Lecture 25

EE 123 Discussion Section 7

Jon Tamir

March 28, 2016

Based on notes from Giulia Fanti, Frank Ong, Michael Lustig

Announcements

- HW8 – to be released later today, due Friday
- HW7 self grade – due today
- Lab 4 – due Thursday March 31
- Midterm 2 – Monday, April 4
  - Same policies as those for Midterm 1
- HAM radios passed out during lab sessions
- Questions?

Plan

- Poles and zeros
- Midterm 2 topics

Poles and zeros

	Magnitude	Phase	Group delay
Poles	push up	go down	delay (positive)
Zeros	push down	go up	advance (negative)

Problem 1

Consider $$h_1[n] = \begin{cases} 1 & |n| \leq 1 \\ 0 & else \end{cases}$$

- Compute its z-transform and plot the poles and zeros
- Sketch its magnitude response

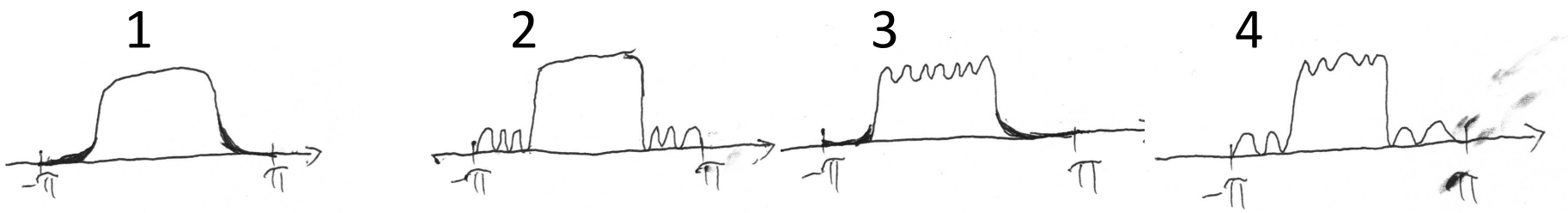
What about the pole zero diagram for

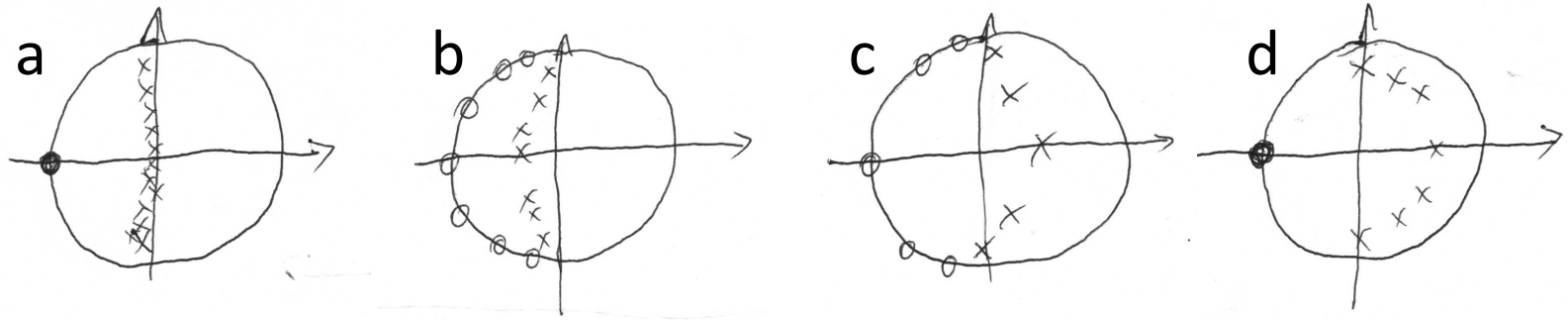
$$h_1[n] = \begin{cases} 1 & |n| \leq 3 \\ 0 & else \end{cases}$$

