Projects

- Some no shows on Monday
- Today everyone has to meet with me -- I’ll add more and post.
Projection Slice Theorem (Bracewell)

\[ \mathcal{F}_{1D}\{p(\rho, \theta)\} = F(\rho \cos \theta, \rho \cos \theta) \]

\[ p(\rho, 0) = p(x) \]

1D FT

\[ F(f_x, 0) \]
Projection Slice Theorem (Bracewell)

\[ \mathcal{F}_{1D} \{ p(\rho, \theta) \} = F(\rho \cos \theta, \rho \cos \theta) \]

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Projection Slice Theorem (Bracewell)

\[ F_{1D}\{p(\rho, \theta)\} = F(\rho \cos \theta, \rho \cos \theta) \]
Projection Slice Theorem (Bracewell)

\[ p(x) = \int_{-\infty}^{\infty} m(x,y) \, dy \]

\[ M(k_x,0) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} m(x,y) e^{-i\pi(k_x x + k_y y)} \, dx \, dy = \]

\[ = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} m(x,y) e^{-i\pi k_x x} \, dx \, dy = \]

\[ = \int_{-\infty}^{\infty} \left[ \int_{-\infty}^{\infty} m(x,y) \, dy \right] e^{-i\pi k_x x} \, dx = \]

\[ = \int_{-\infty}^{\infty} p(x) e^{-i\pi k_x x} \, dx = \frac{1}{2\pi} \mathcal{F}\{p(x)\} \]
Partly Discrete Reconstruction

• Let’s assume continuous angle $\Theta$, discrete $\rho$

$$f(x, y)$$

$$F(\kappa_x, \kappa_y)$$

$$k = \frac{\omega}{2\pi}$$

$\mathbf{DTFT}$

$p_\theta[n] \Rightarrow P_\theta[k]$
Partly Discrete Reconstruction

• Let’s assume continuous angle $\Theta$, discrete $\rho$

\[ f(x, y) \]

\[ F(\kappa_x, \kappa_y) \]

\[ \kappa = \frac{\omega}{2\pi} \]

\[ P_\theta[n] \Rightarrow P_\theta[\kappa] \]
Reconstruction From Polar Coordinates

\[ f[n, m] = \int_{-0.5}^{0.5} \int_{-0.5}^{0.5} F(\kappa_x, \kappa_y) e^{2\pi j (\kappa_x n + \kappa_y m)} d\kappa_x d\kappa_y \]

\[ = \int_0^\pi \int_{-0.5}^{0.5} F(\rho, \theta) e^{2\pi j (\rho \cos(\theta) n + \rho \sin(\theta) m)} |\rho| d\rho d\theta \]

- Polar frequency data must be multiplied by |\rho|
- Also called a rho filter
Discrete Reconstruction

• Let’s assume discrete angle $\Theta_m$, discrete $\rho$

\[ f(x, y) \]

\[ y \]

\[ x \]

\[ F(\kappa_x, \kappa_y) \]

\[ \kappa = \frac{\omega}{2\pi} \]

\[ p_{\theta_m}[n] \Rightarrow P_{\theta_m}[l] \]

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Discrete Reconstruction

- Let’s assume discrete angle $\Theta_m$, discrete $\rho$

$$f(x, y) \rightarrow y$$

$$F(\kappa_x, \kappa_y)$$

$$\kappa = \frac{\omega}{2\pi}$$

DFT

$$p_{\theta_m}[n] \rightarrow P_{\theta_m}[l]$$

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Filtered Back Projection

• Replace integrals with sums. Sum over radius and angle

• Define a (filtered) backprojection:

\[
C_{\theta_m}[n_x, n_y] = \sum_{l=-N/2}^{(N/2)-1} F[l, \theta_m] e^{2\pi j (l/N \cos(\theta_m)n_x + l/N \sin(\theta_m)n_y)} |l/N| \leq \rho
\]

So,

\[
f[n_x, n_y] = \sum_{m} C_{\theta_m}[n_x, n_y]
\]
Example
Example Convolution Back Projection

- For $\Theta=0$

\[
C_0[n_x, n_y] = \sum_{l=-N/2}^{(N/2)-1} F[l, 0]|l/N|e^{2\pi j (l/N n_x)}
\]

\[F[l, 0]|l/N|\]

\[\text{FFT} \quad \text{IFFT} \quad \text{Rho} \quad \text{back proj.}\]
Example Convolution Back Projection

- For $\Theta = \pi / 2$
  
  \[
  C_{\pi / 2}[n_x, n_y] = \sum_{l=-N/2}^{(N/2)-1} F[l, \pi / 2]|l/N|e^{2\pi j(l/N n_y)}
  \]
Convolution Back Projection
Filtered Back Projection

Back projection  Filtered Back projection
How Many Projections?

\[ F(\kappa_x, \kappa_y) \]

FOV

1/FOV
How Many Projections?

256 Proj.  128 Proj  64 Proj
Fan Beam CT

• Single Source
• Many detectors

• How to reconstruct?
Fan Beam CT

- Single Source
- Many detectors

- How to reconstruct?

- Re-binning!
Fan Beam CT

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- How to reconstruct?
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