  - Must adapt Heisenberg box to frequency

- Back to continuous time for a bit.....