Solution 1

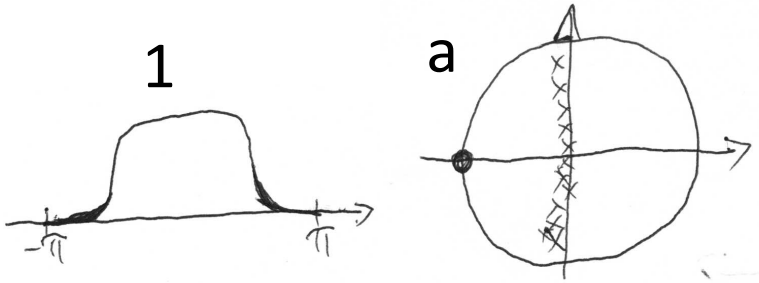
Problem 2

Match the following magnitude response to their pole-zero plot

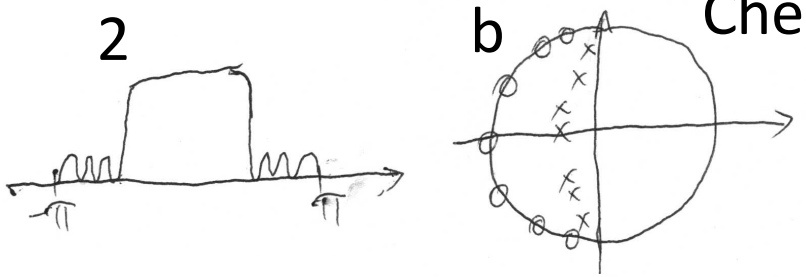




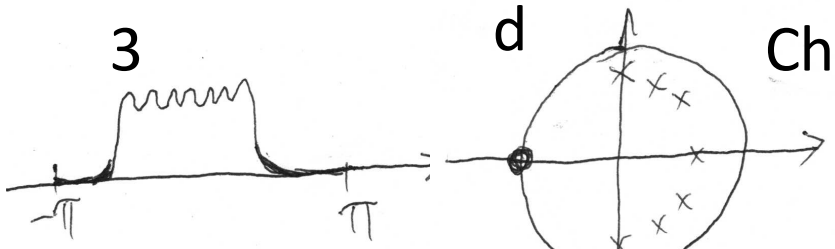
Solution 2



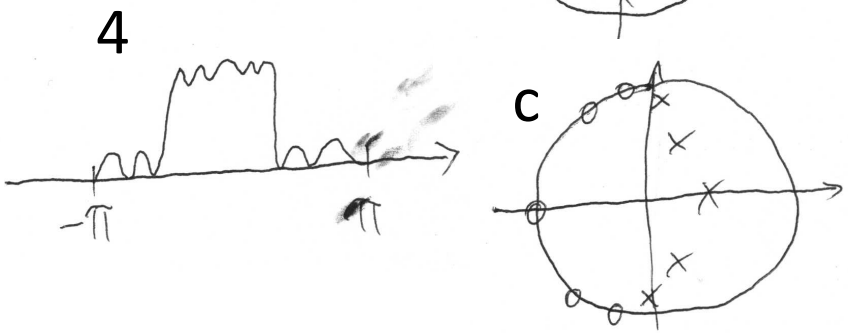
Butterworth



Chebyshev Type II



Chebyshev Type I



Elliptic

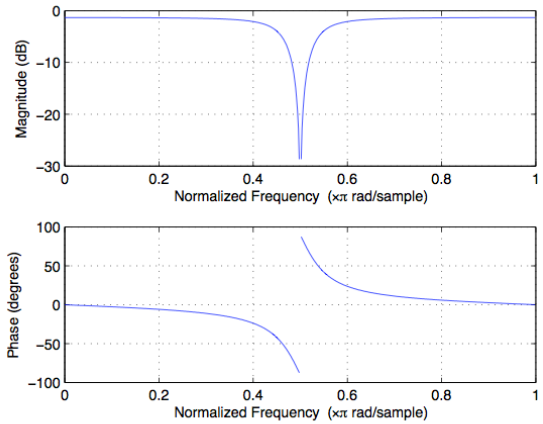
Problem 3

b) (10 points) Match the frequency responses below with the transfer functions:

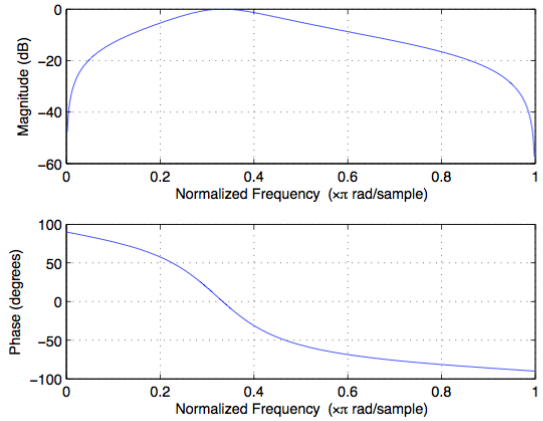
$$H_1(z) = 0.25\frac{1-z^{-2}}{1-0.75z^{-1}+0.5z^{-2}} \qquad H_2(z) = 0.75\frac{1+z^{-2}}{1+0.75z^{-2}}$$

$$H_3(z) = \frac{\frac{4}{9}+\frac{2}{3}z^{-1}+z^{-2}}{1+\frac{2}{3}z^{-1}+\frac{4}{9}z^{-2}} \qquad H_4(z) = 0.2(1+z^{-1}+z^{-2}+z^{-3}+z^{-4}).$$

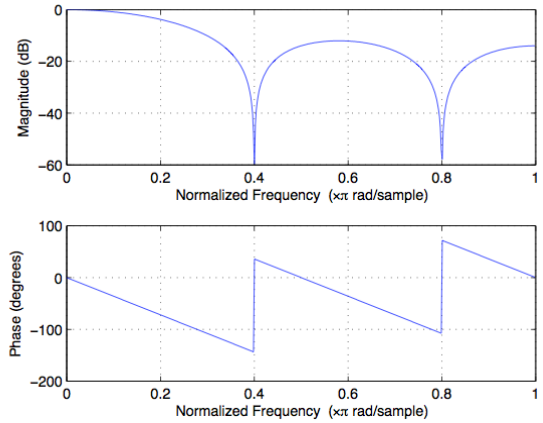
A



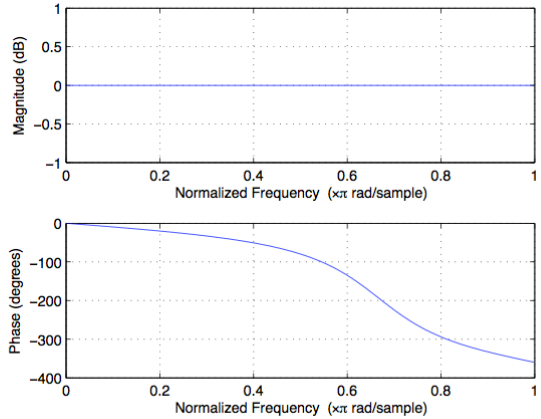
B



C



D



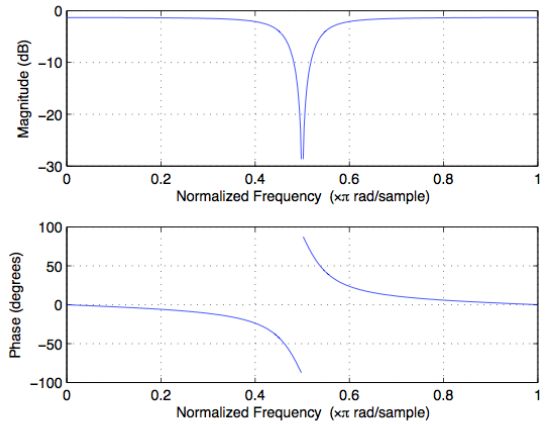
Solution 3

b) (10 points) Match the frequency responses below with the transfer functions:

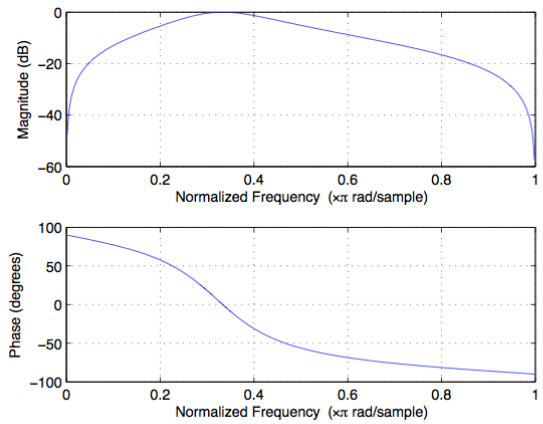
B $H_1(z) = 0.25\frac{1-z^{-2}}{1-0.75z^{-1}+0.5z^{-2}}$ $H_2(z) = 0.75\dfrac{1+z^{-2}}{1+0.75z^{-2}}$ A

D $H_3(z) = \frac{\frac{4}{9}+\frac{2}{3}z^{-1}+z^{-2}}{1+\frac{2}{3}z^{-1}+\frac{4}{9}z^{-2}}$ $H_4(z) = 0.2(1+z^{-1}+z^{-2}+z^{-3}+z^{-4})$. C

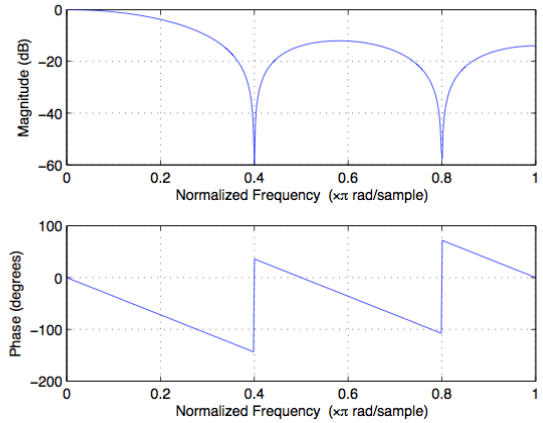
A



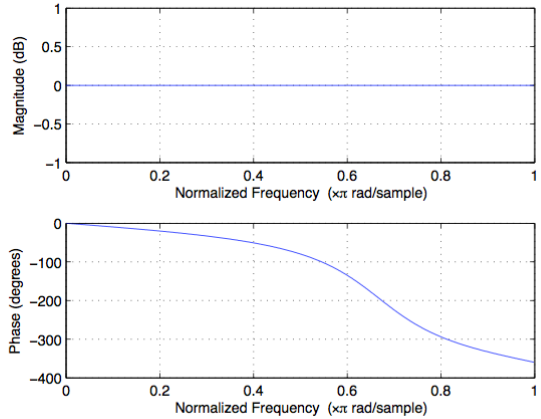
B



C



D



Problem 4

Consider the problem of reconstructing the signal from only its Fourier magnitude $|H(e^{jw})|^2$

a How does the pole-zero plot of $|H(e^{jw})|^2$ look like compared to $H(e^{jw})$?
Hint: the z-transform of $|H(e^{jw})|^2$ is $H(z)H^*(1/z)$

b If the system $h[n]$ is causal and stable, can you uniquely recover $h[n]$ from $|H(e^{jw})|^2$

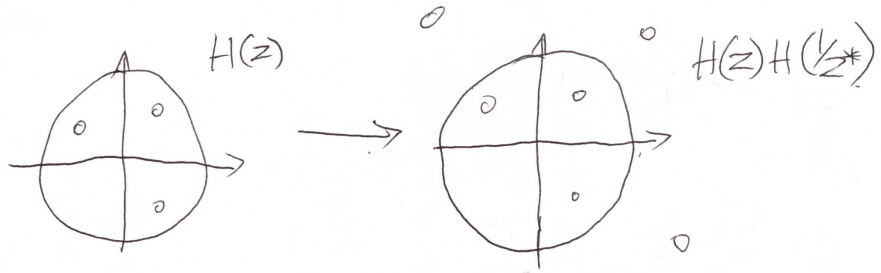
c Assume that $h[n]$ is causal and stable and that, in addition, you know that the system function has the form

$$H(z) = \frac{1}{1 - \sum_{k=1}^{N} a_k z^{-k}}$$

for some finite a_k. Can you uniquely recover $h[n]$ from $|H(e^{jw})|^2$

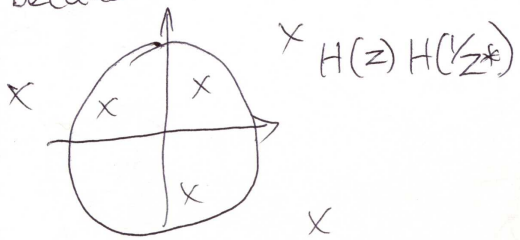
Solution 4

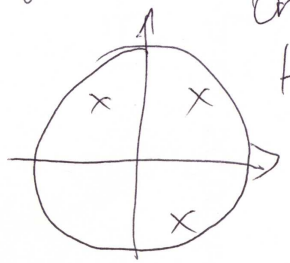
a)



b) h[n] causal and stable does not help
The above example is a counterexample.

c) h[n] causal and stable + all pole system
can uniquely recover the signal.
because





Only solution
H(z)

Problem 5

Consider the linear time-invariant discrete-time system represented by the rational system function

$$H(z) = \frac{1}{1 - \alpha z^{-N}}$$

a) How many poles are there in this system?
b) Write the corresponding difference equation and sketch the system (i.e., adders, multipliers, and delays).
c) If the sampling rate is 16 kHz, N = 16000, α = .5, and the input is a short speech segment of someone saying "ba", what would the output sound like (assuming all the necessary machinery like antialiasing filters, A/D, D/A, etc.)?
d) For N=4, α slightly less than 1, sketch the frequency response.

Solution 5

a) $$H(z) = \frac{1}{1-\alpha z^{-N}}$$

Poles: $z_k = \alpha \cdot e^{j\frac{2\pi}{N}k}$ $\quad k = 0, \cdots N-1$

$\boxed{N \text{ poles}}$

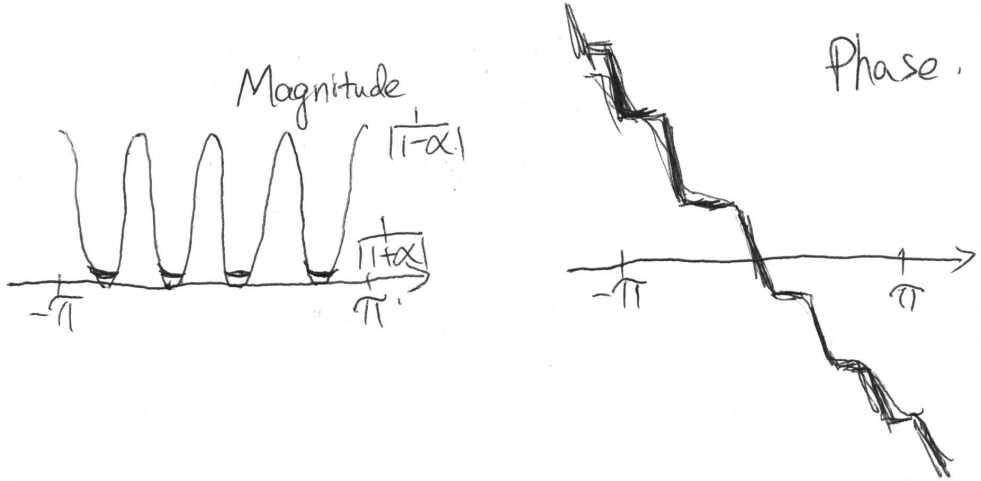
b) $$y[n] - \alpha \cdot y[n-N] = x[n]$$

Solution 5

c) Assuming "ba" lasts less than 1 second.
that means it ~~lasts~~ takes less than
16000 sample time. Since $y[n] = x[n] + \alpha . y[n-16000]$

$\Rightarrow$ ba ba ba ba ba ba ... —

d)



Midterm 2 topics

- Time-frequency tiling
- Time-frequency decomposition
- Sampling
- Resampling
- Multi-rate
- Polyphase decomposition
- Non-ideal sampling
- Filter design, TBW
- Phase response
- Min-phase, all-pass, linear-phase systems

HW8 and previous midterms

Practice problem

4.53. Consider the analysis–synthesis system shown in Figure P4.53-1. The lowpass filter $h_0[n]$ is identical in the analyzer and synthesizer, and the highpass filter $h_1[n]$ is identical in the analyzer and synthesizer. The Fourier transforms of $h_0[n]$ and $h_1[n]$ are related by

$$H_1(e^{j\omega}) = H_0(e^{j(\omega+\pi)}).$$

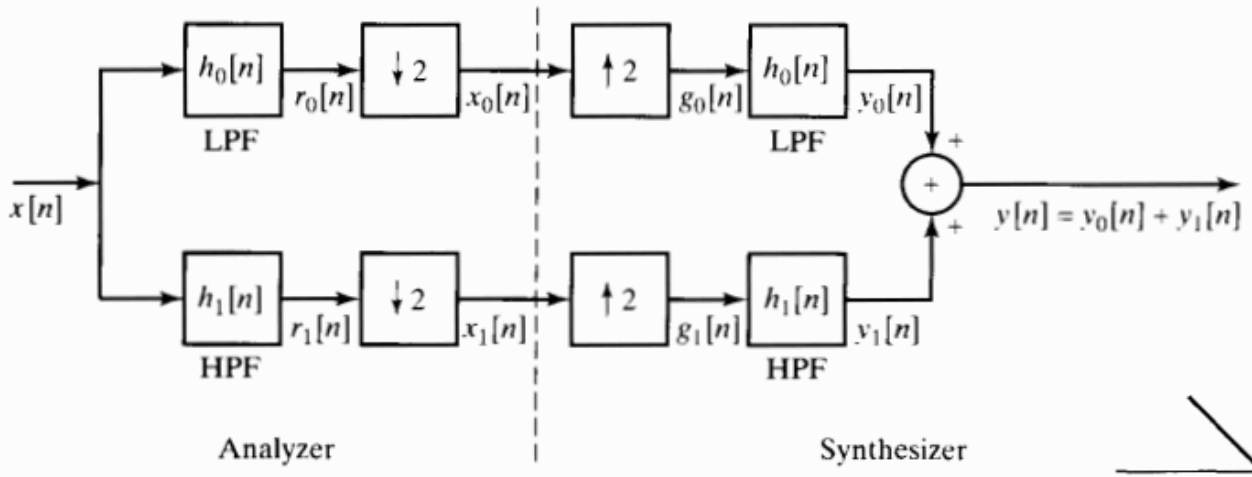
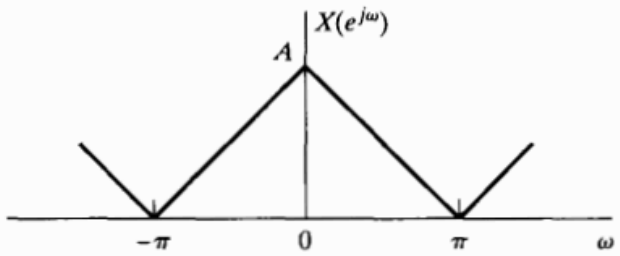


Figure P4.53-1



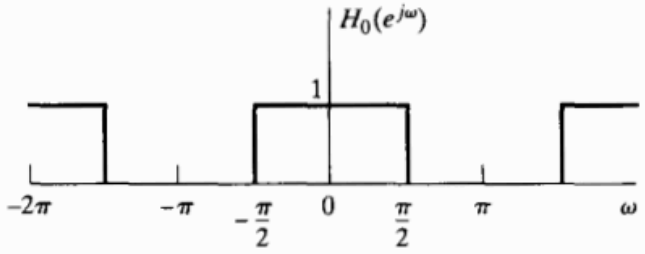
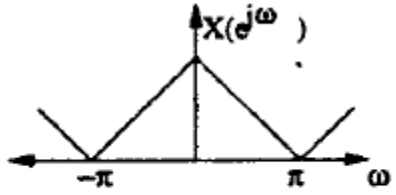


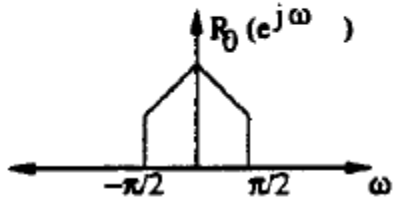
Figure P4.53-2

(a) If $X(e^{j\omega})$ and $H_0(e^{j\omega})$ are as shown in Figure P4.53-2, sketch (to within a scale factor) $X_0(e^{j\omega})$, $G_0(e^{j\omega})$, and $Y_0(e^{j\omega})$.

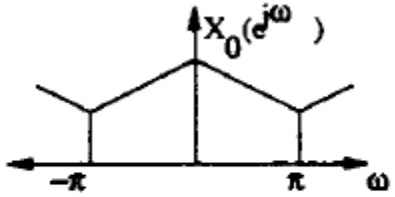
(a) First, $X(e^{j\omega})$ is plotted.



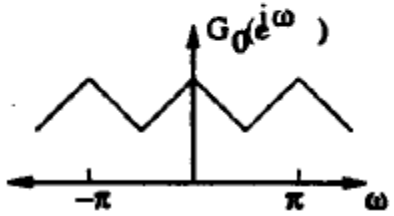
The lowpass filter cuts off at $\frac{\pi}{2}$.



The downsampler expands the frequency axis. Since $R_0(e^{j\omega})$ is bandlimited to $\frac{\pi}{M}$, no aliasing occurs.



The upsampler compresses the frequency axis by a factor of 2.



The lowpass filter cuts off at $\frac{\pi}{2} \Rightarrow Y_0(e^{j\omega}) = R_0(e^{j\omega})$ as sketched above.

(b) Write a general expression for $G_0(e^{j\omega})$ in terms of $X(e^{j\omega})$ and $H_0(e^{j\omega})$. Do *not* assume that $X(e^{j\omega})$ and $H_0(e^{j\omega})$ are as shown in Figure 4.53-2.

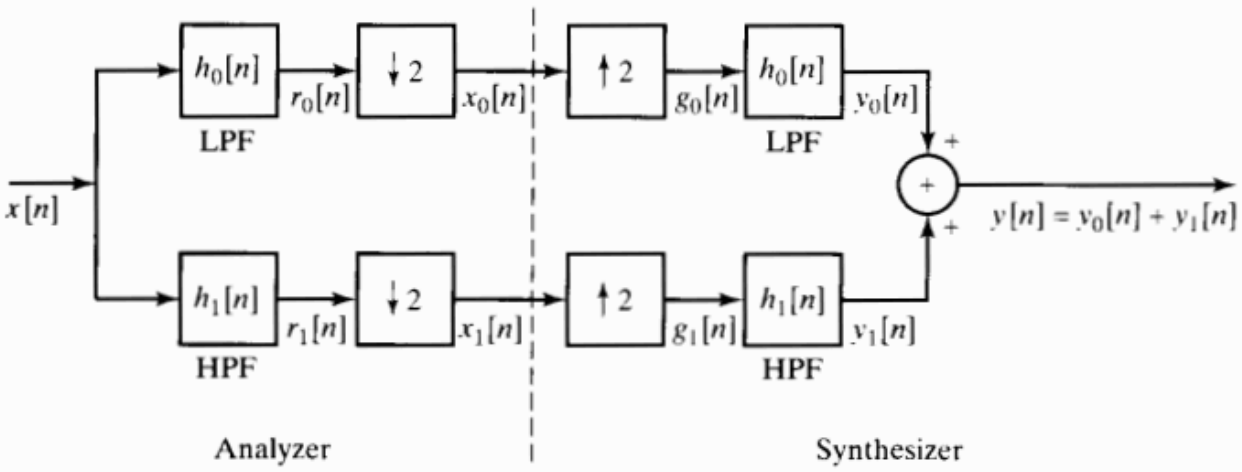


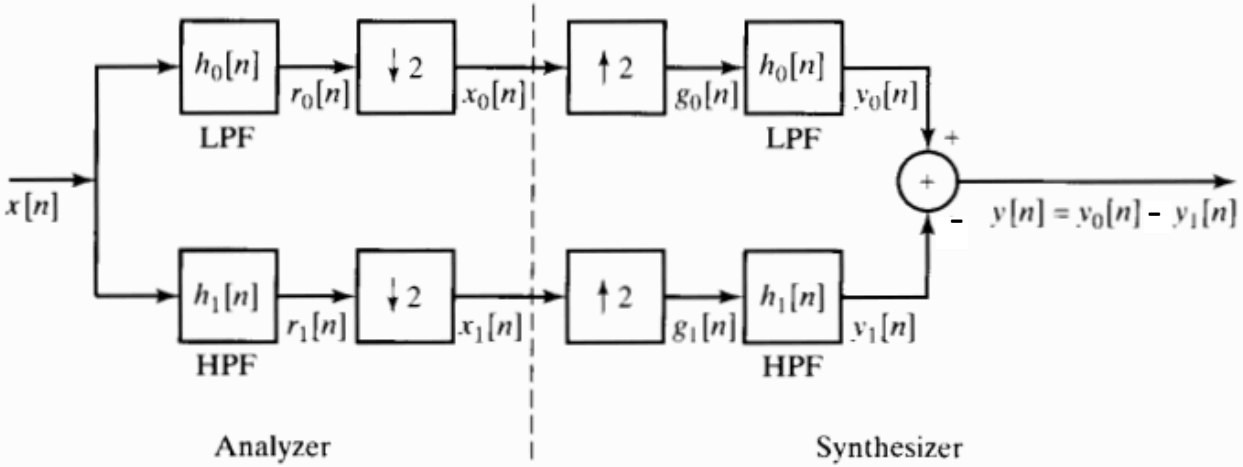
Figure P4.53-1

(b) $G_0(e^{j\omega}) = \frac{1}{2}\left(X(e^{j\omega})H_0(e^{j\omega}) + X(e^{j(\omega+\pi)})H_0(e^{j(\omega+\pi)})\right)$

(c) Determine a set of conditions on $H_0(e^{j\omega})$ that is as general as possible and that will guarantee that $y[n]$ is proportional to $x[n-n_d]$ for any stable input $x[n]$.

Note: Analyzer–synthesizer filter banks of the form developed in this problem are very similar to quadrature mirror filter banks. For further reading, see Crochiere and Rabiner (1983), pp. 378–392.

$$H_1(e^{j\omega}) = H_0(e^{j(\omega+\pi)}).$$



(c)

$$\begin{aligned}
Y_0(e^{j\omega}) &= \frac{1}{2}H_0(e^{j\omega})\left(X(e^{j\omega})H_0(e^{j\omega}) + X(e^{j(\omega+\pi)})H_0(e^{j(\omega+\pi)})\right)\\
Y_1(e^{j\omega}) &= \frac{1}{2}H_1(e^{j\omega})\left(X(e^{j\omega})H_1(e^{j\omega}) + X(e^{j(\omega+\pi)})H_1(e^{j(\omega+\pi)})\right)\\
Y(e^{j\omega}) &= Y_0(e^{j\omega}) - Y_1(e^{j\omega})\\
&= \frac{1}{2}X(e^{j\omega})\left[H_0^2(e^{j\omega}) - H_1^2(e^{j\omega})\right]\\
&\quad + \frac{1}{2}X(e^{j(\omega+\pi)})\underbrace{\left[H_0(e^{j\omega})H_0(e^{j(\omega+\pi)}) - H_1(e^{j\omega})H_1(e^{j(\omega+\pi)})\right]}_{=0}
\end{aligned}$$

The aliasing terms always cancel. $Y(e^{j\omega})$ is proportional to $X(e^{j\omega})$ if $[H_0^2(e^{j\omega}) - H_1^2(e^{j\omega})]$ is a constant.